

# Nonlinear Model Updating for Dynamics of Riveted Joints

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## Abstract

Nonlinear modeling of joints is often difficult to do accurately because of various small-scale and nonlinear features, such as local geometry, preload and friction. These features cause a local change in stiffness and energy dissipation which result in changes in the effective natural frequency and damping of the whole structure. Past studies have modeled bolted joints by constraining the nodes near a joint to a single pair of nodes and then using a nonlinear Iwan Element to capture the energy dissipation and stiffness loss in the joints. In this paper, a similar approach is proposed for modeling riveted joints, which seeks to capture the vibrational response of a test structure including changes in the effective modal frequency and damping with vibration amplitude. A four parameter, nonlinear Iwan element is used to model each rivet joint in the test structure. The rest of the structure is modeled using a linear finite element model (FEM), which is reduced using interface spiders and the Hurty/Craig-Bampton method to speed up simulations. Various modeling practices, including how the geometry is approximated, element types and constraint types, are explored to find which options produce the best results. The Quasi-Static Modal Analysis (QSMA) approach is used to find the effective modal natural frequencies, damping ratios, and mode shapes as a function of vibration amplitude. Optimization is used to find the parameters of the Iwan elements that brings the nonlinear behavior of the first mode into agreement with measurements. The same Iwan parameters are then used to predict the nonlinearity in the other modes of the assembly.

**Keywords:** Nonlinear joints, Iwan Element, Rivet, Quasi-linearity

## 1 Introduction

Joints in structures serve a practical purpose by allowing large, complex structures to be built from smaller, simpler pieces and making it possible to disassemble components for maintenance or to replace a part. Joints also add friction and damping to the structure [1], reducing the vibration amplitudes and potentially avoiding catastrophic failure. Quite a bit of research has been performed seeking to understand how these interfaces [2, 3], and in particular, bolted joints in structures, influence the effective damping and natural frequencies of vibration modes in assembled structures. Many recent works have found that the nonlinearity that they introduce causes these effective properties to change with vibration amplitude or with the level of excitation applied [4, 5].

Several different strategies have been developed to model nonlinearities in bolted joints and many improvements have been proposed in recent years. One method of modeling joint nonlinearities is to develop a detailed Finite Element Model (FEM) of the structure including specific contact elements for the interface of the joint. Friction and contact at the interface can then be captured using node-to-node or node-to-surface contact [6], and there are several important subtleties regarding how these are enforced, for example using Lagrange or Penalty methods [6]. Other FEM techniques involve the use of “thin layer” elements or “zero thickness” elements that represent the behavior of the contact zone with finer detail than the rest of the model [7]. When using FEMs to model nonlinearities in jointed structures, the mesh must be dense relative to the size of the bolt to capture contact accurately [8]. Hence, this approach produces extremely large models that are generally too expensive to simulate dynamically for more than a few cycles of vibration.

Another common approach is a whole joint (or lumped parameter) approach, in which a simplified model is used to represent the entire area of the interface. Generally, the area of the joint is rigidly attached to a single point on the surface in order to reduce the number of degrees-of-freedom involved in the nonlinearity and hence the complexity of the model [9]. The joint is approximated using springs or beam elements, and occasionally linear dampers. If the nonlinear behavior is of interest then a Coulomb friction model is often used. The Jenkins model, which consists of a spring and a Coulomb friction slider, can capture

the elasto-plastic response of a joint [10], but these elements are always either elastic or slipping and hence they can't capture micro-slip. A parallel arrangement of Jenkins elements, called an Iwan element [11], provides the ability to capture micro-slip nonlinearity by allowing a certain fraction of the Jenkins elements to slip under a given load while the rest remain stuck and contribute to the stiffness of the joint [12]. These are two of many models that capture the hysteresis that is often exhibited by bolted joints. They also have certain computational advantages because they satisfy Masing's assumptions [13]. The Bouc-Wen model [14] and the LuGre model [15] are examples of other hysteretic models that can be used to model a whole joint; they have not proven capable of capturing the macro-slip that is often present in bolted structures [16], so they are not explored here.

This work explores the feasibility of modeling the riveted joints in a structure of interest using Segalman's four parameter Iwan element [12]. While the four parameter Iwan element has been successful at modeling joints in structures, the identification of the four parameters in the model tends to be quite difficult. Fortunately, the optimization used to find these parameters can be performed very efficiently using a procedure called quasi-static modal analysis (QSMA) [17, 18, 19]<sup>1</sup>, as elaborated in Section 2. They showed that it could be applied to a reduced model with Iwan elements in place of the bolted joints, allowing optimization to be used to very efficiently adjust the parameters of four parameter Iwan elements until the model captured the measured effective natural frequency and damping of the structure.

Singh et al. [21] developed the modeling procedure that is employed here. In [22] Singh et al. introduced a simple benchmark structure with two bolted joints, called the S4 beam. The S4 beam was tested at various amplitudes of excitation and yielded significant nonlinearities in natural frequency and damping of different modes of the structure. Building on the work of Lacayo et al. [19], Singh et al. [21] modeled the S4 beam and matched the model's amplitude-dependent frequency and damping to the previously measured experimental data for two specific modes of interest, with different Iwan parameters for the joint degrees of freedom that were active in each mode [22]. Specifically, Singh et al. [21] began with a full-order finite element model of the S4 beam. They then applied the Hurty/Craig-Bampton method to reduce the model and created multi-point constraints to simplify the joint interfaces. Linear springs were inserted into the reduced model at the joint locations to match the low amplitude experimental mode frequencies. A Monte Carlo optimization then found the parameters for the four parameter Iwan elements used in the model. In [21], the focus of the work was how different multi-point constraints in the FEM affect the model and the Iwan element stiffness parameter,  $K_T$ . One important finding was that some multi-point constraints yielded higher values of  $K_T$  during the linear optimization. These high spring stiffnesses made it impossible to find a set of Iwan parameters that were capable of matching the nonlinear response of the model to the experimental measurements.

This paper builds on that in [21], seeking to understand how Iwan elements perform when modeling rivets instead of bolts. Whereas bolts apply a large normal pressure due to the bolt pretension, rivets primarily apply a radial pressure to the rivet hole [23]. Using the previously developed QSMA methods, an Iwan element is used to model each rivet in a simple test structure, and then optimization is used to match the nonlinear frequency and damping behavior of the first mode to experimental measurements. One key question is: what type of spider and what spider area are optimal in the case of riveted joints? Optimization is performed for a large range of models to evaluate the model fidelity as the spider type and radius are varied. Because optimization is performed only for the first mode and all of the rivets are treated as being equal, one may question how well the identified model would transfer to other modes or loadings. To evaluate this, the model obtained here is exercised to see how well it captures the dynamics of the first six modes of the assembly.

The following section reviews the the modeling and model updating approach, as well as optimization approach that is used, reviewing improvements that were made relative to that of Lacayo et al. [19] and Singh et al. [21]. Section 3 presents the riveted structures that are studied, and the specific finite element modeling approach. The results of optimization of the linear properties of the structure are presented in Section 3.5, followed by the nonlinear properties in Section 4.

## 2 Theory

This section discusses the process that was used in this study to find a nonlinear model for the riveted joints. A FEM was first built in the Abaqus commercial finite element analysis (FEA) package. The FEM was reduced using the Hurty/Craig Bampton (HCB) method and the resulting mass and stiffness matrices were exported to Matlab for further analysis. Using Matlab's *fminsearch* function, the reduced model's linear natural frequencies were matched to test data by optimizing the stiffnesses of a set of linear springs, which were inserted in every degree of freedom of each joint in the reduced stiffness matrix. This first step is termed "linear optimization" in this work. The linear spring stiffnesses were then used as one of four Iwan parameters in a nonlinear model for the joints. The rest of the parameters were found using the optimization approach used by Lacayo [19] and Singh [21], which uses a Monte Carlo approach to iterate on the rest of the parameters in the Iwan model until the best agreement is obtained between the measured natural frequency and damping versus amplitude and that predicted by the model. This second step is called "nonlinear optimization."

<sup>1</sup>A very similar procedure appears to have been independently devised in [20].

The procedure used by Lacayo [19] and Singh [21] is summarized in Figure 1 in blue. In those works, the process was sequential, starting with FEA modeling and proceeding to the linear optimization and then to the nonlinear optimization. Their approach presumed that linear optimization could be used to find the stiffness of the joint, and that this stiffness could then be used for the nonlinear model of the joint. Our recent works have found this to be problematic. In particular, the results of the nonlinear optimization have been found to be very sensitive to the value of the joint stiffness  $K_T$ , and the value that best reproduces the linear modal parameters often does not work to capture the nonlinearity. Hence, in this work the Iwan parameter  $K_T$  is treated as a parameter and given a range of possible values. Optimization is performed for each possible value of  $K_T$  and the user can then select the value that balances the errors in the linear and nonlinear optimization. This approach is basically that used in multi-objective optimization [24]. These adjustments are marked in orange in Figure 1. A more detailed discussion of each step in the procedure can be found in the following sections.

