Engineering Dynamics, 2nd Edition

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Answers to Selected Homework Problems

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1.2 $c_1 = -244.00,\ c_2 = 0,\ c_3 = 455.48$ rad/s,
$\bar{e}_2 = -0.1449\bar{i} + 0.9704\bar{j} - 0.1932\bar{k},\ \bar{e}_3 = -0.5822\bar{i} + 0.2415\bar{j} + 0.7763\bar{k}$.

1.5 $\mathcal{V} = 6.116\left(10^{-6}\right)\text{ m}^3$.

1.7 $F_y = 386.18,\ F_x = 336.79$ N.

1.8 $\bar{F} = 3009\bar{i} - 3492\bar{j} + 1937\bar{k}$ N, $\bar{M}_A = 7749\bar{i} - 12034\bar{k},\ M_{\text{shaft}} = -11624$ N-m.

1.10 $\Delta\bar{r}_C = -1.1753\bar{i} + 1.0445\bar{j} - 0.4113\bar{k},\ \Delta\bar{r}_B = 1.1753\bar{i} - 1.0445\bar{j} - 0.4113\bar{k}$

1.12 $\bar{v}_P = \left[\varepsilon\alpha\cos(\alpha t)\cos\theta - R\alpha t\sin\theta\right]\bar{i} + \left[\varepsilon\alpha\cos(\alpha t)\sin\theta + R\alpha t\cos\theta\right]\bar{j}$,
$\bar{v}_{\parallel} = \varepsilon\alpha\cos(\alpha t)\,,\quad \bar{v}_{\perp} = R\alpha t$.

1.16 $\mathcal{P} = \left[F_1 + F_2\cos(\theta_1+\theta_2)\right]\dot{s} + F_2L_1\dot{\theta}_2\cos\theta_2 + F_2L\left(\dot{\theta}_1+\dot{\theta}_2\right)\cos(\theta_1+\theta_2)$.

2.2 (a) $m\ddot{x} = \beta B\dot{y},\quad m\ddot{y} - \beta B,\quad m\ddot{z} = 0$,
(b) $\omega = \dfrac{\beta B}{m}$
(c) $C = 0,\ S = \dfrac{v_0}{\omega},\ D = 0,\ E = \dfrac{v_o}{\omega}$
(d) circular path centered at $x = 0, y = -v_o/\omega$ with radius $= v_o/\omega$

2.3 $\beta_{\max} = 21.419^o$

2.6 (a) $\bar{v} = u\bar{i} + \dfrac{\pi Hu}{L}\cos\left(\dfrac{\pi x}{L}\right)\bar{j},\quad \bar{a} = -\dfrac{\pi^2Hu^2}{L^2}\sin\left(\dfrac{\pi x}{L}\right)\bar{j}$,
(b) $N = mg\left[1 + \left(\pi\dfrac{uH}{L}\right)\cos\left(\dfrac{\pi x}{L}\right)^2\right]^{1/2}\left(1 - \pi^2\dfrac{u^2H}{gL^2}\sin\left(\dfrac{\pi x}{L}\right)\right),\ x = ut$.
(c) $u < \left(\dfrac{gL^2}{\pi^2H}\right)^{1/2}$.

2.11 $F = 3.314$ N, $N = 10.326$ N

2.13 $$\begin{Bmatrix}\bar{v}\cdot\bar{e}_R\\ \bar{v}\cdot\bar{e}_\theta\\ \bar{v}\cdot\bar{e}_z\end{Bmatrix} = c\lambda\begin{pmatrix}1\\ -2.094\gamma\\ 8.3776\end{pmatrix},\quad \begin{Bmatrix}\bar{a}\cdot\bar{e}_R\\ \bar{a}\cdot\bar{e}_\theta\\ \bar{a}\cdot\bar{e}_z\end{Bmatrix} = c\lambda^2\begin{pmatrix}-1.0472\gamma^2\\ -4.6276\gamma\\ 2\end{pmatrix}$$

2.14 $\bar{v} = u\cot\theta\bar{e}_R - L\Omega\sin\theta\bar{e}_\theta - u\bar{e}_z,\quad \bar{a} = -\left(\dfrac{u^2}{L(\sin\theta)^3} + L\Omega^2\sin\theta\right)\bar{e}_R - 2\Omega u\cot\theta\bar{e}_\theta$.

2.15 $$\bar{v} = \left[\frac{3}{4}u\cot\theta\right]\bar{e}_R + \left[\frac{3L}{2}\omega\sin\theta\right]\bar{e}_\phi + \left[-\frac{1}{4}u\right]\bar{e}_z$$
$$\bar{a} = \left[-\frac{3}{8}\frac{u^2}{L(\sin\theta)^3} - \frac{3L}{2}\omega^2\sin\theta\right]\bar{e}_R + \left[\frac{3L}{2}\dot{\omega}\sin\theta + \frac{3}{2}u\omega\cot\theta\right]\bar{e}_\phi$$

2.19 (a) $\bar{a} = -525.89\bar{e}_r - 386.92\bar{e}_\phi$, (b) $\bar{a} = -770.94\bar{e}_R + (0)\bar{e}_\theta - 6\bar{e}_z$

2.21 (a) $s = s_0/4$, (b) $s = s_0/2$, (c) $R = 2s_0/\pi$, (d) $s = 0.3280\pi$.

2.23 $\rho = \dfrac{c^2 + k^2L^2}{k^2L},\ \tau = \dfrac{\left(c^2 + k^2L^2\right)^{\frac{1}{2}}}{ck}$

2.25 $\bar{v} = \left(0.1451\bar{i} - 0.4785\bar{j}\right)\beta,\quad \bar{a} = \left(0.0304\bar{i} - 0.16973\bar{j}\right)\left(\beta^2/k\right)$.

2.26 $v = A\omega\left[1 + 3(\sin\theta)^2\right]^{1/2},\quad \dot{v} = 3\omega^2A\dfrac{\sin\theta\cos\theta}{\left[1 + 3(\sin\theta)^2\right]^{1/2}},\quad \bar{a}\cdot\bar{e}_n = \dfrac{2\omega^2A}{\left[1 + 3(\sin\theta)^2\right]^{1/2}}$.

2.28 $\bar{e}_t = -0.4804\bar{i} + 0.2774\bar{j} + 0.8321\bar{k},\ \bar{e}_n = -0.5160\bar{i} + 0.6769\bar{j} - 0.5241\bar{k}$
$\bar{e}_b = -0.7086\bar{i} - 0.6818\bar{j} - 0.1818\bar{k},\ \rho = 6.392$ m, $\tau = 40.33$ m.

2.33 $v = 12.329$ m/s, $\quad\dot{v} = -8.922\text{ m/s}^2,\quad \rho = 3.719$ m, $\bar{r}_C = -22.071\bar{i} + 27.462\bar{j} - 8.2399\bar{k}$ m.

2.35 $\dot{r} = -350.0$ m/s, $\dot{\lambda} = 0.06351$ rad/s, $\dot{\theta} = -0.03244$ rad/s,
$\ddot{r} = 4.831$ m/s^2, $\ddot{\lambda} = -0.01285$ rad/s^2, $\ddot{\theta} = -0.005277$ rad/s^2.

