A non-parametric Iwan element derived from measurements of amplitude-dependent frequency and damping

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Abstract

Mechanical fasteners are a significant source of nonlinear damping and stiffness variation in built-up structures. Hysteretic models are required to simulate this observed dynamic behavior. One popular constitutive model is the Iwan model, which consists of many Jenkins elements in parallel, each of which consists of a spring and Coulomb slider in series. The slip forces of the Jenkins elements are described by a distribution function that determines the hysteretic force-displacement relationship of the joint. The most common Iwan model is Segalman's four parameter model, where the four parameters define the distribution function. These parameters are generally tuned, using intensive optimization methods, to fit the nonlinear frequency and damping that is experimentally observed. This paper proposes an alternative, non-parametric model that is derived from the experimental data. Given a set of measurements of the effective natural frequency and damping ratio of a mode as a function of vibration amplitude, a method is proposed to derive the corresponding distribution function for a modal Iwan element. Identifying the non-parametric model using the proposed approach is more computationally efficient than using traditional, parametric models since the optimization procedure typically required to estimate the parameters is eliminated. Moreover, the non-parametric model offers more flexibility in fitting the observed nonlinear dynamics since it is not limited to a finite set of parameters. The proposed method will be tested on a simple numerical case study followed by experimental data from a benchmark structure with two bolted joints, which is commonly known as the S4 Beam.

Keywords: Nonlinear dynamics, Bolted joints, hysteresis, Iwan model

1 Introduction

Built-up structures assembled using mechanical fasteners exhibit energy dissipation and loss of stiffness due to friction at the contact surfaces [1]. The dynamic response of a structure subjected to a harmonic or random forcing environment is limited primarily by the damping [2]. Thus, it is important to estimate the energy dissipation, or damping, due to friction at the interfaces. Typically, jointed structures exhibit weakly nonlinear behavior since the stiffness of the joint decreases only slightly with increase in vibration amplitude, while there is significant energy loss, leading to an increase in damping [3]. Correspondingly, the vibration modes of the structure do not change significantly due to friction. Therefore, the structure can be assumed to behave quasi-linearly about a particular mode of interest with a suitable hysteretic model that can capture the effect of friction on the dynamics of that mode. This is known as the modal modeling approach, first proposed in the context of jointed structures by Segalman [4].

While several hysteretic models exist, the Iwan model [5] is commonly used for bolted joint nonlinearity due to its convenient physical interpretation. In the Iwan model, the hysteretic system can be understood as a combination of a large number of ideal elastoplastic elements, also known as Jenkins elements [6], with a distribution function describing the strength of these elements. The constitutive relationship of the Iwan model is defined by this distribution function. There are numerous adaptations of the Iwan model [7, 8, 9, 10]. However, all of these models rely on a fixed number of parameters that define the distribution function. Therefore, they are limited in their ability to capture the hysteretic behavior of a joint. There are some cases where the nonlinearity due to bolted joints cannot be well-represented by the parametric Iwan model [11], thus motivating the development of a more flexible Iwan model. Furthermore, even in systems that can be represented by a parametric form of the Iwan model, the identification of parameters is non-trivial. Typically, the parameters need to be tuned to fit the nonlinear response obtained experimentally using computationally expensive optimization schemes.

This paper proposes a non-parametric form of the Iwan model that can be derived directly from experimental measurements. Segalman and Starr [12] presented a relationship between the force-displacement backbone curve and the distribution function for a Masing model. They applied this relationship to invert different Masing models and represent them as equivalent Iwan models. In this paper, the relationship is extended to extract the non-parametric Iwan model from the dynamic response. The paper shows how the backbone curve of a Masing model can be approximated from the amplitude-dependent frequency obtained experimentally. The inversion relationship can then be utilized to fit a distribution function that best captures the experimental measurements, giving us a modal Iwan model that can be numerically integrated to obtain the system's dynamic response to different loading conditions. The following section derives the relationship between the distribution function and the amplitude-dependent frequency, followed by an implementation methodology. Then, a simple numerical case study comprising a single degree-of-freedom system with a four-parameter Iwan model of known parameters is considered. The conference presentation will include findings from experimental data obtained from impact testing of the S4 Beam.