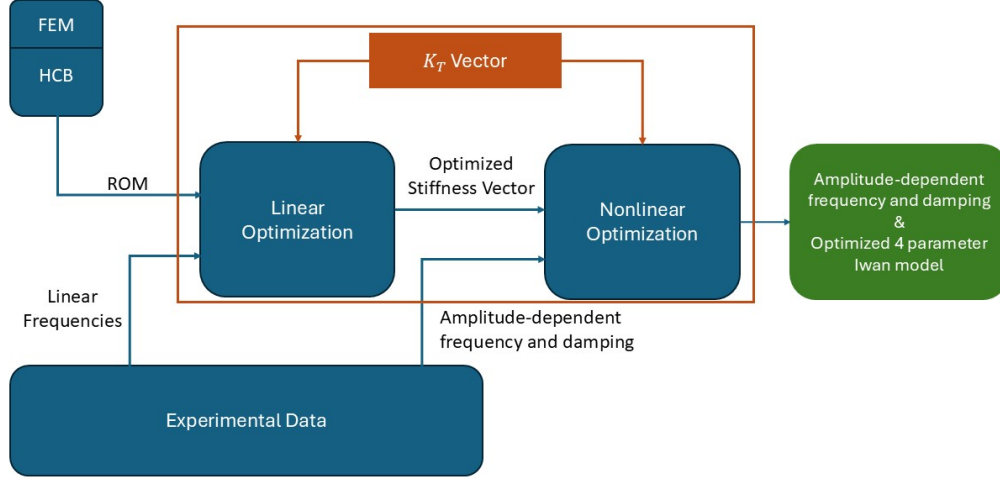


Figure 1: Procedure used in this work for finding an optimized Iwan element to model joint nonlinearity. Previous works [21, 19] followed the steps in blue. This work pursues a multi-objective optimization using the value of  $K_T$ , shown in orange, as a parameter. The output of the optimization is shown in green.

## 2.1 Hurty/Craig Bampton Reduction

The Hurty/Craig Bampton method reduces a large, complicated model to a smaller model that is quicker and more efficient to simulate. A discussion of the general principles of HCB reduction [25, 26] will follow: Consider a full model whose undamped equation of motion is given by Equation (1), where  $\mathbf{M}$  is the full order mass matrix,  $\mathbf{K}$  is the full order stiffness matrix,  $\mathbf{F}$  is a vector of applied forces,  $\mathbf{F}_J(\mathbf{u}, \mathbf{s})$  is the nonlinear force in the system, which is hysteretic and so it depends on the state  $\mathbf{u}$  and on some hidden variables  $\mathbf{s}$ . The vectors  $\mathbf{u}$  and  $\ddot{\mathbf{u}}$  are the nodal displacement and acceleration, respectively.

$$\mathbf{M}\ddot{\mathbf{u}} + \mathbf{K}\mathbf{u} + \mathbf{F}_J(\mathbf{u}, \mathbf{s}) = \mathbf{F} \quad (1)$$

The mass and stiffness matrices are partitioned into boundary and internal degrees of freedom (DOFs) as follows:

$$\begin{bmatrix} \mathbf{M}_{ii} & \mathbf{M}_{ib} \\ \mathbf{M}_{bi} & \mathbf{M}_{bb} \end{bmatrix} \begin{Bmatrix} \ddot{\mathbf{u}}_i \\ \ddot{\mathbf{u}}_b \end{Bmatrix} + \begin{bmatrix} \mathbf{K}_{ii} & \mathbf{K}_{ib} \\ \mathbf{K}_{bi} & \mathbf{K}_{bb} \end{bmatrix} \begin{Bmatrix} \mathbf{u}_i \\ \mathbf{u}_b \end{Bmatrix} + \begin{Bmatrix} 0 \\ \mathbf{F}_{J,b}(\mathbf{u}_b, \mathbf{s}) \end{Bmatrix} = \begin{Bmatrix} 0 \\ \mathbf{F}_b \end{Bmatrix} \quad (2)$$

where the subscript  $i$  denotes the internal DOF and  $b$  denotes a boundary DOF. It is assumed that the nonlinear joints exert forces only on the boundary DOF and that any applied forces  $\mathbf{F}_b$  are applied only to boundary nodes. The interfaces of the structure are described by a set of static constraint modes,  $\psi$ , which are the responses of the structure with each interface DOF displaced one unit. The interior of the structure is defined by a set of fixed interface modes (FIM),  $\phi$ , which are the vibrational modes with every interface DOF fixed, as defined by the following eigenvalue problem.

$$(\mathbf{K}_{ii} - \omega_r^2 \mathbf{M}_{ii})\{\phi_i\}_r = 0 \quad (3)$$

$$\phi = \begin{bmatrix} \{\phi_i\}_1, \dots, \{\phi_i\}_m \\ 0 \end{bmatrix} = \begin{bmatrix} \phi_i \\ 0 \end{bmatrix}$$

Thus a small number of FIMs and constraint modes can accurately and efficiently describe the structure. The combination of the FIMs and constraint modes can be used to form the HCB transformation matrix in Equation (4).

$$\begin{Bmatrix} \mathbf{u}_i \\ \mathbf{u}_b \end{Bmatrix} = \mathbf{T}^{\text{HCB}} \begin{Bmatrix} \mathbf{q}_i \\ \mathbf{u}_b \end{Bmatrix} = \begin{bmatrix} \Phi & \Psi \\ \mathbf{0} & \mathbf{I} \end{bmatrix} \begin{Bmatrix} \mathbf{q}_i \\ \mathbf{u}_b \end{Bmatrix} \quad (4)$$

In this work, the HCB reduction was performed using the *Substructure Generate* step in Abaqus FEA. The code applies the HCB transformation to the FEM, resulting in reduced mass and stiffness matrices  $\hat{\mathbf{M}}$  and  $\hat{\mathbf{K}}$ ,

$$\hat{\mathbf{M}} = (\mathbf{T}^{\text{HCB}})^T \mathbf{M} \mathbf{T}^{\text{HCB}}, \quad \hat{\mathbf{K}} = (\mathbf{T}^{\text{HCB}})^T \mathbf{K} \mathbf{T}^{\text{HCB}}. \quad (5)$$

The user must first specify the set of boundary nodes, which are retained. The set of boundary nodes was reduced by using Multi-Point Constraints (MPCs) to connect all nodes within the joint to one reference (or virtual) node, and these were the only nodes retained in the boundary set. See Figure 2 for an example of how these constraints connect a region of nodes to a reference node.

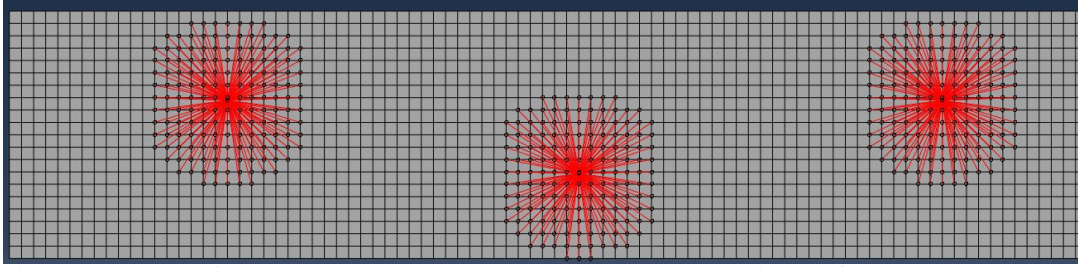


Figure 2: Finite element mesh for model with no holes, showing MPCs applied for the first three rivets. Red lines are the constraints connecting each node within a defined radius to the reference node.

The FEMs used in this work consisted of two beams, each with 24 rivets, with six DOF per rivet. Thirty FIMs were retained in the HCB model, so each FEM was reduced to 318 DOF.

## 2.2 Linear Response Optimization

Once the FEM of the structure was built and the mass and stiffness matrices were reduced, the next step was to insert linear springs in place of the joints and then to adjust their stiffnesses to match the linear mode frequencies of the model to the experimental mode frequencies. This was done by adding the corresponding spring constant into the stiffness matrix at the indices of the degrees of freedom of the joint. Equation (6) shows how the spring stiffness can be added into the reduced stiffness matrices; this was implemented in a Matlab script.

$$\hat{\mathbf{K}}(\mathbf{n}, \mathbf{n}) = \hat{\mathbf{K}}(\mathbf{n}, \mathbf{n}) + K_s \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \quad (6)$$

where  $\mathbf{K}$  is the reduced stiffness matrix,  $\mathbf{n}$  is a length 2 vector representing the indices in the stiffness matrix of the two joints being connected, and  $K_s$  is the spring stiffness that is inserted between that pair of degrees of freedom. This was repeated for every pair of nodes that were joined by rivets, and in all six degrees of freedom. There are 24 rivets in the model, and each node representing a rivet location had 6 degrees of freedom, so there were a total of  $6 \times 24$  springs inserted. The stiffnesses in the  $X$  and  $Y$  directions were assumed to be equal because the rivets are round and have the same geometry in both directions. The rotational stiffnesses in  $X$  and  $Y$  were also assumed to be equal, so there were a total of 4 unknown stiffness parameters. These were collected in a vector  $\boldsymbol{\theta} = [K_x = K_y, K_z, R_x = R_y, R_z]^T$ .

Matlab's *fminsearch* function was used to optimize towards the minimum of the defined function,  $f(\boldsymbol{\theta})$ , in Equation (7). This optimization finds the spring stiffnesses that minimize the root mean square error between experimental and model mode frequencies.

$$f(\boldsymbol{\theta}) = \sum_{i=1}^n \left( \frac{\omega_{\text{model},i}(\boldsymbol{\theta}) - \omega_{\text{exp},i}(\boldsymbol{\theta})}{\omega_{\text{exp},i}(\boldsymbol{\theta})} \right)^2 \quad (7)$$

where  $n$  is the number of vibration modes considered in the optimization.