2.36 $$\bar{v} = \left(\dot{R}\cos\theta - R\dot{\theta}\sin\theta\right)\bar{i} + \left(\dot{R}\sin\theta + R\dot{\theta}\cos\theta\right)\bar{j},$$
$$a = \left(\ddot{R}\cos\theta - 2\dot{R}\dot{\theta}\sin\theta - R\ddot{\theta}\sin\theta - R\dot{\theta}^2\cos\theta\right)\bar{i} + \left(\ddot{R}\sin\theta + 2\dot{R}\dot{\theta}\cos\theta + R\ddot{\theta}\cos\theta - R\dot{\theta}^2\sin\theta\right)\bar{j}.$$

2.38 $$\bar{v} = \dot{\theta}\left(R'\bar{e}_R + R\bar{e}_\theta\right), \quad \bar{a} = \dot{\theta}^2\left[\left(R'' - R\right)\bar{e}_R + 2R'\bar{e}_\theta\right], \quad \rho = \frac{\left[\left(R'\right)^2 + R^2\right]^{3/2}}{R''R - R^2 - 2\left(R'\right)^2}.$$

2.41 (a) $\bar{v} = -0.2828u\bar{i} - 1.1314u\bar{j}$, $\bar{a} = -0.5440u^2\bar{i} - 0.8960u^2\bar{j}$,
(b) $F = m\left(1.1676u^2 - 11.095\right)$.

2.43 $v = 20.62$ m/s, $\dot{v} = 250.0$ m/s^2

3.1 $$[R] = \begin{bmatrix} 0.8321 & -0.5547 & 0 \\ 0.3714 & 0.5571 & 0.7428 \\ -0.412 & -0.618 & 0.6695 \end{bmatrix}, \quad \bar{r}_{C/A} = 0.4828\bar{j} - 0.5356\bar{k} \text{ m}.$$

3.4 $$[R] = \begin{bmatrix} -0.9285 & 0.3714 & 0 \\ -0.1564 & -0.3910 & 0.9070 \\ 0.3369 & 0.8422 & 0.4211 \end{bmatrix}, \quad \bar{r}_{O/A} = 46.42\bar{i} + 7.82\bar{j} - 16.84\bar{k} \text{ m}.$$

3.8 (a) $[x_E \ \ y_E \ \ z_E] = [-75.09 \ \ -48.32 \ \ -42.73]$ mm,
(b) $[X_E \ \ Y_E \ \ Z_E] = [-1.62 \ \ -98.80 \ \ -6.06]$ mm.

3.14 $[X_C \ \ Y_C \ \ Z_C] = [0.1465 \ \ 0.3357 \ \ 0.0766]$ m.

3.17 $\phi = 77.14°$ about $\bar{K}' = 0.9265\bar{I} - 0.3258\bar{J} - 0.1881\bar{K}$.

3.18 $[R]$ is equivalent to rotating 151.48° about $\bar{e} = 0.5644\bar{I} - 0.1780\bar{J} - 0.8961\bar{K}$

3.21 $\Delta\bar{r}_C = -406.9\bar{I} - 378.6\bar{J} - 505.1\bar{K}$ mm.

3.23 $\Delta\bar{r}_A = -113.19\bar{I} + 124.14\bar{J} - 13.76\bar{K}$ mm, $\Delta\bar{r}_B = 95.83\bar{I} - 25.66\bar{J} + 13.76\bar{K}$ mm.

3.25 $\Delta\bar{r}_F = 39.689\bar{I} + 5.236\bar{J} - 13.818\bar{K}$ mm, $\Delta\bar{r}_F = 33.757\bar{i} + 24.417\bar{j} + 7.6023\bar{k}$ mm,

3.30 $\bar{\omega} = 1000\pi\bar{i} + 0.16667\bar{k}$ rad/s, $\bar{\alpha} = 166.7\pi\bar{j}$ rad/s^2,
where $\bar{i} = \bar{e}_t$ and $\bar{j} = \bar{e}_n$ for the airplane's path.

3.31 $$\bar{\omega} = -0.4330\left(\dot{\theta} + 2\dot{\beta}\right)\bar{i} + 0.5\left(\dot{\theta} + 2\dot{\gamma}\right)\bar{j} + 0.250\left(3\dot{\theta} - 2\dot{\beta}\right)\bar{k},$$
$$\bar{\alpha} = -0.250\left(\dot{\theta}\dot{\beta} + 3\dot{\theta}\dot{\gamma} - 2\dot{\beta}\dot{\gamma}\right)\bar{i} - 0.8660\dot{\theta}\dot{\beta}\bar{j} + 0.4330\left(\dot{\theta}\dot{\beta} - \dot{\theta}\dot{\gamma} - 2\dot{\beta}\dot{\gamma}\right)\bar{k}.$$

3.32 $\bar{\omega} = -2\bar{i} + 5241\bar{j} - 19.32\bar{k}$ rad/s, $\bar{\alpha} = 101\,210\bar{i} + 12.76\bar{j} - 10370\bar{k}$ rad/s^2.

3.33 $$\bar{a}_C = \left[L\ddot{\theta}\sin 2\theta - L\dot{\theta}^2\left(9 + \cos 2\theta\right)\right]\bar{i} + \left[L\ddot{\theta}\left(3 + \cos 2\theta\right) + L\dot{\theta}^2\sin 2\theta\right]\bar{j}, \ \bar{i} = \bar{e}_{C/B}.$$

3.37 $$\bar{a}_{B/A} = \left(-\Omega^2 H + 2\Omega\dot{\theta}W\sin\theta\right)\bar{i} - \left(\Omega^2 + \dot{\theta}^2\right)W\left(\cos\theta\right)\bar{j} - \dot{\theta}^2 W\sin\theta\bar{k}.$$

3.39 $$\bar{v}_E = -\left(3.77L + 127.3R\right)\bar{j} - 10\left(L + R\right)\bar{k},$$
$$\bar{a}_E = -\left(106.3L + 16315R\right)\bar{i} + 25.13\left(L + R\right)\bar{j} - \left(495.3L + 182.1R\right)\bar{k}.$$

3.42 $\bar{v}_D = -6.566\bar{i} - -2.347\bar{j} + 11.638\bar{k}$ m/s, $\bar{a}_D = -314.9\bar{i} - 237.4\bar{j} - 317.4\bar{k}$ m/s^2,
where $\bar{j} = \bar{e}_{C/B}$ and $\bar{k} = \bar{e}_{B/A} \times \bar{e}_{C/B} / \left|\bar{e}_{B/A} \times \bar{e}_{C/B}\right|$.

3.44 $$\dot{u} = \left(50 - \sin\theta\right)g + \dot{\theta}^2 s + \Omega^2 s\left(\cos\theta\right)^2,$$
$$N_{\text{horizontal}} = 2m\Omega\left(u\cos\theta - \dot{\theta}s\sin\theta\right), \quad N_{\text{vertical}} = m\left(g\cos\theta + \Omega^2\frac{s}{2}\sin 2\theta + 2\dot{\theta}u\right)$$

3.46 $$\bar{v}_C = \left(v\cos\beta - b\dot{\theta}\cos\theta\right)\bar{i} - b\dot{\beta}\sin\theta\bar{j} + b\dot{\theta}\sin\theta\bar{k}$$
$$\bar{a}_C = \left(-b\ddot{\theta}\cos\theta + \left(a + b\sin\theta\right)\dot{\beta}^2 + b\dot{\theta}^2\sin\theta - 2u\dot{\theta}\cos\theta\right)\bar{i} - \left(\left(a + b\sin\theta\right)\ddot{\beta} + 2u\dot{\beta}\sin\theta + 2b\dot{\theta}\dot{\beta}\cos\theta\right)\bar{j} + \left(b\ddot{\theta}\sin\theta + b\dot{\theta}^2 + 2u\dot{\theta}\sin\theta\right)\bar{k}$$