2 Theory

The restoring force due to an Iwan element can be written as

$$f_{\mathrm{nl}}(x,t,\tilde{\phi}) = \int_0^\infty k\tilde{\rho}(\tilde{\phi})[x(t) - \tilde{x}_{\mathrm{s}}(t,\tilde{\phi})]d\tilde{\phi} \tag{1}$$

where $x(t)$ is the imposed displacement, $\tilde{x}_{\mathrm{s}}(t)$ is the current displacement of the sliders, k is the stiffness of each Jenkins element, and $\tilde{\rho}(\tilde{\phi})$ is the population density of sliders having strength $\tilde{\phi}$. Note that $\tilde{\phi}$ here has units of force. Equation 1 can be written as the sum of the forces due to sliders that have slipped and the forces due to the sliders that remain stuck, given by Eq. 2.

$$f_{\mathrm{nl}}(x,t,\tilde{\phi}) = \int_0^{kx} \tilde{\phi}\tilde{\rho}(\tilde{\phi})d\tilde{\phi} + kx\int_{kx}^\infty \tilde{\rho}(\tilde{\phi})d\tilde{\phi} \tag{2}$$

Here, the first term on the right hand side is the force due to the sliders that have slipped when the imposed displacement is x, and the second term is the force due to the sliders that are stuck. Segalman [7] showed how the stiffness k can be eliminated from Eq. 2 by a change of variables,

$$\begin{aligned} \phi &= \frac{\tilde{\phi}}{k} \\ \rho(\phi) &= k^2\tilde{\rho}(k\phi) \end{aligned} \tag{3}$$

where ϕ is now the displacement at which $\rho(\phi)$ number of sliders slip. Due to this change of variables, Eq. 2 can be rewritten as

$$f_{\mathrm{nl}}(x,t,\phi) = \int_0^x \phi\rho(\phi)d\phi + x\int_x^\infty \rho(\phi)d\phi. \tag{4}$$

Thus, $\rho(\phi)$, known as the distribution function, completely characterizes the Iwan model. Two derivatives of the nonlinear force with respect to x ultimately results in Eq. 5, thus giving a relation between the distribution function and the force-displacement data. Iwan [13] presented a similar result for the distribution function in terms of the stress-strain curve.

$$\frac{\partial^2 f_{\mathrm{nl}}}{\partial x^2} = -\rho(x) \quad \rightarrow \quad \rho(\phi) \quad = -\left.\frac{\partial^2 f_{\mathrm{nl}}}{\partial x^2}\right|_{x=\phi} \tag{5}$$

Therefore, if the force-displacement relation is known, the underlying distribution function that defines the corresponding Iwan model can be calculated using Eq. 5. This relationship is applicable in the modal domain as well, with modal displacement amplitude q replacing the physical displacement x in Eq. 5. A dynamic test, however, gives us amplitude-dependent frequency and damping measurements. To estimate the force-displacement backbone curve, we exploit the secant approximation. The secant stiffness K_{sec} can be calculated from the backbone curve as

$$K_{\mathrm{sec}} = \omega_{\mathrm{n}}^2(q) = \frac{f_{\mathrm{nl}}(q,\phi)}{q} \tag{6}$$

Therefore, the nonlinear force can be estimated if the natural frequency $\omega_{\mathrm{n}}(q)$ is known as a function of modal displacement amplitude q, as shown in Eq. 7.

$$f_{\mathrm{nl}}(q,\phi) = \omega_{\mathrm{n}}^2(q) \times q \tag{7}$$

3 Implementation

Consider the natural frequency obtained for discrete values of displacement amplitude, shown in Fig. 1a. This data could either be obtained from impact hammer tests (i.e. transient data) [14] or controlled steady-state testing methods such as force appropriation [15]. The corresponding restoring force backbone curve can be estimated using Eq. 7, and the result of applying this equation to the data in Fig. 1a is shown in Fig. 1b.

In order to estimate the distribution function of the underlying Iwan model, a cubic spline is first fit to the points shown in Fig. 1b. Fitting a cubic spline allows us to smooth over any noise while ensuring that the second derivative is continuous.