The procedure above mimics that used by Lacayo [19] and Singh [21], but this work ended up deviating from that procedure. It was found that the  $K_x$  stiffness parameter had a significant effect on the mode frequencies of the model and that the standard

*fminsearch* optimization would force the  $K_x$  parameter to extremely high values, with little improvement in the accuracy of the mode frequencies. As a result, the optimization was modified as described below, adopting a multi-objective approach.

Rather than include  $K_x$  in the set of parameters to be found through optimization, it was instead treated as a parameter. A vector of possible  $K_x$  values was specified, spanning about three orders of magnitude. For each value of  $K_x$ , the optimization problem mentioned above was solved with  $\hat{\theta} = [K_z, R_x = R_y, R_z]^T$ . The resulting spring constants  $\hat{\theta}$  were stored, along with the natural frequencies of the optimal model. The nonlinear optimization, discussed in the next section, was also repeated for various values of  $K_x$ , which becomes the tangential stiffness parameters  $K_T$  in the Iwan element, and the accuracy of both was considered simultaneously, as discussed later and shown in Figure 7. Further discussion of this issue can be found in Section 3.5.

## 2.3 Nonlinear Response Optimization

The nonlinear model is created by replacing certain springs joining the two beams with four-parameter Iwan elements [12]. For a structure such as this, one would expect slip to occur primarily in the  $x$ - and  $y$ -directions, although the latter was found to have no effect on any of the modes of interest, so in the end only the springs  $K_x$  were replaced with Iwan elements.

An Iwan element is a single model that contains a distribution of Jenkins [27] elements in parallel, each of which contains a spring connected to a Coulomb friction slider. The preload in each slider is different, so under load only some fraction of the Jenkins elements slip; this is how the Iwan element captures micro-slip. Segalman's Iwan element is governed by four parameters, the total stiffness of the joint for small loads ( $K_T = K_x$  in this study), the force  $F_s$  at which the joint slips completely, and two non-dimensional parameters  $\chi$  and  $\beta$ . When the joint is in microslip, the log damping ratio versus log vibration amplitude is a line with slope  $1 + \chi$ . Let  $\mathbf{p}_k = [K_T, F_s, \chi, \beta]^T$  denote the parameters of the  $k$ th Iwan element.

As explained previously, spiders were used to reduce each rivet to two interface nodes, and the Hurty/Craig-Bampton method was used to create a model defined by a certain number of fixed-interface modes  $\mathbf{q}_i$  and spider nodes  $\mathbf{u}_b$ , so the state vector  $\mathbf{u} = [\mathbf{q}_i^T, \mathbf{u}_b^T]^T$ . The equation of motion for the reduced model is then,

$$\hat{\mathbf{M}}\ddot{\mathbf{u}} + \hat{\mathbf{K}}\mathbf{u} + \sum_{k=1}^{N_J} \mathbf{F}_{J,k}(\mathbf{u}, \mathbf{p}_k) = \mathbf{F}_{\text{ext}} \quad (8)$$

where  $N_J$  is the number of joints,  $\hat{\mathbf{F}}_{\text{ext}}$  is a vector of applied forces and  $\mathbf{F}_{J,k}(\mathbf{u}, \mathbf{p}_k)$  defines the force exerted by the  $k$ th Iwan element. It is a vector of zeros except for two entries that contain the equal and opposite force in the element. The dynamic response of the system can be found by integrating the equation of motion using the Newmark method, as explained in [3, 28].

### 2.3.1 QSMA

The idea behind Quasi-Static Modal Analysis (QSMA) was first presented by Festjens et al. [17] as an alternative to dynamic time integration of a FEM. In QSMA, a distributed force is applied at increasing amplitudes to excite a particular mode in question. The resulting force-displacement curve is then used to find the effective natural frequency and damping of the structure as it vibrates in the specific mode of interest over a range of vibration amplitude. Specifically, Masing's rules [29] are used to infer the response over one loading and unloading cycle from the quasi-static force-displacement behavior. In effect, the static response is used to infer the unforced response of the structure as the vibration decays in the mode in question. This process is much faster than a dynamic simulation of a FEM, yet it only reveals the response of the structure for the special case in which it vibrates only in the mode in question.

In [17], Festjens et al. split their model into a linear portion and a nonlinear portion and iterated on each until both solutions agreed at the interface. Lacayo and Allen [30] adjusted their approach by simply treating the whole model as the nonlinear portion, thus avoiding the need to iterate, and dubbed the resulting approach QSMA. This potentially increases the computational cost, so Lacayo and Allen used Hurty/Craig-Bampton reduction and modeled the joints as four parameter Iwan elements [12]; In contrast, Festjens et al. modeled the preload and contact in detail using Coulomb friction.

The process used by Lacayo and Allen [30] is the same process used in this work and will be summarized here. The assumption is made that the structure is vibrating at the  $r$ th mode such that  $\mathbf{u}(t) = \varphi_r \sin(\omega_r t)$ , where  $\varphi_r$  is the mode shape of the  $r$ th mode. The inertial term in Equation (8) is now proportional to  $\mathbf{M}\varphi_r$ . The structure is allowed to vibrate freely such that  $\mathbf{F} = 0$ , and so the inertial term can be moved to the right hand side and treated as an applied distributed load that only excites one mode in the linear structure. This yields Equation (9).

$$\mathbf{K}\mathbf{u} + \mathbf{F}_J(\mathbf{u}, \theta) = -\alpha \mathbf{M}\varphi_r \quad (9)$$

which is a nonlinear quasi-static problem that can be solved for the displacement  $\mathbf{u}$  in terms of a monotonically increasing amplitude,  $\alpha$ .

After solving for the quasi-static response,  $\mathbf{u}(\alpha)$ , the amplitude-dependent modal frequency and damping ratio can be found using the same approach as Festjens et al. which comes from assumptions made using Masing's rules and the force-deflection hysteresis curve. In brief summary, the amplitude dependent stiffness is related to the natural frequency which comes from the slope of the secant line in the hysteresis plot, as given in Equation (10).

$$\omega_r(\alpha_j) \triangleq \sqrt{\frac{\alpha_j}{q_r(\alpha_j)}} \quad (10)$$

where  $\omega_r(\alpha_j)$  is the natural frequency of the mode and  $q_r(\alpha_j)$  is the modal displacement, which are both dependent on the amplitude of excitation,  $\alpha_j$ . The amplitude dependent damping in the mode in question can be found using Equation (11).

$$\zeta_r(\alpha_j) \triangleq \frac{D(\alpha_j)}{2\pi\alpha_j q_r(\alpha_j)} \quad (11)$$

where  $D(\alpha_j)$  is the total energy dissipated per cycle of vibration or the area enclosed by the hysteresis curve after applying Masing's rules.

### 2.3.2 Monte Carlo Optimization

Optimization is used to find the four Iwan parameters that cause the natural frequency and damping versus amplitude of the reduced order model to best agree with measurements. The optimization has been found to be prone to getting stuck in local minima, so a Monte Carlo optimization strategy is used. Bounds are defined around each of the parameters, and then  $N_{MC}$  random samples are drawn from a uniform distribution defined by those bounds. QSMA is used to evaluate the weighted average error between the frequency and damping curves predicted by the model and the measured ones and the set of parameters that gives the best results is retained. The bounds are then contracted around the set of parameters that gave the best results and the optimization is repeated. The process ends when the optimization either reaches a prescribed accuracy or ceases to improve.

In the studies in this paper, we have typically used  $N_{MC} = 100$  and after each set of iterations defined a new range for each parameter that is  $\pm 3\%$  of the optimal value found in the previous step. The power-law slope  $\chi$  has a clear effect on the damping curve so it typically only varies between  $-1 < \chi < 1$ . In contrast, the other two parameters  $F_S$  and  $\beta$  may need to be varied by several orders of magnitude to find the optimal value.

The approach above has worked well in some prior studies [21], yet it loses effectiveness if the parameter ranges are very large. When certain parameters are varied over a large range the response sometimes changes very little, but if the parameter is near the optimum it may be very sensitive to small changes. In the course of this work the authors have found it helpful to manually explore the search space using very wide parameter ranges before starting the optimization. This helps to ensure that the Monte Carlo algorithm will search in a space where the changes to the parameters can improve the response of the model.

## 3 Case Study: Thin Riveted Beams

The test structure consists of two long and slender beams joined together by 24 steel rivets set in an offset pattern along the length of the beams. This test structure and its model are referred to as the riveted beams. Various iterations of the model were created in Abaqus to understand the effect of different modeling choices that can be made, such as whether to model the beams with shell or solid elements, what type of multi-point constraints to use, and other settings specific to the constraints and HCB substructuring procedure. This section describes the different factors that were investigated and their respective effects on the accuracy of the model compared to experimental data. All experimental data used in this work was collected as reported in [31].

### 3.1 Material and Geometric Properties

The riveted beams are made of Inconel® nickel-chromium alloy 625 which has properties defined in [32]. A single beam without rivets or holes of the same material was also tested, as shown in Figure 3, and is referred to as the single beam. This was used to verify the material properties and geometric dimensions of the beam.

### 3.1.1 Sensitivity Analysis

A sensitivity analysis was performed to quantify the effect of each material and geometric property on the modes of the beam. For the sensitivity analysis, a shell model was used with linear, reduced integration elements with the refined mesh that is discussed later.

A 1% change in beam width resulted in a corresponding 1% shift in the natural frequencies of both stiff bending and torsion modes. The frequencies of all other bending modes remained virtually unaffected. Increasing beam thickness by 1% lead to a 1% increase in natural frequencies for all modes except stiff bending, which stayed constant. A 10% alteration in the beam's modulus induced a 4.6% change in the natural frequencies of all modes. Finally, a 10% change in Poisson's ratio produced a 1% shift in torsion mode frequencies while leaving other modes largely unchanged.