3.49 $$\bar{v}_B = \dot{\xi}\bar{i} + \Omega\xi\sin\theta\bar{j} + \dot{\theta}\xi\bar{k},$$
$$\bar{a}_B = \left[\ddot{\xi} - \dot{\theta}^2\xi - \Omega^2\xi\left(\sin\theta\right)^2\right]\bar{i} + \left[2\Omega\dot{\xi}\sin\theta + 2\Omega\dot{\theta}\xi\cos\theta\right]\bar{j} + \left[\ddot{\theta}\xi - \Omega^2\xi\sin\theta\cos\theta + 2\dot{\theta}\dot{\xi}\right]\bar{k}.$$

3.51 $$\bar{\omega} = 0.9397\Omega\cos\phi\bar{i} - 0.9397\Omega\sin\theta\bar{j} + \left(-0.3420\Omega + \dot{\phi}\right)\bar{k},$$
$$\bar{\alpha} = -0.9397\Omega\dot{\phi}\sin\phi\bar{i} - 0.9397\Omega\dot{\phi}\cos\phi\bar{j} + \ddot{\phi}\bar{k},$$
$$\bar{v}_G = -1.47\Omega L\sin\phi\bar{i} + L\left[-\left(0.171 + 1.47\cos\phi\right)\Omega + 0.5\dot{\phi}\right]\bar{j} + 0.4698\Omega L\sin\phi\bar{k},$$
$$\bar{a}_G = L\left[\left(-0.5 - 0.5027\cos\phi + 0.8415\left(\cos\phi\right)^2\right)\Omega^2 - 0.5\dot{\phi}^2 + 0.342\Omega\dot{\phi}\right]\bar{i}$$
$$+L\left[0.5\ddot{\phi} + \left(0.5027\sin\phi - 0.2208\sin 2\phi\right)\Omega^2\right]\bar{j}$$
$$+L\left[-\left(1.381 + 0.1607\cos\phi\right)\Omega^2 + 0.9397\Omega\dot{\phi}\cos\phi\right]\bar{k}$$

3.53 $$\bar{\omega} = \left(\frac{v}{\tau} + v\frac{d\beta}{ds}\right)\bar{e}_t + \frac{v}{\rho}\bar{e}_b,$$
$$\bar{\alpha} = \left(\frac{\dot{v}}{\tau} - \frac{v^2}{\tau^2}\frac{d\tau}{ds} + \dot{v}\frac{d\beta}{ds} + v^2\frac{d^2\beta}{ds^2}\right)\bar{e}_t + \frac{v^2}{\rho}\frac{d\beta}{ds}\bar{e}_n + \left(\frac{\dot{v}}{\rho} - \frac{v^2}{\rho^2}\frac{d\rho}{ds}\right)\bar{e}_b.$$

3.54 $(\bar{v}_B)_{x_2y_2z_2} = -46.57\bar{I} + 41.16\bar{J}$ m/s, $(\bar{a}_B)_{x_2y_2z_2} = -19668\bar{I} - 35112\bar{J}$ m/s^2.

3.56 $(\bar{a}_{can})_{xyz} = (0.25s - 2.503)\,i - 0.918\bar{j} - (s + 5.553)\,\bar{k}.$

3.59 $s = -\left(\dfrac{\omega_e\sin\lambda}{u}\right)d^2$ to the right.

4.1 $$\bar{v}_D = \left(R\omega_1\cos\beta - R\omega_2 - L\dot{\beta}\right)\bar{i} + L\omega_1\sin\beta\bar{j} - R\omega_1\cos\beta\bar{k},$$
$$\bar{a}_D = \left(-L\omega_1^2\sin\beta\cos\beta - L\ddot{\beta}\right)\bar{i} + \left[-R\left(\omega_1^2 + \omega_2^2\right) + 2\omega_1\left(L\dot{\beta} + R\omega_2\right)\cos\beta\right]\bar{j}$$
$$-\left[\omega_1^2 L\left(\sin\beta\right)^2 + L\dot{\beta}^2 + 2R\omega_2\dot{\beta}\right]\bar{k}.$$

4.3 $\dot{\psi} = -98.266$ rad/s, $\dot{\gamma} = -21.484$ rad/s, $\dot{\eta} = 144.77$ rad/s,
$\bar{\omega} = -21.406\bar{i}' - 21.406\bar{j}' + 44.633\bar{k}'$ rad/s.

4.5 $\theta = \pi/3,\ \phi = 0,$
$\dot{\psi} - 40$rad/s, $\dot{\theta} = 10$rad/s, $\dot{\phi} = -800$rad/s,
$\ddot{\psi} = 9007$rad/s^2, $\ddot{\theta} = -1001$rad/s^2, $\ddot{\phi} = 3811$rad/s^2,

4.9 $v = 5.313$ m/s, $\dot{v} = 724.1$ m/s^2.

4.12 $$\bar{v}_A = b\dot{\theta}\cos\theta\bar{J},\quad \bar{a}_A = b\left(\ddot{\theta}\cos\theta - \dot{\theta}^2\sin\theta\right)\bar{J},$$
$$\bar{v}_B = -b\dot{\theta}\sin\theta\bar{I},\quad \bar{a}_B = -b\left(\ddot{\theta}\sin\theta + \dot{\theta}^2\cos\theta\right)\bar{I},$$
$$\bar{v}_G = \frac{\sqrt{2}}{2}b\dot{\theta}\cos\left(\theta + \frac{\pi}{4}\right)\left(\bar{I} + \bar{J}\right),$$
$$\bar{a}_G = \frac{\sqrt{2}}{2}b\left[\ddot{\theta}\cos\left(\theta + \frac{\pi}{4}\right) - \dot{\theta}^2\sin\left(\theta + \frac{\pi}{4}\right)\right]\left(\bar{I} + \bar{J}\right).$$

4.15 $\theta = 60^\circ:\ \bar{\omega}_{BC} = -0.1111\dot{\theta}\bar{k},\quad \bar{\alpha}_{BC} = -0.4467\dot{\theta}^2\bar{k},\quad \bar{k}$ is inward.
$\theta = 120^\circ:\ \bar{\omega}_{BC} = -0.2727\dot{\theta}\bar{k},\quad \bar{\alpha}_{BC} = -0.1232\dot{\theta}^2\bar{k}.$

4.17 $\bar{\omega}_{BC} = 0.8660\omega_{AB}\bar{k},\ \bar{\omega}_{CD} = -0.50\omega_{AB}\bar{k},\ \bar{\alpha}_{BC} = 0.250\omega_{AB}^2\bar{k},$
$\bar{\alpha}_{CD} = 1.616\omega_{AB}^2\bar{k},\quad \bar{k}$ is outward.

4.18 $\bar{v}_B = 0.866v_A\bar{i} + 0.866L\Omega\bar{j} + 0.5v_A\bar{k},\quad \bar{v}_G = 0.433v_A\bar{i} + 0.433L\Omega\bar{j} - 0.25v_A\bar{k},$
where $\bar{k}$ is upward and $\bar{i}$ is radial.

4.20 $\bar{v}_C = -R\Omega\sin\theta\bar{i} + u\bar{j} - u\tan\theta\bar{k},\quad \bar{a}_C = -2\Omega u\bar{i} - R\Omega^2\sin\theta\bar{j} - \dfrac{u^2}{R}\dfrac{1}{\left(\cos\theta\right)^3}\bar{k}$
where $\bar{k}$ is upward and $\bar{j}$ is radial.