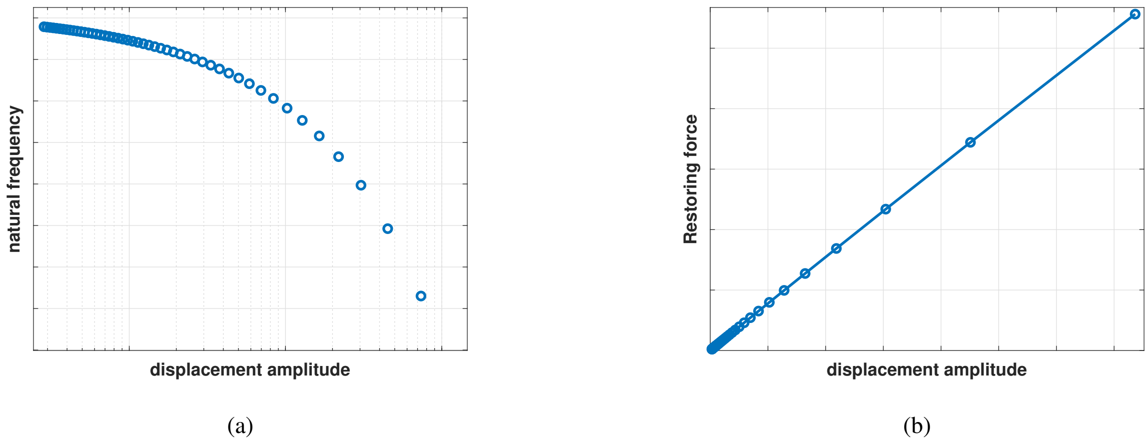


Figure 1: (a) Frequency as a function of amplitude, from experimental measurement, and (b) Restoring force as a function of amplitude calculated from (a)

A cubic spline comprises piece-wise cubic functions, but the resultant distribution function for the Iwan model is still non-parametric because there can be as many segments or knots as one desires. Once a cubic spline is fit, the second derivative of force with respect to displacement amplitude can be evaluated for any slider strength ϕ ranging from zero to the maximum displacement amplitude measured. For future numerical computations, this range is divided into a set of discrete values at which the corresponding slider distribution, $\rho(\phi)$ is calculated. Segalman [7] proposed breaking up the interval $(0, \phi_{\max})$ into N intervals with the lengths forming a geometric series. Therefore, the spacing between consecutive values of ϕ can be given by Eq. 8.

$$\triangle\phi_{\mathrm{m+1}} = \alpha\triangle\phi_{\mathrm{m}},\ \forall\ (m+1) < N \tag{8}$$

A value of N between 30 and 100 is typically sufficient to capture the nonlinearity in bolted joints. In Eq. 8, $\alpha = 1$ results in uniform spacing between consecutive points while $\alpha > 1$ results in the density of points decreasing as ϕ increases. The value of α must be chosen such that the density of points is higher in regions where the distribution function is changing rapidly. Generally, α is chosen to be slightly greater than 1 (typically 1.2) to ensure that the low-amplitude nonlinearity is accurately captured. Once the geometric series ϕ is formed, the distribution of sliders for each element of ϕ can be determined by evaluating the second derivative of the cubic spline.

The distribution function must be truncated at some slider displacement, ϕ_{tmax}. In theory, this point of truncation would correspond to the displacement at which all the sliders forming the Iwan model have slipped, i.e. at the initiation of macroslip. However, it is often difficult, if not impossible, to experimentally obtain macroslip measurements. Joints are typically designed to maintain integrity, meaning that they are expected to always operate in the microslip regime. Therefore, the macroslip displacement is challenging to determine and may not even be essential. Instead, the maximum displacement amplitude measured can be set as the truncation point ϕ_{tmax}. However, when this is done the Iwan system reproduces the underlying nonlinear behavior only up to some maximum force [12]. The coefficient of the corresponding stiffness term, K, is chosen such that the total stiffness of the system equals the measured low-amplitude, linear stiffness. Thus, K must satisfy the equation

$$\left.\frac{df}{dq}\right|_{q=0} = K_0 = \int_0^{\phi_{\mathrm{tmax}}} \rho(\phi)d\phi + K, \tag{9}$$

where K_0 is the low-amplitude stiffness (or linear natural frequency of the mode squared) and the integral term is the stiffness due to all the Jenkins elements of the Iwan model. In this way, the discrete distribution function, along with the truncation point ϕ_{tmax} and the stiffness K completely define the non-parametric Iwan model. Note that this model is only applicable in the amplitude range that has been measured; the nonlinear behavior cannot be extrapolated beyond the truncation point.