### 3.1.2 Model Updating for Single Beam

Two corresponding FEMs of the single beam were created, one with shell elements and one with solid elements. The geometric dimensions, defined in Figure 4, and material properties of these models were modified to align their natural frequencies, obtained by solving the eigenvalue problem on the full FEM in Abaqus, to the experimental frequencies. Normalized nominal values for the material properties and normalized geometric dimensions of the single beam are given in Table 1. The nominal dimensions were also checked by measuring the dimensions of the beam using calipers and a measuring tape; the measured dimensions are displayed in Table 1. The properties obtained after updating the single beam model to correlate with the measured natural frequencies are also shown. The main changes were increasing the Poisson ratio by 4% and the width by 2%. The density of the beam was left at the nominal value, which was verified by weighing the beam.

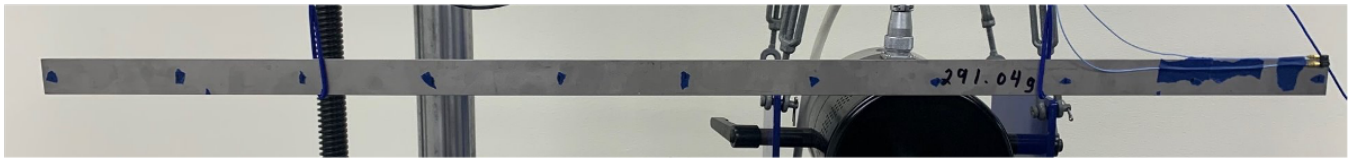


Figure 3: A modal test was performed on a single beam to obtain the linear natural frequencies of the modes, which were used to determine the material properties and to update the geometric dimensions.

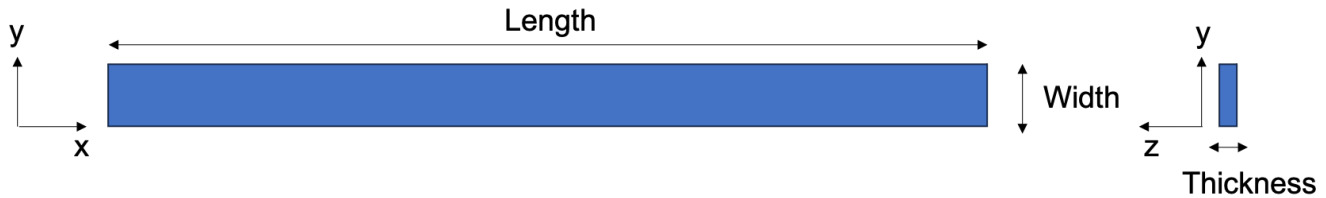


Figure 4: Dimensions of the single beam.

Table 1: Normalized nominal and measured dimensions of the single beam along with the normalized properties used in the final model.

| Property              | Nominal | Measured | Final Model Property |
|-----------------------|---------|----------|----------------------|
| Length                | 1.00    | 1.00     | 1.00                 |
| Width                 | 0.0281  | 0.0287   | 0.0286               |
| Thickness             | 0.0017  | 0.0016   | 0.0016               |
| Modulus of Elasticity | 1.00    | -        | 1.00                 |
| Poisson's Ratio       | 0.0105  | -        | 0.0109               |

The natural frequencies of the beam are compared to those of the models in Table 2. The solid model mesh had nine elements through the thickness of the beam, and the results suggest that this isn't quite enough for the soft direction bending modes to converge. Quadratic elements were also used to test this hypothesis, and the mesh with quadratic elements was found to reproduce the measured natural frequencies more accurately. The shell element models were less sensitive to whether linear or quadratic elements were used; Table 2 shows that the natural frequencies are almost indistinguishable for those models.

Table 2: Frequency error between single beam finite element model and experimental data acquired as shown in Fig. 3. Negative error means that the model natural frequency is less than the experimental frequency.

| Mode              | Lin Shell Error (%) | Quad Shell Error (%) | Lin Solid Error (%) | Quad Solid Error (%) |
|-------------------|---------------------|----------------------|---------------------|----------------------|
| 1st Bending       | 0.02                | 0.02                 | -1.98               | 0.02                 |
| 2nd Bending       | -0.31               | -0.31                | -2.30               | -0.31                |
| 3rd Bending       | 0.08                | 0.08                 | -1.92               | 0.08                 |
| 4th Bending       | 0.18                | 0.18                 | -1.82               | 0.18                 |
| 5th Bending       | -0.13               | -0.14                | -2.13               | -0.14                |
| 1st Stiff Bending | 0.14                | 0.26                 | 0.13                | 0.26                 |
| 6th Bending       | -0.03               | -0.05                | -2.02               | -0.05                |
| 1st Torsion       | 0.63                | 0.50                 | 2.87                | 0.50                 |
| 7th Bending       | 0.11                | 0.10                 | -1.88               | 0.10                 |
| 8th Bending       | 0.12                | 0.10                 | -1.87               | 0.10                 |
| 9th Bending       | -0.08               | -0.10                | -2.06               | -0.10                |
| 2nd Torsion       | -0.06               | -0.19                | 2.16                | -0.18                |
| 2nd Stiff Bending | 1.16                | 1.27                 | 1.15                | 1.28                 |
| 10th Bending      | -0.22               | -0.24                | -2.19               | -0.24                |

### 3.2 Modeling the Riveted Assembly

While several FEMs were created in debugging the modeling process, the results of only three of those are reported here and they are referred to as: Shell with Holes, Shell without Holes, and Solid without Holes. These are shown in Figure 5. Many parameters exist within these models that can be used to tune the model. In this section, the three models are compared to each other with the same dimensions, material properties, element types and mesh densities.

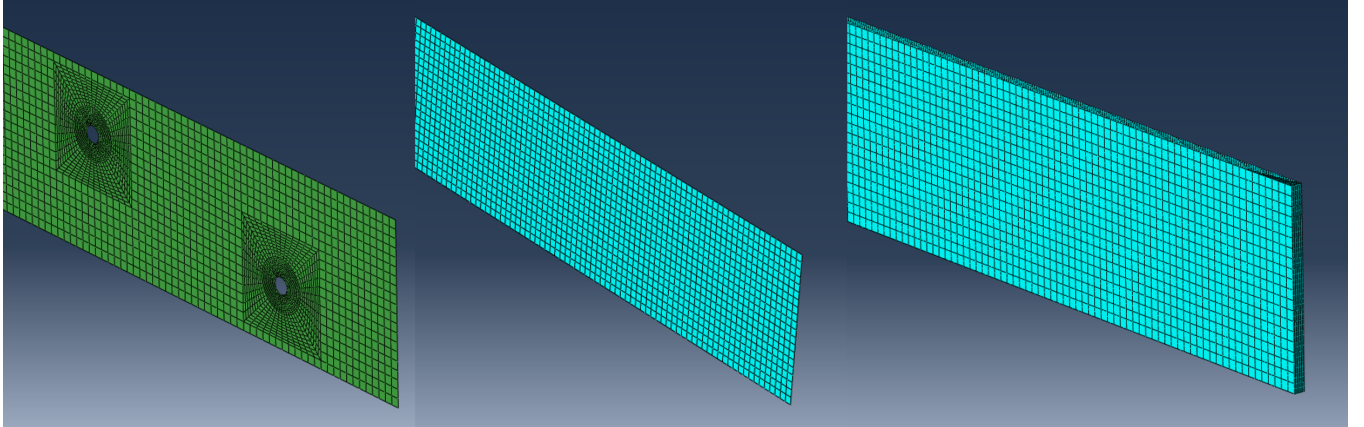


Figure 5: Riveted beam models: The left model used shell elements and includes the rivet holes, the center model used shell elements but ignores the holes, and the right model used solid elements neglects the holes.

Each model consists of two copies of the structure shown in Figure 5, with spiders at the 24 rivet locations and springs joining the corresponding spiders. The linear optimization outlined in Sec. 2.2 was performed on each model to find the spring constants that brought the model into closest agreement with the measured linear natural frequencies of the riveted beam assembly. In order to better compare the models, one model was optimized and the resulting spring stiffness values were inserted into the other two models. The frequency error was then calculated. Note that the spring constants reported in this work are all **normalized** by the  $K_x$  spring constant that was judged to be optimal in the end. All three models used linear, reduced integration elements and the same spider radius and type.

Table 3 shows the frequency error for all models when the Shell without Holes model was optimized, using RBE3 type Multi-Point Constraints (MPC). The effects of different constraint types were investigated and are compared in Section 3.3.2, however the trends presented in this section regarding model design were consistent no matter which type of constraint elements were used. Additionally, the results were similar no matter which model was optimized.

In this and several other cases studied, the model that included the holes had larger errors than the other models, while also having 2.5 times as many degrees of freedom. Considering this, and the high accuracy of the other models, it was judged to be easier and more efficient to ignore the holes so only the “no holes” models are used going forward. All models had single-digit errors in the bending modes and double-digit errors in the torsion mode. Additionally, the fact that the solid and shell models

gave similar natural frequencies suggests that the error in the torsion mode was not due to the shell elements nor a problem with the offsets in the shell model. Hence, that model, which had one sixth as many DOF as the solid model, proved to be the most efficient.

While investigating the effect that changes in the stiffness values have on the model, it was found that these stiffness values place all models in a regime where the stiffness can be changed by a few orders of magnitude without changing the natural frequencies noticeably. This is further discussed in Section 3.5.