4.23 $v_D = 18.138$ m/s, $\bar{\omega}_{CD} = 17.490\bar{i} + 141.22\bar{j} - 90.88\bar{k}$ rad/s,
$a_D = -13802$ m/s^2, $\alpha_{CD} = 950.7\bar{i} + 4718\bar{j} - 4940\bar{k}$ rad/s^2.

4.26 (a) No unique solution, (b) $\omega_{AB} = -\omega_{CD} = 1.20v_A\bar{J} + 1.60v_A\bar{K}$ rad/s, where $\bar{I} = \bar{e}_{A/C}$.

4.28 $$\bar{v}_C = \tfrac{1}{2}\left[\left(\omega_1 r_1 + \omega_2 r_2\right) + \left(\omega_2 r_2 - \omega_1 r_1\right)\cos\theta\right]\bar{i} + \tfrac{1}{2}\left(\omega_2 r_2 - \omega_1 r_1\right)\sin\theta\bar{j}$$
$$\bar{a}_C = -\frac{\left(\omega_2 r_2 - \omega_1 r_1\right)^2}{2(r_2 - r_1)}\sin\theta\bar{i} - \left[\frac{\left(\omega_2 r_2 + \omega_1 r_1\right)^2}{2(r_2 + r_1)} + \frac{\left(\omega_2 r_2 - \omega_1 r_1\right)^2}{2(r_2 - r_1)}\cos\theta\right]\bar{j}$$
where $\bar{j}$ is radial and $\bar{i}$ is downward to the right

4.30 $\bar{\omega}_A = 0.7273\dfrac{v}{R}$ clockwise, $\bar{\alpha}_A = 0.1172\dfrac{v^2}{R^2}$ clockwise.

4.32 $\bar{\omega} = \dfrac{v(\cos\theta)^2}{R(\cos\theta)^2 + h}$ clockwise, $\bar{\alpha} = \dfrac{2v^2 h(\cos\theta)^3 \sin\theta}{\left[R(\cos\theta)^2 + h\right]^3}$ counterclockwise.

4.35 $\bar{\omega} = -\dfrac{v}{R}\bar{i} + \dfrac{v}{R}\cos\beta\bar{j}, \quad \bar{\alpha} = \dfrac{v^2}{R^2}(1+\cos\beta)\sin\beta\bar{k},$
$\bar{i}$ parallel to the cone generator and $\bar{j}$ upward.

4.38 $\bar{\omega} = \left[\Omega_1\cos(\beta+\gamma) + (\Omega_1 - \Omega_2)\dfrac{\sin\beta}{\sin\gamma}\right]\bar{i} - \Omega_1\sin(\beta+\gamma)\bar{j},$
$\bar{\alpha} = \left[\dot{\Omega}_1\cos(\beta+\gamma) + \left(\dot{\Omega}_1 - \dot{\Omega}_2\right)\dfrac{\sin\beta}{\sin\gamma}\right]\bar{i} - (\Omega_1 - \Omega_2)\Omega_1\dfrac{\sin\beta}{\sin\gamma}\sin(\beta+\gamma)\bar{j}$
$-\dot{\Omega}_1\sin(\beta+\gamma)\bar{k}$
where $\bar{i}$ is along axis of gear A and $\bar{k}$ is upward to the left

4.40 Precession : $\dot{\psi} = \Omega_1 + \dfrac{(\Omega_1 - \Omega_2)}{(\sin\beta)^2}\left[\left(\dfrac{R}{r} - 1\right)\cos\beta - 1\right],$
Spin: $\dot{\phi} = \dfrac{(\Omega_1 - \Omega_2)}{(\sin\beta)^2}\left(\dfrac{R}{r} - 1 - \cos\beta\right),$
$\bar{\omega} = \left(\dot{\psi}\cos\beta + \dot{\phi}\right)\bar{i} + \dot{\psi}\sin\beta\bar{k}, \quad \bar{\alpha} = -\dot{\phi}\dot{\psi}\sin\beta\bar{j},$
$\bar{i}$ parallel to the cone generator, $\bar{k}$ upward.

4.42 $\psi = 33.69°, \quad \dot{\psi} = -15.428$ rad/s, $\quad \dot{\theta} = 0.6934$ rad/s, $\quad \dot{\phi} = 13.699$ rad/s.

4.44 $\bar{\omega} = 2\Omega(1+\cos\beta)\bar{i} + \dot{\beta}\bar{j} - \Omega\cos\beta\bar{k}, \quad \dot{\beta} = \dfrac{u}{R}\left(\dfrac{1}{\sin\beta - 2\cos\beta}\right)$
$\bar{\alpha} = -2\Omega\dot{\beta}\sin\beta\bar{i} - \left[\dfrac{u^2}{R^2}\dfrac{1}{(\sin\beta - 2\cos\beta)3} + \Omega^2\cos\beta(2 + 2\cos\beta - \sin\beta)\right]\bar{j}$
$-2\Omega\dot{\beta}(1+\cos\beta - \sin\beta)\bar{k}, \quad \bar{i} = \bar{e}_{C/B}, \quad \bar{k}$ upward.

5.2 (a) $\bar{H}_O = 8.3138\bar{k}$ kg-m^2/s
(b) $\bar{H}_O = 8.3138\left(\sin\phi\bar{J} + \cos\phi\bar{K}\right)$ kg-m^2/s
(c) $\dfrac{d}{dt}\bar{H}_O \approx 2452.357\bar{J} - 454.517\bar{K}$ kg-m^2/s^2
$\dfrac{d}{dt}\bar{H}_O = 2452.388\bar{J} - 454.523\bar{K}$ kg-m^2/s^2

5.4 $\bar{\Gamma}_C = \left[-\dfrac{3}{2}mgL\sin\theta + \dfrac{5}{4}L^2mL^2\theta - \dfrac{5}{4}mL^2\Omega^2\cos\theta\sin\theta\right]\bar{j} - \dfrac{5}{2}mL^2\Omega\dot{\theta}(\cos\theta)\bar{k}$
$\bar{F} = 2mg\left[\cos\theta + \dfrac{3}{2}L\Omega^2\cos\theta\sin\theta\right]\bar{i} - 3mL\Omega\dot{\theta}(\sin\theta)\bar{j} - \dfrac{3}{2}m\left[L\Omega^2(\cos\theta)^2 + L\dot{\theta}^2\right]\bar{k}$

5.6 $m = 6.369$ kg, $\quad x_G = 1.1078$ m, $\quad y_G = 0.5847$ m,
$I_{xx} = 3.544, \quad I_{yy} = 9.931, \quad I_{zz} = 13.475$ kg-m^2, $\quad I_{xy} = 5.776$ kg-m^2.

5.8 $x_G = \dfrac{4R}{3\pi}, \; z_G = \dfrac{2L}{3}, \; I_{xx} = \dfrac{mL^2}{4}\left[\dfrac{1+(\cos\psi)^2}{(\cos\psi)^2}\right], \; I_{xz} = \dfrac{mL^2}{\pi}\tan\psi$

5.10 $m = \dfrac{7}{12}\pi\rho R^2 L, \quad z_G = \dfrac{11}{28}L, \quad I_{xx} = I_{yy} = m\dfrac{128L^2 + 93R^2}{560}, \quad I_{zz} = m\dfrac{93R^2}{280}.$

5.11 $m = 414.1$ kg, $\quad \bar{r}_{G/O} = 2.26\bar{j} + 200\bar{k}$ mm, centroidal $\hat{x}\hat{y}\hat{z}$ are principal axes with
$I_{\hat{x}\hat{x}} = 9.495, \quad I_{\hat{y}\hat{y}} = 9.498, \; I_{\hat{z}\hat{z}} = 9.800$ kg-m^2.

