

Unconstrained: θ Constrained: x, y Constr. eq: $x^2 + y^2 = R$

$$T = \frac{1}{2} I_0 \dot{\theta}^2 \quad I_0 = mL^2$$

$$V = -mgL \cos \theta$$

Lagrange: unconstrained:

$$mL^2 \ddot{\theta} + mgL \sin \theta = 0$$

Lagrange: constrained

$$T = \frac{1}{2} m \dot{x}^2 + \frac{1}{2} m \dot{y}^2 \quad V = -mgy$$

$$\frac{d}{dt} \left(\frac{\partial T}{\partial \dot{q}_n} \right) - \frac{\partial T}{\partial q_n} + \frac{\partial V}{\partial q_n} = Q_n + \sum_{j=1}^J a_{jn} \lambda_j$$

$$\sum_{n=1}^N a_{in} \dot{q}_n \quad (8.1.6)$$

$$x^2 + y^2 = R \rightarrow 2x\dot{x} + 2y\dot{y} = 0$$

Constr. eq.

$$q_1 = x, \quad q_2 = y \quad a_{11} = x, \quad a_{12} = y$$

$$a_{11} \dot{q}_1 + a_{12} \dot{q}_2 = 0$$

$$m\ddot{x} + 0 = x\lambda_1$$

$$m\ddot{y} - mg = y\lambda_1$$

$$= \begin{bmatrix} m & 0 \\ 0 & m \end{bmatrix} \begin{bmatrix} \ddot{x} \\ \ddot{y} \end{bmatrix} = \begin{bmatrix} 0 \\ mg \end{bmatrix} + [x, y]^T \lambda$$

derivative of constr. eq:

$$[0] \{ \ddot{q} \} + \left[\frac{da}{dt} \right] \{ \dot{q} \} + \frac{db}{dt} = 0 \rightarrow \dot{x}^2 + x\ddot{x} + \dot{y}^2 + y\ddot{y} = 0$$

$$\rightarrow [x \ y] \begin{bmatrix} \ddot{x} \\ \ddot{y} \end{bmatrix} + [\dot{x} \ \dot{y}] \begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} = 0$$

$$x = R \sin \theta$$

$$y = R \cos \theta$$

$$\frac{x}{y} = \tan \theta \rightarrow \text{atan2} \rightarrow$$

Augmented Method