If it were not for the high error in the torsion mode, one could end the investigation here, as the frequency errors in the bending modes for the Shell without Holes and Solid without Holes models are within typical aerospace standards. However, later efforts in the nonlinear optimization phase of the work found that it was impossible to accurately capture the nonlinearity in the riveted beams with large spring constants. Hence, the models were further investigated with two goals: to decrease  $K_x = K_y$  values and to improve the accuracy of the torsion mode.

Table 3: Normalized spring constants and frequency errors for all models using the optimized stiffnesses for the Shell without Holes model. All three models had RBE3 type MPCs with a radius about 2 times greater than the radius of the rivets.

| Mode        | Shell Holes Error (%) | Shell w/out Holes Error (%) | Solid w/out Holes Error (%) | DOF   | Stiffness |
|-------------|-----------------------|-----------------------------|-----------------------------|-------|-----------|
| 1st Bending | -0.61                 | 0.00                        | -0.50                       | $K_x$ | 20695     |
| 2nd Bending | -1.39                 | -0.28                       | -0.79                       | $K_y$ | 20695     |
| 3rd Bending | -2.03                 | -0.21                       | -0.76                       | $K_z$ | 531.9     |
| 4th Bending | -2.76                 | -0.40                       | -0.97                       | $R_x$ | 9863      |
| 5th Bending | -4.37                 | -0.77                       | -1.39                       | $R_y$ | 9863      |
| 1st Torsion | -23.05                | -15.72                      | -13.42                      | $R_z$ | 737.3     |

### 3.3 Spidering

Spidering involves constraining a group of nodes to a single reference node using MPCs. Two MPC types, rigid bar elements, called RBE2 in Nastran or RBAR in [21], and averaging-type or RBE3 in Nastran, were investigated. RBAR MPCs impose rigid body motion on the constrained region relative to the reference node, while RBE3 MPCs force the slave node to match the average displacement and rotation of the surface but do not introduce additional stiffness [6]. The region of nodes to be constrained is defined by a specified radius centered on the reference node, as illustrated in Figure 2. Both the type of MPC and size of the radius used were found to have a large effect on the result of the model. They are discussed in the following sections.

#### 3.3.1 Spider Radius

The Shell without Holes model with RBE3 and RBAR MPCs was used to analyze the effects of changing the spider radius, and the results are shown in Figure 6. For each MPC, the set of optimized stiffnesses presented in Section 3.5 were used to join the beams. Additionally, all radius data in Figure 6 has been normalized to the radius of the rivet hole in the model.

As seen in Figure 6a, increasing the spider radius for the RBE3 models decreased the error in all modes. The first five bending modes have relatively low error, although the torsion mode has more than 15% error. This large torsion mode error is of particular interest because the torsion mode has proven difficult to model using the techniques discussed here. It may seem that the spider radius could be increased until all frequency errors approached zero. Unfortunately, in this model there is a limit to how large the spider radius can become before it exceeds the size of the beam. Hence, the optimal radius for the RBE3 model would be as large as it can be before exceeding the width of the beam, which in this case is a radius of about 4.5 times the radius of the rivet hole.

The RBAR models follow an opposite pattern where the frequency error tends to increase as the spider radius increases. Thus, an optimal radius for the RBAR constraint was one that was as small as possible. However, this smaller radius model had a greater spread in the error of the first five bending modes when compared to the optimal RBE3 model. Another interesting finding was that the RBAR model with the smallest radius had the least error in frequency in the 4th bending mode, whereas the RBE3 models always had the least error in the 1st bending mode. Similar to the RBE3 models, the RBAR model struggles to accurately model the torsion mode of the riveted beams. It appears that the torsion mode could be modeled accurately with a RBAR radius of around 3.25 times the radius of the rivet holes, however, this introduces a significant amount of error in each of the bending modes.

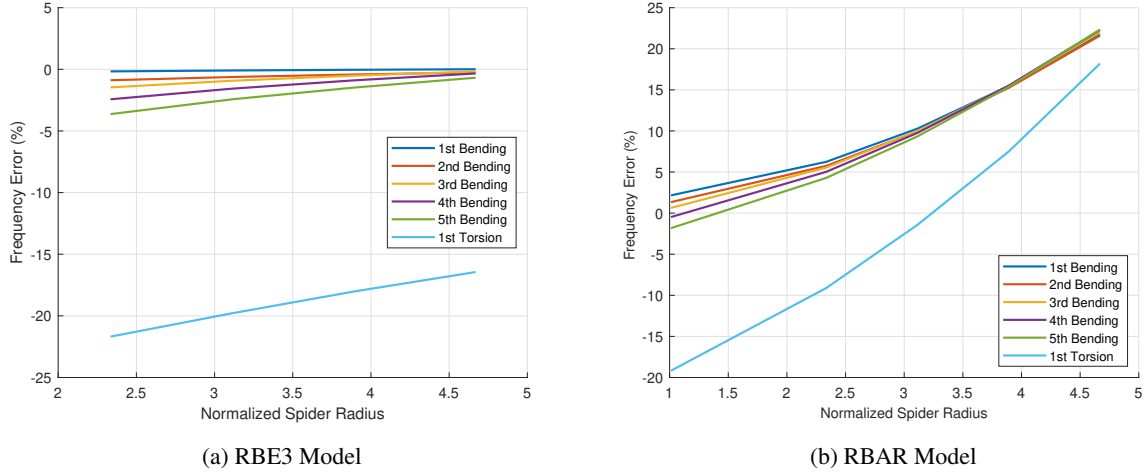


Figure 6: Mode frequency errors for various radius values of riveted beam models using RBAR and RBE3 MPCs

This reveals one strategy that could be used to bring the torsion mode into agreement with measurements. RBAR spiders could be used with a large radius (i.e 3.5). This would produce errors in each of the bending mode frequencies of about 12%, with 4% error in the torsion mode. Then, according to the sensitivity analysis in Section 3.1.1, the young's modulus could be reduced by  $10 * (12/4.6) = 26\%$  to bring the bending modes to nearly zero error. Unfortunately, this would decrease the torsion mode natural frequency, resulting in about -8% error. An 80% shift in the Poisson Ratio would then be needed to bring the torsion mode error back to near zero. All of these changes in the material properties were judged to be too large to be reasonable, and so the material properties were kept at the values found in Section 3.1.2.

### 3.3.2 Spider Type and Spring Stiffness

As explained in Sec. 2, rather than optimizing sequentially as done in [19, 21] this work treated the  $K_x$  spring stiffness as a parameter in the linear and nonlinear optimizations. The prior sections established that the Shell without Holes model provided the best accuracy/efficiency and revealed the optimal spider radius to use depending on whether RBE3 or RBAR spiders were used. Hence, the natural frequencies of the Shell without Holes model were found for both RBE3 and RBAR models, each with the optimal spider radius, over a range of  $K_x$  values. The results are summarized in Figure 7, which shows the RMS error for the first 5 bending modes and the error in the 1st torsion mode for each model.

The  $K_x$  stiffness values used are normalized relative to the optimal value selected in Section 3.5 and span a range of a few orders of magnitude. The error in the bending modes levels out near a normalized stiffness of 1.0, suggesting that this value balances the two objectives of keeping  $K_T$  small while minimizing the errors in the natural frequencies. As expected, both models accurately captured the bending modes and were unable to model the torsion mode with any  $K_x$  stiffness.

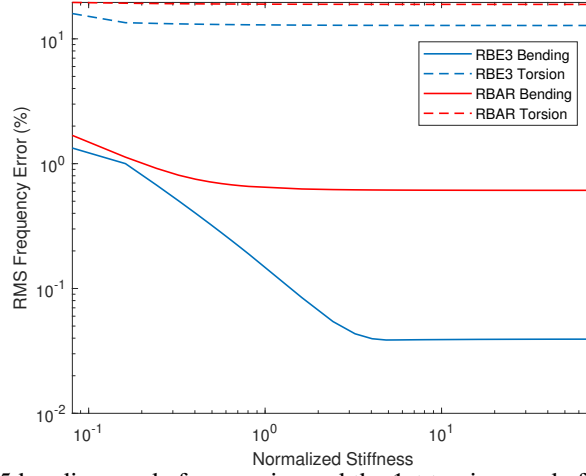


Figure 7: RMS Error of the first 5 bending mode frequencies and the 1st torsion mode frequency for RBAR and RBE3 modes with increasing  $K_x$  stiffness. The RBE3 model has a spider radius of about 4.7 times the radius of the rivet hole. The RBAR model has a spider radius about the same size as the rivet hole.

### 3.4 Torsion Mode

Evident in the results above, the torsion mode has proven difficult to capture accurately with these reduced models. Interestingly, the results given in Section 3.1 show that models for the single beam were able to accurately model not just the bending modes but all modes. In those results the frequency errors of the torsion modes were higher than those of the other modes, but they were still within an acceptable range. This suggests that the difficulty in modeling the torsion mode accurately in the riveted beams is not in the individual beam models themselves but rather in the way they are connected to each other and possibly in the approximations made in the spidering process. Hence, much of the parameter exploration presented above was focused on the way the two beams were connected.

The type of MPC used in the model seems to have a large effect on the accuracy of the torsion mode. Models with the RBAR MPCs were capable of minimizing error in the torsion mode but it came at the expense of accuracy in the bending modes. Models with RBE3 MPCs were incapable of minimizing torsion mode error at all. Interestingly, the torsion mode could be underestimated or overestimated in the RBAR models unlike the RBE3 models which always underestimated the natural frequency of the torsion mode.