5.13
$$[I] = mR^2\begin{bmatrix} 0.542 & 0.042 & 0 \\ 0.042 & 2.542 & 0 \\ 0 & 0 & 1.292 \end{bmatrix}.$$

5.15 $I_{xx} = 9.265\sigma R^3, \quad I_{yy} = 2.649\sigma R^3, \quad I_{zz} = 8.773\sigma R^3$
$I_{xy} = 0.745\sigma R^3, \quad I_{yz} = 2\sigma R^3, \quad I_{xz} = 0$

5.18 $I_1 = 26.61(10^{-6}), \quad I_2 = 456.7(10^{-6}), \quad I_3 = 483.3(10^{-6})$ kg-m^2.

5.20
$$[I] = \begin{bmatrix} 188.5 & 228.9 & 0 \\ 228.9 & 1687.5 & -21.5.1 \\ 0 & -21.5 & 1616.0 \end{bmatrix} \text{ kg-m}^2 \text{ for } \theta = 30.03°$$

5.22 $I_1 = 304.88(10)^{-6},\ I_2 = 1221.8(10)^{-6},\ I_3 = 1462.7(10)^{-6}$ kg-m^2

$$[R] = \begin{bmatrix} -0.9723 & -0.2104 & -0.04106 \\ -0.2292 & 0.9591 & -0.08656 \\ -0.04644 & -0.1894 & 0.9954 \end{bmatrix}$$

5.24 $\bar{H}_C = \frac{1}{9} mL^2 \Omega \sin\theta \bar{j},\ \ d\bar{H}_C/dt = \frac{1}{9} mL^2 \Omega^2 \sin\theta \cos\theta \bar{k}$
with x aligned with bar AB, y upward in the vertical plane.

5.27 $d\bar{H}_O/dt = 0.090411\bar{k} = 0.090411\bar{k}'$ kg-m^2/s^2

5.29
$$\bar{F}_O = -mL\left[\dot{\psi}^2 (\sin\theta)^2 + \dot{\theta}^2\right]\bar{i} - mL\left[\ddot{\theta} - \dot{\psi}^2 \sin\theta\cos\theta\right]\bar{j} - mL\left[\ddot{\psi}\sin\theta + 2\dot{\psi}\dot{\theta}\cos\theta\right]\bar{k},$$
$$\bar{M}_O = \left[I_1\ddot{\psi}\cos\theta - (I_1 - I_2 + I_3)\dot{\psi}\dot{\theta}\sin\theta\right]\bar{i} + \left[I_2\ddot{\psi}\sin\theta - (I_1 - I_2 - I_3)\dot{\psi}\dot{\theta}\cos\theta\right]\bar{j}$$
$$- \left[I_3\ddot{\theta} + (I_1 - I_2)\dot{\psi}^2 \sin\theta\cos\theta\right]\bar{k},\quad \bar{i} = \bar{e}_{G/O}.$$

5.31 $\Sigma\bar{F} = -1.44L\bar{k},\ \Sigma\bar{M} = (-15.71\cos\beta + 0.001836\sin\beta\cos\beta)\bar{k}$ Nm with $\bar{k} = \bar{e}_{D/C}$

5.33 $\bar{H}_G = mR^2\omega_1\left[0.125\lambda\bar{i} + (0.433\lambda - 0.5)\bar{k}\right],\quad \dfrac{d\bar{H}_G}{dt} = mR^2\omega_1^2\left(0.25\lambda - 0.10825\lambda^2\right)\bar{j}$
$\lambda = 2.309$ for no dynamic reactions,
$\bar{i}$ is the axis of the disk, $\bar{j}$ is perpendicular to the diagram.

5.35 $\bar{F}_B = 3mR\Omega^2\bar{I} + mg\bar{K},\quad \bar{M}_B = \dfrac{1}{4}mR^2\left[-2\Omega\dot{\phi}(\cos\theta)^2 + \left(\Omega^2 - \dot{\phi}^2\right)\sin\theta\cos\theta\right]\bar{j}$
$\bar{i}$ is the axis of the disk, $\bar{j}$ is perpendicular to the diagram.

5.38 $\theta = 89.416°,\ \beta = 28.369°,\ |\bar{w}| = 2.4658,\ \dot{\psi} = |\bar{\omega}|\dfrac{\sin\beta}{\sin\theta},\ \dot{\phi} = |\bar{\omega}|\dfrac{\sin(\theta - \beta)}{\sin\theta}$

5.40 (a) $\bar{H}_G = m(0.3147\bar{i} - 0.0906\bar{j} + 0.0315\bar{k})$ kg-m^2/s
(b) $\psi = 0,\ \theta = 84.506°,\ \phi = 163.93°$
(c) $\dot{\psi} = 1.2139$ rad/s, $\dot{\theta} = -0.78386$ rad/s, $\dot{\phi} = -0.097324$ rad/s

6.1 $\Omega = \left(\dfrac{3g}{2L\cos\theta}\right)^{1/2}.$

6.5 $\dot{v} = 0,\quad \bar{F}_{\text{left}} = -\dfrac{mv^2}{6R}\cos\theta\bar{I} + \left(\dfrac{3}{2}mg + \dfrac{mv^2}{6R}\sin\theta\right)\bar{K},$
$\bar{F}_{\text{right}} = \dfrac{mv^2}{6R}\cos\theta\bar{I} + \left(\dfrac{3}{2}mg - \dfrac{mv^2}{6R}\sin\theta\right)\bar{K},\quad \bar{K}$ upward.

6.7 $\bar{F}_A = \left[10g\cos\gamma - 2.5\dot{\psi}^2(\sin\gamma)^2\right]\bar{i} + \left[10g\sin\gamma - 2.5\dot{\psi}^2\sin\gamma\cos\gamma\right]\bar{j} - 2.5\ddot{\psi}\sin\gamma\bar{k}$ N,
$\bar{M}_A = 0.4\ddot{\psi}\cos\gamma\bar{i} + 0.225\ddot{\psi}\sin\gamma\bar{j} + \left(2.5g\sin\gamma - 24\pi\dot{\psi}\sin\gamma - 0.175\dot{\psi}^2\sin\gamma\cos\gamma\right)\bar{k}$ N-m,
$\bar{i}$ along the axis of symmetry, $\bar{k}$ horizontal.

6.10 $\dfrac{1}{3}\ddot{\theta} + \Omega^2\left(\dfrac{1}{2} + \dfrac{1}{3}\cos\theta\right)\sin\theta = \dfrac{g}{2L}\cos\theta.$

6.14 $\omega_1^2 > \dfrac{g/R}{0.4 + 1.25\sin 2\theta}.$

6.15
$$N\left[L(1 + \cos\gamma) - R\sin\gamma\right] = mgL(1 + \cos\gamma) - mR^2\Omega^2\left[\frac{L^2}{R^2}\sin\gamma\right.$$
$$\left. + \left(\frac{1}{4} - \frac{L}{2R}\right)\cos\gamma\right]\sin\gamma,\quad \gamma = \frac{\pi}{6}.$$

6.19 Front wheel drive: $\dot{v} = \mu g\dfrac{L - b}{L + \mu h}$, rear wheel drive: $\dot{v} = \mu g\dfrac{b}{L}$,
all wheel drive: $\dot{v} = \mu g$.