4 Numerical Case Study: Four-parameter Iwan Model

A single degree-of-freedom (SDOF) system with a linear spring, linear viscous damper and a four-parameter Iwan model has been used as a test case in this paper. The four-parameter Iwan model [7] is commonly used in bolted joint dynamics. The model is defined by a power-law distribution function, given by Eq. 10.

$$\rho(\phi) = R\phi^{\chi}[\mathrm{H}(\phi) - \mathrm{H}(\phi - \phi_{\max})] + S\delta(\phi - \phi_{\max}) \tag{10}$$

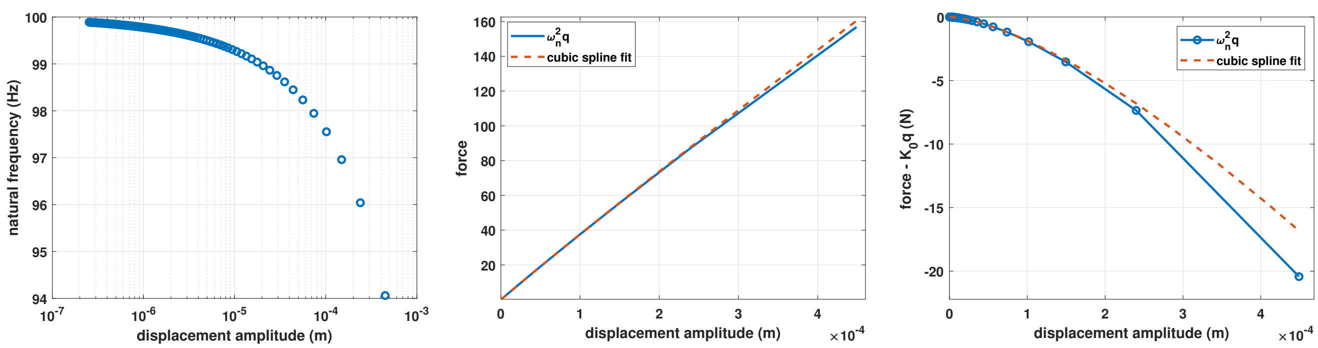
The four-parameter model can thus be represented by the parameter set $[\phi_{\max},\ \chi,\ R,\ S]$,where $\phi_{\max}$ is the displacement at which all sliders slip (i.e. macroslip occurs), χ is a dimensionless quantity that determines the power-law energy dissipation,

and R and S can be understood as the stiffness of the power-law portion of the distribution and the delta function portion of the distribution respectively. Since the parameters R and S have fractional dimension, Segalman proposed using another set of more intuitive parameters, $[F_{\mathrm{S}}, K_{\mathrm{T}}, \chi, \beta]$, with F_{S} being the force required to cause macroslip, K_{T} being the tangential stiffness of just the joint at small applied loads, and χ and β being dimensionless parameters that determine the energy dissipation. Since the true solution of the four-parameter model is known, it has been used as a test case here.
The linear and nonlinear parameters of the SDOF system can be found in Table 1. These parameters results in a linear, low-amplitude natural frequency, f_{n0} of 100 Hz, representative of a nonlinear mode of a bolted structure.

Table 1: Properties of the nonlinear SDOF system used

Parameter	Value
Mass (m)	1 kg
Linear spring stiffness (K_{lin})	1.42×10^5 N/m
Material damping (ζ_{lin})	1×10^{-4}
Iwan joint $[F_{\mathrm{s}}, K_{\mathrm{T}}, \chi, \beta]$	[400 N,2.53×10^5 N/m,-0.5,1]