### 3.5 Optimization Results for Linear Modal Properties

Although the torsion mode was never able to be accurately modeled, the deformations in the structures of interest are more like the bending motions of these beams. Therefore, the most accurate model at capturing the bending modes was used for the remainder of the work. This model was the Shell without Holes design using RBE3 type MPCs with as large a radius as possible to ensure minimum frequency error in all bending modes and minimum  $K_x$  stiffness for reasons that will be illustrated in Sec. 4.1. Table 4 shows the normalized frequencies and optimized stiffnesses. This value for  $K_x$  balances the two objectives of minimizing both the linear mode frequency error while being suitable for the nonlinear optimization discussed in Sec. 4.1. As noted previously, the optimization forces  $K_x = K_y$  and  $R_x = R_y$ .

Table 4: Normalized mode frequency error for the first 6 bending modes and normalized spring stiffness for each degree of freedom used for all 24 rivets. Shell without Holes model using RBE3 type MPCs with radius of 4.7 times the rivet radius.

| Mode Type | Experimental Freq. (Hz) | Model Freq. (Hz) | Error (%) | DOF   | Stiffness |
|-----------|-------------------------|------------------|-----------|-------|-----------|
| Bending   | 1.0000                  | 1.0001           | 0.0078    | $K_x$ | 1         |
| Bending   | 2.7462                  | 2.7392           | -0.25     | $K_y$ | 1         |
| Bending   | 5.3266                  | 5.3176           | -0.17     | $K_z$ | 0.0264    |
| Bending   | 8.7025                  | 8.6746           | -0.33     | $R_x$ | 0.4468    |
| Bending   | 12.8347                 | 12.7502          | -0.66     | $R_y$ | 0.4468    |
| Bending   | 17.5646                 | 17.4679          | -0.55     | $R_z$ | 0.0375    |

To understand whether these results represent a global optimum, the effect of changes in spring stiffnesses across all degrees of

freedom was investigated. This was done by sweeping one stiffness parameter across a large range of values above and below the optimal value, while holding the rest of the stiffness parameters constant at their optimized values. Figure 8 shows the results of this investigation. In this model, only  $K_x$  seemed to have a significant effect on the accuracy of the mode frequencies.  $R_z$  was also important, but only for a few modes. There is an interesting trend as  $K_x$  is increased, where one by one, each mode reaches a minimum error and then the error increases towards a bound as the stiffness continues to increase. This suggests a tradeoff in error between the modes themselves, where, for example, having a perfectly accurate mode 1 frequency will cause higher error in other modes.

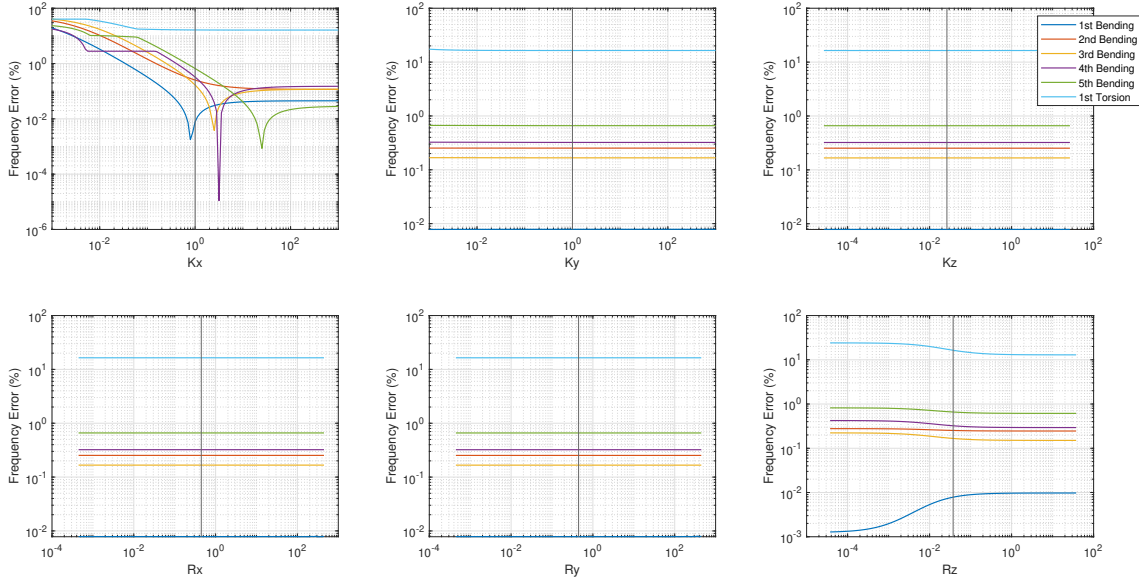


Figure 8: Plots of mode frequency error due to changes in one spring stiffness at a time. Spring stiffnesses are normalized to  $K_x$  and the vertical lines represent the optimized value for each stiffness parameter.

## 4 Optimization Results for Nonlinear Modal Properties

As explained previously, the four Iwan parameters ( $K_T$ ,  $F_s$ ,  $\chi$ ,  $\beta$ ) were equal for all 24 Iwan elements and were varied using a Monte Carlo optimization routine, with the goal of maximizing correlation between the measured natural frequency and damping versus amplitude and those of the model.

### 4.1 Results for the First Bending Mode

The Shell without Holes model with RBE3-type MPCs and a spider radius of about 4.7 times the rivet radius was used in the Monte Carlo optimization to find a set of Iwan parameters that yielded the best fit nonlinear frequency and damping versus amplitude. The experimental data that was used for the optimization came from a series of tests with a large range in amplitude, splicing the various tests together. This allowed for the optimized model to have a good fit for both the low-amplitude, linear response and the high-amplitude, nonlinear regime. This model's nonlinear frequency and damping versus amplitude can be seen in Figure 9. This model predicted that the frequency would change by less than 1 Hz and that the damping would increase by an order of magnitude over the range of amplitudes tested. The model had reasonable accuracy in matching the changes in frequency and damping, although the match to the curvature of these plots could be improved. It should be noted, however, that the experimental data had a significant amount of variability, which will be explored in the following section. For this reason, the agreement shown here was deemed to be sufficient.

Two of the same models were also optimized, but using a value for  $K_x = K_T$  that was an order of magnitude too large or small, compared to the optimal value. In our initial efforts, damping results such as these were frequently encountered, eventually leading us to the optimal value of  $K_T$ . The results shown here demonstrate the extreme effect that  $K_T$  has on the performance of the model, and why it was important to treat  $K_T$  as a parameter rather than to merely accept the outcome of the linear optimization. As shown in Figure 9, a  $K_T$  value that is smaller than optimal results in an increase in the damping of the Iwan

element and an effective natural frequency that decreases sharply relative to the measured behavior. A  $K_T$  value that is too large has the opposite effect, and the damping of this Iwan element gets closer to the material damping of the model. This range of optimal  $K_T$  values is surprisingly small. If the value of  $K_T$  is too small or too large, the optimization is unable to obtain reasonable agreement even when varying all of the other parameters of the Iwan element.

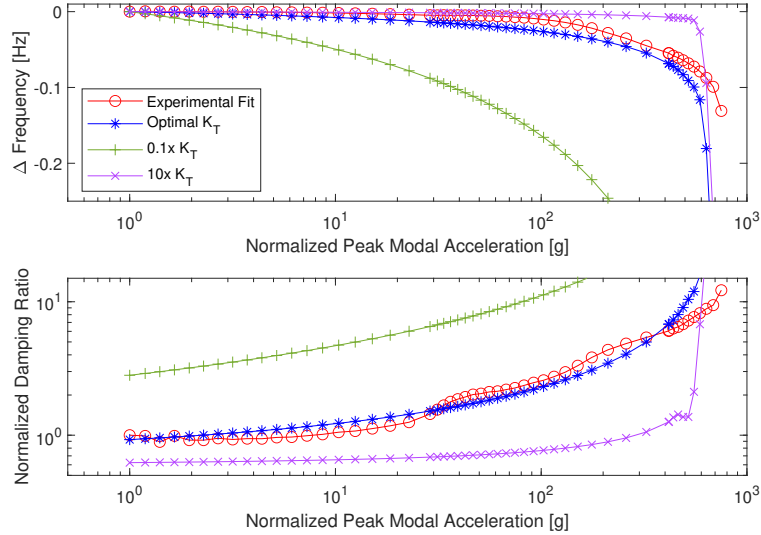


Figure 9: Frequency and damping of Iwan joint model with parameter set A in Table 6. Also plotted are results using  $K_T$  values that are 0.1 times and 10 times the optimal value.

The most accurate model using RBAR MPCs, which had a spider radius about equal to the rivet radius, was then given the same set of Iwan parameters as the RBE3 model. The nonlinear frequency and damping response of the RBAR model was indistinguishable from that shown in Figure 9, with the only difference in the two models being the linear mode frequencies, which are shown in Table 5. In terms of the linear natural frequencies, the RBE3 model has the least error starting with the first bending mode while the RBAR model has the smallest error centered in between the third and fourth bending modes.