6.22 $\ddot{\theta} = 0.978\dfrac{g}{L}.$

6.25 $\ddot{\phi} = 22.45$ rad/s^2.

6.32 $\omega_A - \omega_B = \dfrac{gL}{R^2\dot{\psi}}.$

6.34 $\Delta N = 63.4$ N, increase at front wheels, decrease at rear wheels.

6.38 $v^2 = \frac{1.7194}{1+\kappa^2/R^2}\left(\frac{FR}{m}\right).$

6.40 $v^2 = \frac{2FR^3}{m\left(R^2+\kappa^2\right)}\left[\theta+\sin\theta-\sqrt{5}+\sqrt{8-2\cos\theta-(\cos\theta)^2}\right].$

6.42 $\max\phi = 37.49°.$

6.44 (a) $\dot{\theta} = 4.3503\sqrt{\frac{g}{L}}$, (b) $\max(\theta) = 133.8°$ above horizontal.

6.52 $\omega_2 = \frac{mvh}{2I_A}$ where point A is the corner where impact occurs.

6.54 $\bar{v}_G = 15.459$ m/s downward, $\omega_2 = 17.441$ rad/s counterclockwise.

6.58 $\left(\bar{v}_B\right)_2 = 0.8589v_1$ at 34.19° below the left direction.

6.63 $P = 1593.2$ J. Power minimized when $\theta = 90°$.

7.1 $M = \frac{1}{2}mgL.$

7.4 Part (a): $Q_1 = -\frac{(FLH+\Gamma H)}{y^2+H^2} - k(y-H) - \frac{mgLH^2}{2\left(y^2+H^2\right)^{3/2}}$

Part (b): $Q_1 = -\left[FL+\Gamma+\frac{kH^2(\tan\theta-1)}{(\cos\theta)^2}+\frac{1}{2}mgL\cos\theta\right]$

7.10 $Q_1 = -\sqrt{2}kb^2\sin\frac{\theta}{2} - F = Q_{\text{cons}} + Q_{\text{nc}}$

7.14 $3m\ddot{u} + m\ddot{x}_m = F$

$m\ddot{u} + m\left[1+4\beta^2x_m^2\right]\ddot{x}_m + 4\beta^2x_m\dot{x}_m^2 + 2\beta mgx_m = 0$

7.18 $\frac{1}{3}mL^2\ddot{\phi} + \frac{1}{2}mL\varepsilon\Omega^2\sin\phi + k\phi = 0.$

7.21 $4mR^2\left(\ddot{\theta}+\omega^2\sin\theta\cos\theta\right) + 2mgR\sin(2\theta+\omega t) = 2FR\sin\theta.$

7.24 $(m_1+m_2)\ddot{x} - m_2(R-r)\left(\ddot{\theta}\cos\theta-\dot{\theta}^2\sin\theta\right) + kx = F,$

$\frac{3}{2}(R-r)\ddot{\theta} - \ddot{x}\cos\theta + g\sin\theta = 0.$

7.25 $m\left[2\ddot{R}_1+\ddot{R}_2-(2R_1+R_2)\dot{\theta}^2\right] + k(R_1-L) - 2mg\cos\theta = 0,$

$m\left[\ddot{R}_1+\ddot{R}_2-(R_1+R_2)\dot{\theta}^2\right] + k(R_2-L) - mg\cos\theta = 0.$

7.30 $(m_1+m_2)\ddot{s} + m_1(R+\varepsilon\cos\theta)\ddot{\theta} - m_1\varepsilon\dot{\theta}^2\sin\theta = (m_1+m_2)g\sin\beta,$

$m_1\left(R^2+\varepsilon^2+\kappa^2+2R\varepsilon\cos\theta\right)\ddot{\theta} + m_1(R+\varepsilon\cos\theta)\ddot{s}$

$-m_1R\varepsilon\dot{\theta}^2\sin\theta = m_1g\left[R\sin\beta+\varepsilon\sin(\beta+\theta)\right].$

7.33 $\left[1+8(\cos\theta)^2\right]\ddot{\theta} - \left(9\Omega^2+8\dot{\theta}^2\right)\sin\theta\cos\theta = \frac{(2mg-4F)}{mL}\sin\theta.$

7.36 $(m_1+m_2)\ddot{z} + \frac{1}{2}m_2L\left(\ddot{\theta}\sin\theta+\dot{\theta}^2\cos\theta\right) + (m_1+m_2)g + kz = F,$

$\frac{1}{3}L\ddot{\theta} + \frac{1}{2}\ddot{z}\sin\theta - \frac{1}{3}L\dot{\psi}^2\sin\theta\cos\theta + \frac{1}{2}g\sin\theta = 0,$

$\frac{1}{3}m_2L^2\left[\ddot{\psi}(\sin\theta)^2+2\dot{\psi}\dot{\theta}\sin\theta\cos\theta\right] = M.$

7.39 $\frac{2}{3}mL^2\ddot{\theta} + \frac{1}{\sqrt{2}}mgL\sin\theta = 0,\quad \frac{1}{3}mL^2\ddot{\psi} = M.$

7.40 $m_1\ddot{s} - m_1\left[\dot{\theta}^2+\dot{\psi}^2(\sin\theta)^2\right]s + ks + m_1g\sin\theta\cos\psi = 0,$

$\left(I_1+m_1s^2\right)\ddot{\theta} + 2m_1s\dot{s}\dot{\theta} - m_1s^2\dot{\psi}^2\sin\theta\cos\theta + m_1gs\cos\theta\cos\psi = 0,$

$\left[I_2+m_1s^2(\sin\theta)^2\right]\ddot{\psi} + 2m_1\dot{\psi}s\left[\dot{s}(\sin\theta)^2+s\dot{\theta}\sin\theta\cos\theta\right] - m_1gs\sin\theta\sin\psi = \Gamma.$

7.41 $$\left[I_p + m\left(L^2 + \tfrac{1}{4}R^2\right)\left(1 + (\cos\beta)^2\right) + 2mL^2\cos\beta\right]\ddot{\psi}$$
$$-2m\sin\beta\left[L^2 + \left(L^2 + \tfrac{1}{4}R^2\right)\cos\beta\right]\dot{\beta}\dot{\psi} + \tfrac{1}{2}mR^2\dot{\phi}\dot{\beta}\sin\beta = \Gamma,$$
$$m\left(L^2 + \tfrac{1}{4}R^2\right)\ddot{\beta} + m\sin\beta\left[L^2 + \left(L^2 + \tfrac{1}{4}R^2\right)\cos\beta\right]\dot{\psi}^2 - \tfrac{1}{2}mR^2\dot{\psi}\dot{\phi}\sin\beta - mgL\cos\beta = 0.$$

8.5 Configuration Constraints:
$L(\sin\theta\sin\psi) - H = 0$
$-\xi + L\cos(\theta) = \eta\cos(\Omega t)$
$L(\cos\psi\sin\theta) = \eta\sin(\Omega t)$
Velocity Constraints:
$L\left(\dot{\theta}\cos\theta\sin\psi + \dot{\psi}\cos\psi\sin\theta\right) = 0$
$\dot{\eta}(\cos\Omega t) - \eta\Omega(\sin\Omega t) = -\dot{\xi} - \left[L\left(\dot{\theta}\sin\theta\cos^2\psi + \dot{\theta}\sin\theta\sin^2\psi\right)\right]$
$\dot{\eta}(\sin\Omega t) + \eta\Omega(\cos\Omega t) = L\left(\dot{\theta}\cos\theta\cos\psi - \dot{\psi}\sin\theta\sin\psi\right)$