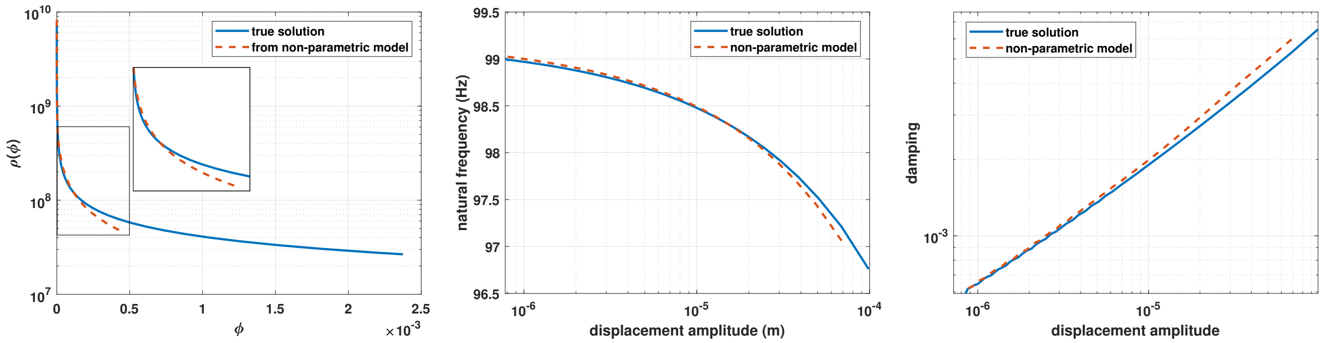
The system's response to an impulsive fore of 200 N amplitude was simulated. The system is in the microslip regime at this amplitude. The impulsive force was applied as one half cycle of a sinusoid with a width equal to the time period of linear oscillation, i.e. $1/f_{n0}$. The time response for a simulation period of 10 s was obtained, which was then processed using the Hilbert transform, as described in [16], to estimate the amplitude-dependent damping and natural frequency. This is the simulation equivalent of performing an impact hammer test and extracting the amplitude-dependent parameters for a particular nonlinear mode.



(a) (b) (c)

Figure 2: (a) Frequency vs displacement amplitude obtained from the Hilbert transform of an impulsively excited response for a system with 4-parameter Iwan model, (b) Restoring force vs. amplitude calculated from (a), and (c) Nonlinear restoring force backbone curve, obtained by subtracting the linear force term $K_0 q$, showing some error in the cubic spline fit at high amplitudes

Figure 2 shows the amplitude-dependent natural frequency, estimated by post-processing the time response using the Hilbert transform. The solid line in Fig. 2b shows the resultant backbone curve, while the dashed line shows the cubic spline fitting the backbone curve. The cubic spline is inaccurate at high amplitudes; the inaccuracy is more pronounced when viewing the purely nonlinear component of the restoring force, shown in Fig. 2c. This inaccuracy is due to insufficient data points when trying to fit a spline at higher amplitudes. Since the system damping is high at high amplitudes, the amplitude decays rapidly during these initial cycles resulting in fewer amplitude points. Additional work needs to be done to improve the robustness of the spline fitting tool used. Regardless, the distribution function can be calculated by taking two derivatives of the cubic spline. Figure 3a shows the resultant distribution function for the non-parametric Iwan model, given by a dashed line, compared to the true distribution function for the four-parameter model being fit. As expected, the distribution function cuts off at the highest displacement amplitude that was simulated (i.e. the maximum displacement amplitude in Fig. 2a). Other than that, the distribution function appears to follow the true solution well. For a better comparison, the response of the non-parametric Iwan model to a sine beat force input was simulated. The sine beat has a force amplitude of 50 N, centered at 100 Hz. The response for the four-parameter as well as non-parametric model was simulated using the Newmark-β integration scheme. The response was then post-processed using Hilbert transform to get the amplitude-dependent frequency and damping, shown in Fig. 3b and 3c respectively. The non-parametric model is, again, less accurate at higher amplitudes. Overall, the frequency estimate has a mean error of 0.04% and the damping has a mean error of 4.86%.



(a) (b) (c)

Figure 3: (a) Distribution function, (b) amplitude-dependent frequency, and (c) amplitude-dependent damping for the non-parametric Iwan model (dashed line) compared to the true solution (solid line)

5 Conclusions

This paper proposed a non-parametric form of the Iwan model in which the distribution function is directly extracted from the amplitude-dependent frequency measurements. The backbone curve is estimated from the frequency by utilizing the secant approximation. The distribution function can then be calculated from the second derivative of the restoring force with respect to displacement. The proposed modeling approach offers two main advantages over the existing methods. Firstly, the accuracy of the model is not limited to the definition of the distribution function by a finite set of parameters. Secondly, it eliminates the need for computationally intensive optimization schemes, typically used for parameter identification in the standard Iwan element. A simple case study comprising the four-parameter Iwan model was presented as a proof of concept. While the resultant distribution function closely matched the true solution, it is limited by the effectiveness of the curve-fitting algorithm used to fit the backbone curve. The amplitude-dependent frequency and damping estimated by the non-parametric Iwan model closely matches the true solution, with a mean error of 0.04% in frequency and 4.86% in damping.
The conference presentation will also show results from testing the proposed modeling approach on experimental measurements of the S4 Beam.

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