Table 5: Normalized linear mode frequencies of the RBAR and RBE3 models using optimized Iwan parameters for the RBE3 model. Both models are Shell No Holes design. The RBE3 model spider radius is about 4.7 times the rivet radius and the RBAR model spider radius is about equal to the rivet radius.

| Experimental Freq. ( ) | RBE3 Freq. ( ) | RBE3 Error (%) | RBAR Freq. (Hz) | RBAR Error (%) |
|------------------------|----------------|----------------|-----------------|----------------|
| 1.000                  | 1.000          | 0.008          | 1.022           | 2.163          |
| 2.746                  | 2.739          | -0.253         | 2.783           | 1.345          |
| 5.327                  | 5.318          | -0.167         | 5.361           | 0.646          |
| 8.703                  | 8.675          | -0.322         | 8.662           | -0.461         |
| 12.835                 | 12.750         | -0.660         | 12.603          | -1.807         |
| 17.565                 | 17.468         | -0.549         | 17.090          | -2.700         |

From these results it was found that two models using two different MPC types (and the best corresponding MPC radius) could achieve similar nonlinear frequency and damping. Note that this contradicts the findings in [21], which found that the nonlinear response could only be captured when using RBAR spiders. In this study, it was possible for both models to have the same value for  $K_T$  and reasonably similar errors in the linear natural frequencies, and it seems that the nonlinear response is primarily dictated by the value of the Iwan parameters and in particular the  $K_T$  stiffness. The natural frequencies of the various modes are influenced much more by the selection of MPC type. While this does not provide a clear answer to which MPC type should be used in all situations, it is useful to know that changing the MPC type will not necessarily change the nonlinear frequency and damping of the model and that it can be possible to find accurate models for different MPC types and radii.

## 4.2 Results for Higher Order Bending and Torsion Modes

The optimization in the prior section was all performed using one set of data for Mode 1. Other data was not available as those studies were being performed. After that was done, a larger set of data was made available, which excited Mode 1 to higher amplitudes and excited several other modes. The nonlinear model was adjusted to capture this improved data set for Mode

1, and the result from both is shown in Figure 11 along with the full set of updated measurements. The results show that the nonlinearity is activated at slightly lower amplitudes than in the prior data set, and that the material damping has decreased; it is possible that the joints have loosened slightly since the first tests were performed.

Note that in this and the following figures, the experimental data is colored according to the drive point that was used. The drive points and their colors are defined in Fig. 10. The drive points 301, 302 and 303 are on the axis of the beam, so they excite the torsional modes minimally. No data for Drive point 203 is included in this paper. See [31] for additional details.

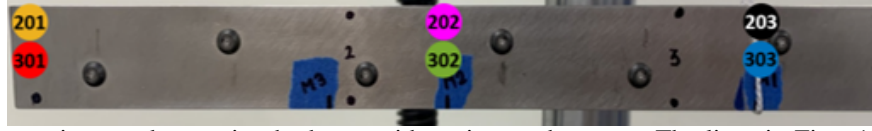


Figure 10: Drive points used to excite the beam with an impact hammer. The lines in Figs. 11-15 are colored according to drive point as noted in this figure. Drive points 202 and 302 are on a node line for Mode 2 while 203 and 303 are on a node line for Mode 1.

Table 6 shows the normalized Iwan parameters for the RBE3 model fit to both the original impact data (Set A) and the new, larger set of data (Set B). Note that the slip force parameter,  $F_s$ , for the high amplitude range is about  $3/4$  that of the value for the full amplitude range and that the parameter  $\beta$  is about  $1/5$  of its original value. The parameter  $\chi$  is about the same and the stiffness parameter,  $K_T$ , was kept the same. Figure 11 shows the difference between the models using Set A and Set B Iwan parameters when compared to the experimental data. As expected, Set B, which was the set of parameters optimized specifically for the new data set, produces a response that matches more closely.

Table 6: Normalized Iwan parameters using the original experimental data (Set A) and larger set of experimental data (Set B).

| Parameter | Set A | Set B  |
|-----------|-------|--------|
| $F_s$     | 1.00  | 0.755  |
| $K_T$     | 1.00  | 1.000  |
| $\chi$    | -1.00 | -1.003 |
| $\beta$   | 1.00  | 0.222  |

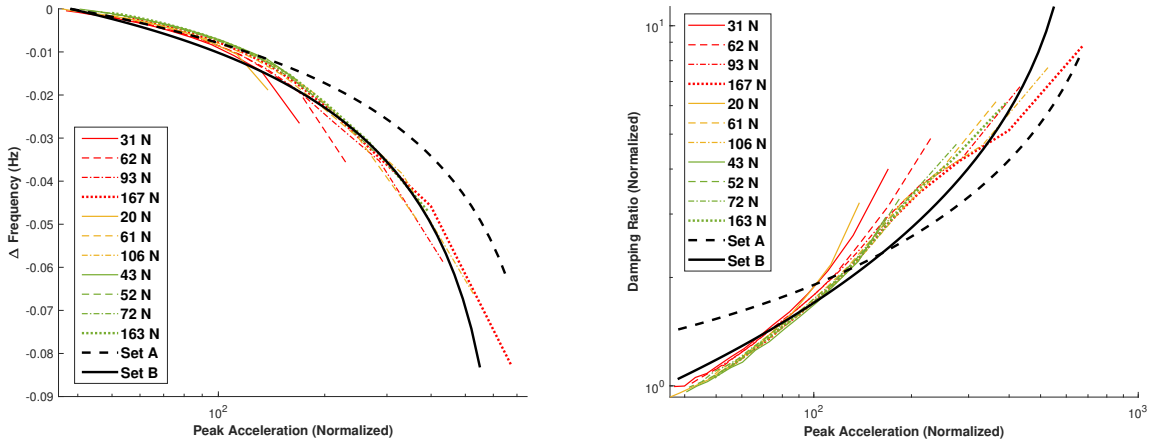


Figure 11: Frequency and damping plots for the 1st bending mode of the RBE3 Model using Set A Iwan parameters. The various colors represent different locations for the hammer impact during testing.

The Set B Iwan parameters were then used for QSMA in order to see how the model would predict the frequency and damping of higher order modes including the first torsion mode. Figures 12 - 15 compare the model to the experimental data for the second, third, fourth, and fifth bending modes of the structure. The measurements for all of these modes show much more variability than was present in Mode 1. The variability seen in these measurements is very similar to that reported by Wall et al. in [33], where it was attributed to mode coupling. In essence, Wall found that one tended to obtain minimal damping when a single mode was excited in isolation. If the excitation point was changed so that other modes were excited simultaneously, the mode in question always exhibited higher damping. Similarly, the shift in the effective natural frequency was larger when

multiple modes were excited. Hence, we presume that the variability seen in the measurements for Modes 2-6 is due to modal coupling, and that the minimal damping would be observed if the mode in question could be excited in isolation. This can be seen, to some extent, in Fig.12, where the curves shown in blue were obtained by exciting the beam at the node line of Mode 1, making Mode 2 the dominant mode in that response; these curves show minimal damping for Mode 2, which one can presume to be the uncoupled Mode 2 behavior.

Quasi-static modal analysis estimates the uncoupled behavior of each mode, so that is what we expect the model (black lines in each of Figures 12 - 15) to predict for each of these modes. Based on the results in [33, 34], one would expect the model to agree with the minimum damping and maximum frequency in each of Figures 12- 15. In general, the results show that the predicted frequency and damping was located in the middle of the spread of data. The exception to this was bending Mode 3 (see Figure 13), where the experimental damping values were much higher than the predicted values, especially the low-level material damping. The bungee cords suspending the beam were placed at the node lines of Mode 1, so we suspect that the measured damping for Mode 3 may be higher due to the influence of the bungees.

Another trend among all of the bending modes was that the model overestimated the frequency change at the largest amplitudes. The model's prediction of frequency versus amplitude generally shows a steeper slope than the experimental data. In spite of the errors, these results are encouraging considering the variability that is often present in damping and the lack of alternative methods for predicting the damping in a structure such as this.

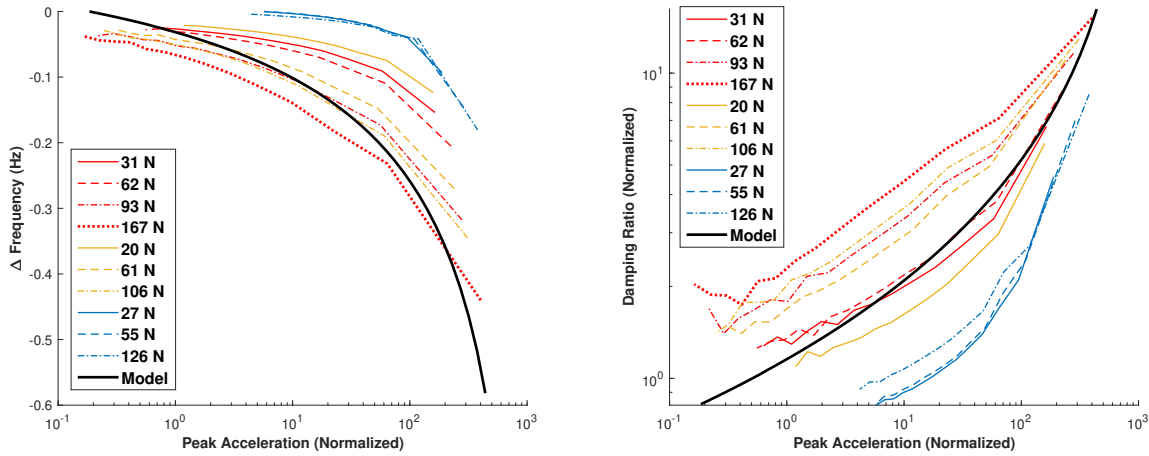


Figure 12: Frequency and damping plots for the 2nd bending mode of the RBE3 Model using Set B Iwan parameters.

