Evaluating New Nonlinear System Identification Methods on Curved Beams

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ABSTRACT
This work explores the application of recent nonlinear identification methods to measurements from structures where traditional methods have failed. Specifically, curved beams that exhibit geometric nonlinearity, softening-hardening and snap-through behavior and strong modal coupling. System identification is performed using steady state input-output measurements, using two recently presented methods, the inverse SNRM-based algorithm by Kwarta and Allen and the sparse identification method by Breunung et al. The latter produces excellent results and new insights regarding what terms are needed to model a structure such as this.

Keywords: Nonlinear System Identification, Geometric Nonlinearity, Sine Testing, Snap Through, Nonlinear Feedback

1 INTRODUCTION

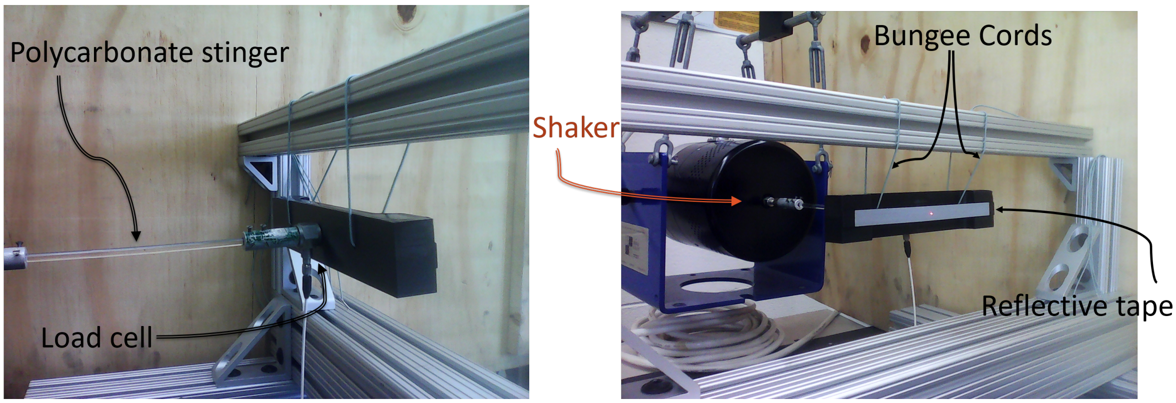
While many algorithms and approaches have been proposed, nonlinear system identification continues to prove challenging for many structures and systems. In particular, geometrically nonlinear structures prove challenging, especially when snap through or softening-hardening nonlinearities are present, as commonly occurs in thin curved panels. Structures with bolted interfaces also prove challenging, as the hysteretic nonlinear functions that govern their behavior are not well known, and these can change the damping markedly and in a manner which depends on the amplitude of several different vibration modes. This work compares the performance of two recently presented system identification methods, in order to understand how these challenges may be addressed.

Breunung et al. [1; 2] present a new system identification method that relies on a steady state measurements. These inputs are transformed into the frequency domain and linear and nonlinear terms are modeled with a set of library functions. Subsequently, spurious coefficients are removed and an iterative selection procedure retains only relevant terms of the equations of motion (EOM). This sparse regression increases the robustness and simplicity of the obtained governing equations. Breunung et al. [1; 2] studied single degree-of-freedom oscillators as well as structures with multiple active modes, including internally resonant systems. For the structures studied the method was found to yield low order models that capture the underlying physics well. This work applies that method to the curved, geometrically nonlinear beams that were tested as part of the study in [3]. The samples were curved beams, 3D printed from PLA onto a massive, relatively stiff backing so that a base excited condition could be approximated. The results are compared to those obtained by the inverse SNRM-based method proposed by Kwarta and Allen in [3]. SNRM is an abbreviation for the Single Nonlinear Resonant Mode method introduced by Szemplińska-Stupnicka in [4].

2 METHODS

2.1 EXPERIMENTAL MEASUREMENTS

Figure 1 shows the experimental setup from [3]. The shaker was driven sinusoidally using a Labview program that allowed the amplitude and frequency of the force applied to the structure to be adjusted until the structure was near quadrature, as observed on a Lissajous figure [5; 6; 7]. The voltage signal sent to the shaker underwent a calibration procedure that utilized low-amplitude measurements and a linearized model. This procedure converted the units of the input signal from volts to newtons. The oscillations of the beam's center point were then measured using a PSV-400M Scanning Vibrometer.



(a) Back view. **(b)** Front view.

Fig. 1: Photographs of the experimental setup reported in [3].

The measurements used in this work were steady-state measurements from the "flat" and "curved" beams mentioned in [3]. Both exhibited some curvature due to thermal stresses during the manufacturing process, but the flat beam showed only hardening behavior while the curved beam had enough curvature to exhibit significant softening and hardening.

2.2 SYSTEM IDENTIFICATION METHODS

The inverse SNRM-based method treats the response near a resonance as that from a linear system, whose natural frequency and damping are allowed to change with vibration amplitude. Specifically, the vibration velocity is approximated as presented in Eq. (1) and (2).

$$v(t) = \mathrm{Re}\{\mathbf{V}e^{i\Omega t}\} \tag{1}$$

$$\mathbf{V} = \frac{\Omega\varphi_j\varphi_j^T\mathbf{F}}{\tilde{\omega}_{0,j}^2 - \Omega^2 + 2i\tilde{\zeta}_j\tilde{\omega}_{0,j}\Omega} + \sum_{\substack{k=1\\k\neq j}}^{N_{lin}} \frac{\Omega\varphi_k\varphi_k^T\mathbf{F}}{\omega_{0,k}^2 - \Omega^2 + 2i\zeta_k\omega_{0,k}\Omega} \tag{2}$$

Because the mode shape of the nonlinear mode, φ_j is assumed to be independent of vibration amplitude, the amplitude dependent natural frequency $\tilde{\omega}_{0,j}$ and damping ratio $\tilde{\zeta}_j$ can be identified from the magnitude and phase of a single steady-state vibration measurement $\mathbf{V}$. All other modes are treated as linear, including rigid body modes if applicable; for near-resonant measurements the summation on the right typically contributes little to the sum. Regarding Eq. (2), the remaining quantities should be considered as follows: Ω represents the forcing frequency, $\mathbf{F}$ signifies the distribution of the steady-state sinusoidal excitation, j is the index of the mode under consideration, and N_{lin} denotes the number of relevant linear modes.