8.9 (a) $\bar{v}_B = L\dot{\theta}\dfrac{\cos\theta}{\sin(\theta+\beta)}\left[(\cos\beta)\,\bar{i} + (\sin\beta)\,\bar{j}\right]$
(b): Nonholonomic

8.11 $$(m_1 + m_2)\ddot{x} - m_2\left(\ddot{R} - R\dot{\theta}^2\right)\cos\theta + m_2\left(R\ddot{\theta} + 2\dot{R}\dot{\theta}\right)\sin\theta + 2k_1 x = \lambda_1\sin\theta,$$
$$m_2\left(\ddot{R} - \ddot{x}\cos\theta - R\dot{\theta}^2\right) + k_2(R - R_0) - m_2 g\sin\theta = 0$$
$$m_2\left(R\ddot{\theta} + \ddot{x}\sin\theta +\right) - m_2 g\cos\theta = \lambda_1, \quad \dot{x}\sin\theta + R\dot{\theta} = 0.$$

8.12 $$mR^2\left\{\left[2\left(1+\kappa^2\right) + 2 + 2\sin\theta\right]\ddot{\theta} + (\cos\theta)\dot{\theta}^2 + \frac{5}{2}\left[\sin\phi - \cos(\theta+\phi)\right]\ddot{\phi}\right\}$$
$$+2.5(\cos\phi)\dot{\theta}\dot{\phi} + \sin(\theta+\phi)\dot{\phi}^2 = RN\cos\theta + RF,(1+\sin\theta)$$
$$mR^2\left[6.25\ddot{\phi} + \left[1.25 - 2.5\cos(\theta+\phi)\right]\ddot{\theta} + 2.5\sin(\theta+\phi)\dot{\theta}^2\right] =$$
$$-2.5RN\cos\phi + 2.5RF\sin\phi$$

8.17 $$\{z\} = \begin{bmatrix}\psi & \beta & \dot{\psi} & \dot{\beta}\end{bmatrix}^{\mathrm{T}}, \quad I_1 = m\left[L^2 + \kappa_1^2(\sin\beta)^2 + \kappa_2^2(\cos\beta)^2\right]$$
$$\begin{bmatrix} I_1 & 0 & -c_1 \\ 0 & m\kappa_2^2 & 1 \\ -c_1 & 1 & 0\end{bmatrix}\begin{Bmatrix}\ddot{\psi} \\ \ddot{\beta} \\ \lambda_1\end{Bmatrix} = \begin{Bmatrix} -m\left(\kappa_1^2 - \kappa_2^2\right)\dot{\psi}\dot{\beta}\sin 2\beta + m\kappa_1^2\Omega_1\dot{\beta}\cos\beta + \Gamma \\ \dfrac{1}{2}m\left(\kappa_1^2 - \kappa_2^2\right)\dot{\psi}^2\sin 2\beta + m\kappa_1^2\Omega_1\dot{\psi}\cos\beta \\ c_2\dot{\psi}\end{Bmatrix}$$

8.20 $q_1 = X_C, \quad q_2 = Y_C, \quad q_3 = \theta, \quad \dot{X}_C\sin\theta - \dot{Y}_C\cos\theta = 0$
$$\left(m + \frac{3}{2}m_w\right)\ddot{X}_C + mL\left(\ddot{\theta}\sin\theta + \dot{\theta}^2\cos\theta\right) = \left(\bar{F}_1 + \bar{F}_2\right)\cdot\bar{I} + \lambda_1\sin\theta$$
$$\left(m + \frac{3}{2}m_w\right)\ddot{Y}_C - mL\left(\ddot{\theta}\cos\theta - \dot{\theta}^2\sin\theta\right) = \left(\bar{F}_1 + \bar{F}_2\right)\cdot\bar{J} - \lambda_1\cos\theta$$
$$\left(I + m_1L^2 + \frac{1}{4}m_wR^2\right)\ddot{\theta} + mL\left(\ddot{X}_C\sin\theta - \ddot{Y}_C\cos\theta\right) =$$
$$\left(\bar{r}_{A/C}\times\bar{F}_1 + \bar{r}_{B/C}\times\bar{F}_2\right)\cdot\bar{k}$$

8.22 $$\left[\frac{15}{8}\cos 2\beta + 4\cos\beta + \frac{27}{8}\right]\ddot{\psi} + (\sin\beta)\ddot{\phi} - \left[\frac{15}{4}\sin\beta - 4\cos\beta\right]\dot{\psi}\dot{\beta} + (\cos\beta)\dot{\beta}\dot{\phi}$$
$$= \frac{\Gamma}{mR^2} + (\sin\beta - 2\cos\beta - 1)\lambda_1$$
$$\left[\frac{3}{4}\cos 2\beta - \sin 2\beta + \frac{11}{2}\right]\ddot{\beta} + \left[\frac{15}{4}\sin 2\beta + 4\sin\beta\right]\dot{\psi}^2 - (\cos\beta)\dot{\psi}\dot{\phi}$$
$$-\frac{g}{R}(6\cos\beta - \sin\beta) = -\frac{F}{mR}(2\cos\beta - \sin\beta)$$
$$\ddot{\phi} + (\sin\beta)\ddot{\psi} + (\cos\beta)\dot{\psi}\dot{\beta} = \lambda_1$$

8.29 $\max(\theta) = 80.73^\circ$ when $t = 1.995$ s.

8.31 $X_G = -111.2$ m, $Y_G = -70.0$ m, speed $= 19.73$ m/s, $\theta = 56.2^\circ$ at $t = 60$ s.

8.33 θ passes 89° when $t = 9.28$ s.

8.40
$$m\ddot{X}_B + \frac{1}{2}mL\left[(\cos\phi)\,\ddot{\phi} - (\sin\phi)\,\dot{\phi}^2\right] = Q_1$$
$$m\ddot{Y}_B + \frac{1}{2}mL\left[\ddot{\phi}\sin\phi + (\cos\phi)\,\dot{\phi}^2\right] - mg = Q_2$$
$$\frac{1}{3}mL^2\ddot{\phi} + \frac{1}{2}mL\,(\cos\phi)\,\ddot{X}_B + \frac{1}{2}mL\,(\sin\phi)\,\ddot{Y}_B - \frac{1}{2}mgL\,(\sin\phi) = Q_3$$
with $\{Q\} = [\sigma]\begin{Bmatrix} N_A \\ N_B \end{Bmatrix} - \mu\,[\gamma]\begin{Bmatrix} f_A \\ f_B \end{Bmatrix}$
$$[\sigma] = \begin{bmatrix} -\cos\phi & 1 \\ -\sin\phi & 0 \\ -\left(H^2 + Y_B^2\right)^{1/2} & 0 \end{bmatrix}, \quad [\gamma] = \operatorname{sgn}\left(\dot{\phi}\right)\begin{bmatrix} \sin\phi & 0 \\ -\cos\phi & -1 \\ 0 & 0 \end{bmatrix}$$

9.1
$$\dot{\psi} = \frac{p_\psi}{m\left(3R^2 - RL + \frac{5}{4}L^2 + \frac{I}{m}\right)}$$
$$\frac{3}{2}m\ddot{R} = \left[\frac{6R - L}{m\left(3R^2 - RL + \frac{5}{4}L^2 + \frac{I}{m}\right)^2}\right]p_\psi^2$$