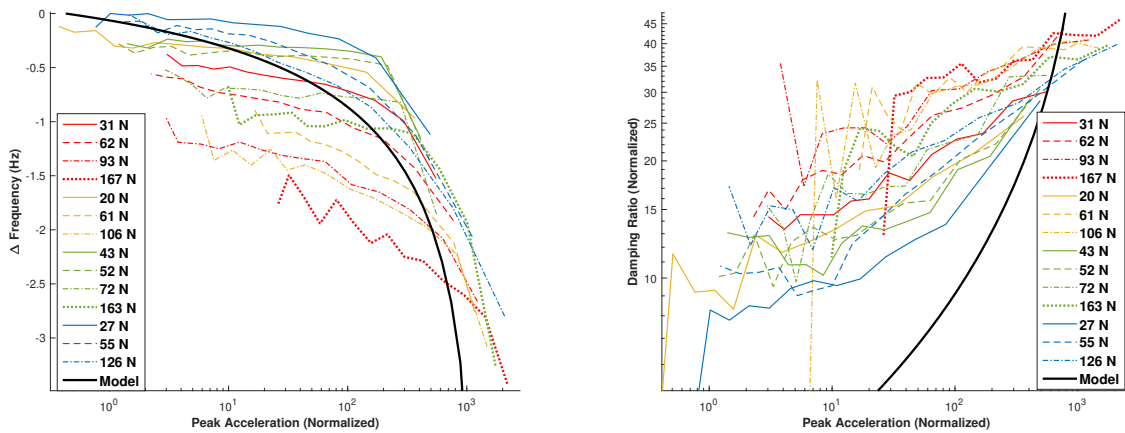


Figure 13: Frequency and damping plots for the 3rd bending mode of the RBE3 Model using Set B Iwan parameters.

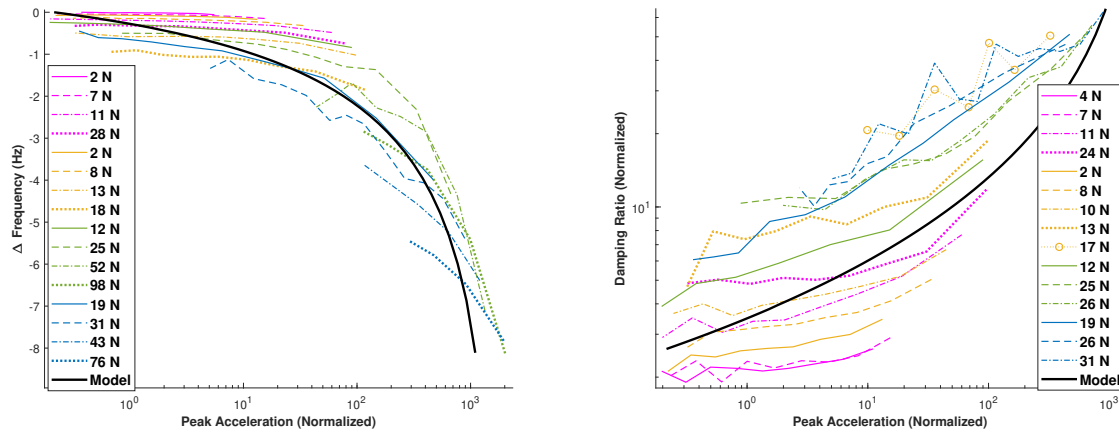


Figure 14: Frequency and damping plots for the 4th bending mode of the RBE3 Model using Set B Iwan parameters.

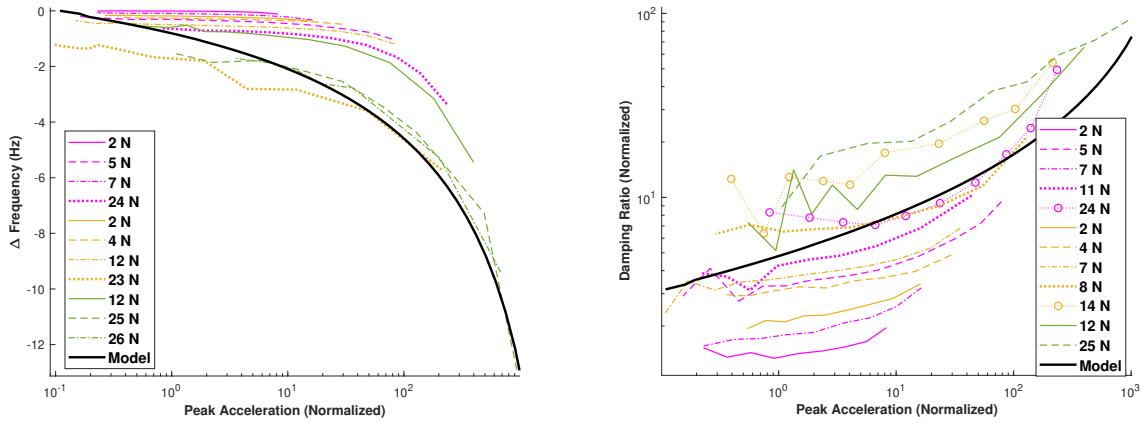


Figure 15: Frequency and damping plots for the 5th bending mode of the RBE3 Model using Set B Iwan parameters.

This same method was used to predict the amplitude dependent frequency and damping the first torsion mode of the model. Figure 16 shows the frequency and damping of the model compared to that of the experimental data. While the experimental data for the torsion mode had a much greater variability than many of the bending modes, it is clear that the model was not nearly as accurate. The model predicted minimal change in frequency and minimal damping for this mode. In fact, this mode was found to exhibit more nonlinearity than any of the bending modes, making it difficult to characterize [31]. The model predicted that this mode would behave somewhat linearly and underpredicted the damping of this mode by a little more than an order of magnitude. The reasons for this are unknown.

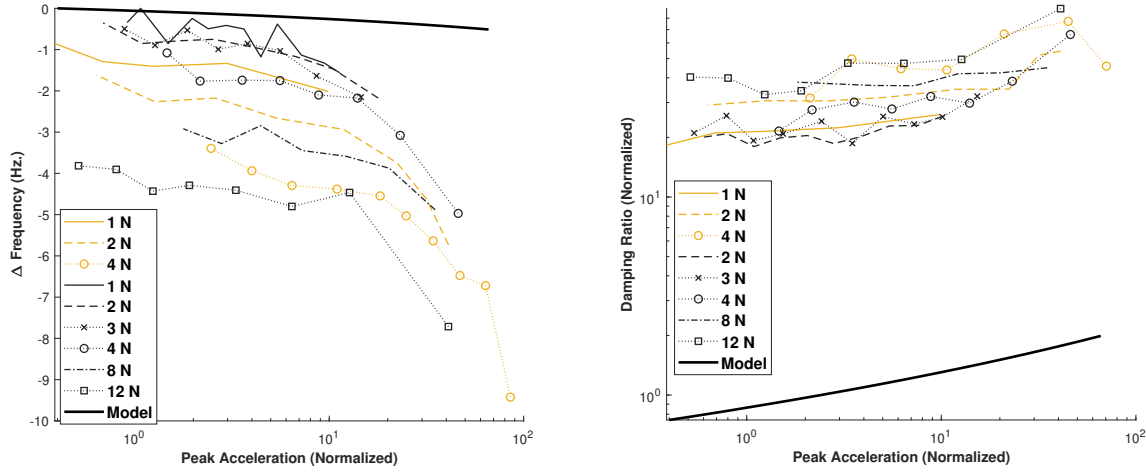


Figure 16: Frequency and damping plots for the 1st torsion mode of the RBE3 Model using Set B Iwan parameters.

## 5 Conclusions

The approach used in this paper was a modified version of that used to model structures with bolted joints in a few prior works [19, 21]. Specifically, a reduced order model of the riveted structure was created by using spiders to replace each rivet with springs in six degrees of freedom. Then the springs in the axial direction were replaced with Iwan elements to capture nonlinearity. Both RBAR and RBE3 MPC types were found to be effective at capturing the linear bending modes of the structure, although the model struggled to capture the torsion modes. When RBE3 spiders were used, a large spider radius was required to bring the model's linear natural frequencies into agreement with measurements, while RBAR spiders required a small spider radius. Both models accurately captured the nonlinearity in the first bending mode of the assembly. This study has shown that the modeling approach in [19, 21] is effective for a structure with many rivets, and good agreement was obtained when the parameters were assumed to be equal for all 24 rivets in the assembly.

This study made one important change to the optimization procedure used in [19, 21], which proved highly beneficial. The x-direction stiffness of the springs (along the axis of the beam), which becomes the  $K_T$  parameter in the Iwan model, was treated a parameter that was varied to study its effect on the results. Before this was done, we found ourselves spending considerable effort trying to use the nonlinear optimization step to find a set of Iwan parameters that would bring the model into agreement with the measurements, when no such set existed. In contrast, with  $K_T$  treated as a parameter, it was straightforward to vary it until the model captured the nonlinear behavior accurately, and one could readily see how this influenced the accuracy of the linear natural frequencies of various modes. Note that this approach was actually first proposed by Bahr et al. [35], and will be documented more thoroughly in an upcoming paper. Additionally, the Monte Carlo procedure that was used to find the Iwan parameters was found to struggle when the parameter bounds were large, as they often need to be. This issue was resolved by manual exploration of the search space before beginning the nonlinear optimization, this could possibly be addressed by drawing the Monte Carlo samples logarithmically from wide bounds; alternatives should be explored in future works.

This paper considered not only the nonlinear behavior of the first mode, but other available test data from [31] for several other modes. The first mode was chosen for optimization because it exhibited the strongest nonlinearity. The model obtained by optimizing to the first was found to be capable of providing a reasonable estimate of the nonlinear behavior seen in many of the higher order bending modes. This model treats each rivet as a discrete nonlinearity, and hence has the potential to capture nonlinear modal interactions. This should be explored in future works.

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