9.3
$$\mathcal{R} = +\frac{1}{2}\left(\frac{3}{2}\right)m_1\,(R-r)^2\,\dot{\theta}^2 - \frac{m_1\,(R-r)\,\dot{\theta}p_x\cos\theta}{(m_1+m_2)} - \frac{1}{2}\frac{p_x^2}{(m_1+m_2)} + m_1 g\,(R-r)\cos\theta$$

9.7 Both ψ and ϕ are ignorable
$$p_\psi = \left\{mL^2\,(1+\cos\beta)^2 + mR^2\left[\frac{1}{4}(\sin\beta)^2 + \frac{1}{2}(\cos\beta)^2\right]\right\}\dot{\psi} + \frac{1}{2}mR^2\dot{\phi}\cos\beta$$
$$p_\phi = \frac{1}{2}mR^2\left(\dot{\phi} + \dot{\psi}\cos\beta\right)$$
$$\begin{Bmatrix} p_\psi \\ p_\phi \end{Bmatrix} = [D]\begin{Bmatrix} \dot{\psi} \\ \dot{\phi} \end{Bmatrix} \to \begin{Bmatrix} \dot{\psi} \\ \dot{\phi} \end{Bmatrix} = [D]^{-1}\begin{Bmatrix} p_\psi \\ p_\phi \end{Bmatrix}$$
$$\mathcal{R} = (T-V) - \begin{bmatrix} p_\psi & p_\phi \end{bmatrix}\begin{Bmatrix} \dot{\psi} \\ \dot{\phi} \end{Bmatrix} = T - V - \begin{bmatrix} p_\psi & p_\phi \end{bmatrix}[D]^{-1}\begin{Bmatrix} p_\psi \\ p_\phi \end{Bmatrix}$$

9.12
$$\mathcal{M} = m\left[\frac{1}{3}L^2 + 2\frac{R^2\,(\cos\theta)^2}{(\sin\theta)^4} - RL\cot\theta\right]$$
$$\dot{\theta} = \frac{p_\theta}{\mathcal{M}}$$
$$\dot{p}_\theta - \frac{p_\theta^2}{2\mathcal{M}^2}\left[LR\left(\cot^2\theta + 1\right) - 4R^2\frac{\cos\theta}{\sin^5\theta}\left(1 + 2\,(\cos\theta)^2\right)\right] - \frac{\cos\theta}{(\sin\theta)^2}\left[kR^2\left(\frac{1}{\sin\theta} - 2\right) + 2mgR\right] + mgL\sin\theta = F\,(L - R\cot\theta)$$

Note: The angle θ mentioned in the problem statement is the same as the angle ψ in Figure P.9.12.

9.14
$$\dot{\psi} = \frac{p_\psi}{mR^2\dot{\psi}\left[\left(\frac{3}{2} - \frac{8}{3\pi}\cos\psi\right)\right]}$$
$$\dot{p}_\psi - \frac{p_\psi^2}{2mR^2\left(\frac{3}{2} - \frac{8}{3\pi}\cos\psi\right)^2}\left(\frac{8}{3\pi}\sin\psi\right) + \frac{4}{3\pi}mgR\sin\psi = 0$$

9.18
$$\dot{\psi} = \frac{3p_1}{mL^2}, \quad \dot{\theta} = \frac{3p_1}{2mL^2}$$
$$\dot{p}_1 = M, \quad \dot{p}_2 + \frac{\sqrt{2}}{2}mgL\sin\theta = 0$$

9.22
$$mR^2\ddot{\gamma}\left(\frac{3}{2} - \frac{8}{3\pi}\cos\psi\right) + \dot{\gamma}^2\left(\frac{4R}{3\pi}\right)(\sin\psi) + \frac{4R}{3\pi}mg\sin\psi = 0, \quad \dot{\gamma} = \dot{\psi}$$

9.24 $$\left[1+\sin\left(\frac{1}{4}\pi-\theta\right)^{2}\right]\ddot{\psi}-\cos(2\theta)\,\dot{\theta}\dot{\psi}=\frac{3M}{mL^{2}}$$
$$\frac{1}{3}\ddot{\theta}-\frac{2}{3}\dot{\psi}^{2}\cos(2\theta)+\frac{\sqrt{2}}{2}mgL\sin\theta=0$$

9.28 $$I_2\ddot{\beta}+I_1\dot{\psi}\Omega\cos\beta+(I_2-I_1)\,\dot{\psi}^{2}(\sin\beta)(\cos\beta)=0$$
$$\left(mL^{2}+I_1\right)\ddot{\psi}+2I_1\dot{\psi}\dot{\beta}(\cos\beta)\sin\beta-I_2\Omega\dot{\beta}\cos\beta-(I_1-I_2)\,\Omega\dot{\beta}\cos\beta=M$$

9.31 $$\dot{\gamma}_1=\dot{R},\quad \dot{\gamma}_2=\dot{\theta},\quad \dot{\psi}=\frac{1}{r}\dot{\gamma}_1,\quad \dot{\phi}=-\frac{R}{r}\dot{\gamma}_2$$
$$\frac{7}{5}\left(\ddot{\gamma}_1-\dot{\gamma}_2^{2}\right)-\frac{2}{5}R\Omega\dot{\gamma}_2=0$$
$$\frac{7}{5}\left(R\ddot{\gamma}_2+2\dot{\gamma}_1\dot{\gamma}_2\right)+\frac{2}{5}\Omega\dot{\gamma}_1=0$$

9.35 $$F\frac{d}{dx}\left[\frac{w'}{1+(w')^{2}}\right]-\mu\ddot{w}-\mu g=0$$

9.38 $$F\left[\left(\frac{\partial^{2}u}{\partial x^{2}}\right)-\frac{3}{2}\left(\frac{\partial u}{\partial x}\right)^{2}\left(\frac{\partial^{2}u}{\partial x^{2}}\right)-\left(\frac{\partial^{2}u}{\partial x^{2}}\right)\left(\frac{\partial w}{\partial x}\right)^{2}-\left(\frac{\partial u}{\partial x}\right)\left(\frac{\partial w}{\partial x}\right)\left(\frac{\partial^{2}w}{\partial x^{2}}\right)\right]-\mu\ddot{u}=0$$
$$F\left[\left(\frac{\partial^{2}w}{\partial x^{2}}\right)-\frac{3}{2}\left(\frac{\partial w}{\partial x}\right)^{2}\left(\frac{\partial^{2}w}{\partial x^{2}}\right)-\left(\frac{\partial^{2}w}{\partial x^{2}}\right)\left(\frac{\partial u}{\partial x}\right)^{2}-\left(\frac{\partial w}{\partial x}\right)\left(\frac{\partial u}{\partial x}\right)\left(\frac{\partial^{2}u}{\partial x^{2}}\right)\right]-\mu\ddot{w}$$
$$-\mu g=0$$

9.42 $$\frac{1}{2}\mu L^{2}\ddot{q}_1+\frac{\pi^{2}}{2}FLq_1-\frac{\pi^{4}}{8}FL\left(\frac{3}{2}q_1+12q_1q_2^{2}\right)+\frac{2}{\pi}\mu gL^{2}=0$$
$$\frac{1}{2}\mu L^{2}\ddot{q}_2+\frac{\pi^{2}}{2}FL\,(4q_2)-\frac{\pi^{4}}{8}FL\left(12q_1^{2}q_2+24q_2^{3}\right)=0$$