2.3 ROBUST IDENTIFICATION OF NONLINEAR OSCILLATORS

Breunung et al. [1; 2] fit the model presented in Eq. (3) to the steady state response measurements. In Eq. (3), the function $N(q,\dot{q})$ is used to model nonlinearities and a polynomial expansion of $N(q,\dot{q})$ is utilized. Subsqeuently, the acceleration $\ddot{q}$, velocities $\dot{q}$ and position q are expanded into Fourier series and equality is imposed for the first N_H harmonics. This procedure yields a set of linear equations and before obtaining a least squares solution, spurious terms below the noise floor are removed and the conditioning of the linear system is increased. Finally, an iterative procedure includes only the most dominant terms in Eq. (3). This yields simple and interpretable equations which accurately capture the measured frequency response.

$$\ddot{q} + c\dot{q} + kq + N(q, \dot{q}) = f(t) \tag{3}$$

3 RESULTS

3.1 INVERSE SNRM-BASED METHOD

The inverse SNRM-based method is applied by taking a set of near-resonant steady-state measurements and using those to identify a model for the amplitude dependent natural frequency and damping. The details are reported in [3], but in summary a set of approximately thirty measurements were taken near resonance, such that the phase of the velocity was typically about 20 degrees away from being in phase with the force. These were used to estimate the response at resonance, and then a model was fit to the natural frequency versus amplitude, which is shown with a black dashed line in Fig. 2. The same was done for the damping, as shown in [3]. These models were then used with Eq. (2) to reconstruct the steady-state response velocity at three forcing levels, and those are compared with steady-state measurements at those levels.

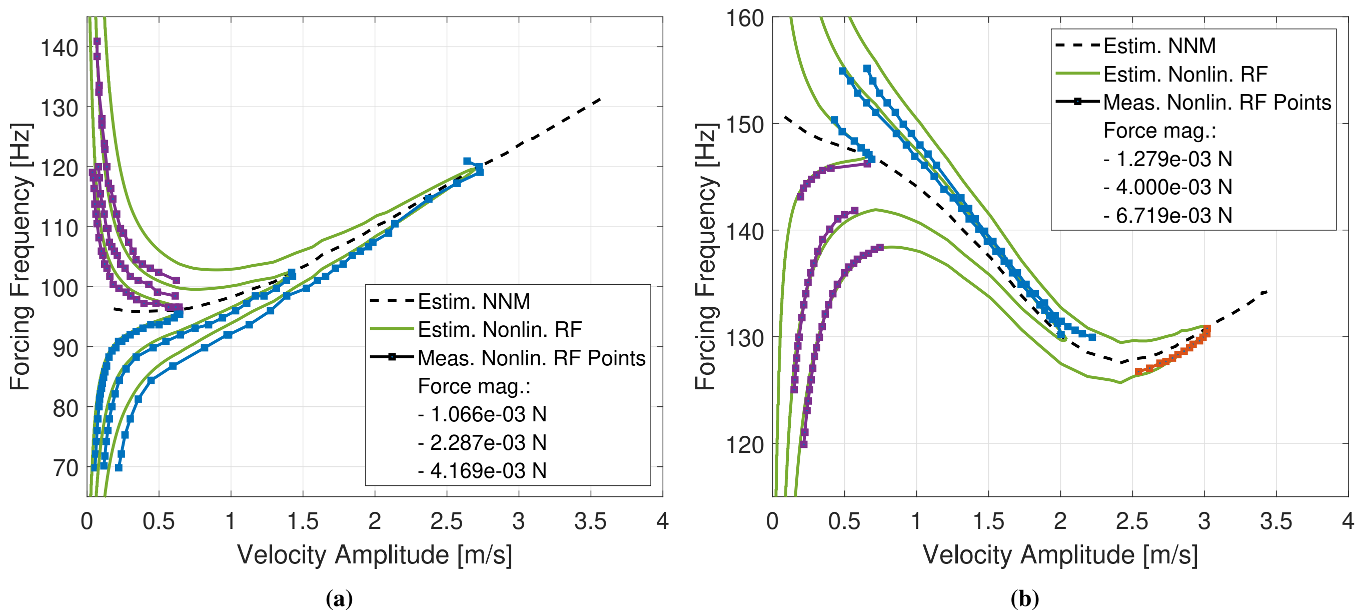


Fig. 2: Results obtained using the inverse SNRM-based method for the **(a)** flat and **(b)** curved beams presented in [3].

The results obtained by the inverse SNRM-based method do an excellent job of enveloping the Nonlinear Normal Mode (NNM) measurements (i.e. the black dashed line in Fig. 2). The method is efficient and conceptually simple. Moreover, the inverse SNRM-based method also did very well at reconstructing many of the steady-state response measurements, some of which are quite far from the resonant data that was used to identify the SNRM model. However, some of the measurements show noticeable discrepancies, especially those for the curved beam at the $F = 6.719$mN forcing level for frequencies above the resonance and for the flat beam for $F = 4.169$mN also above the resonance. Note that the measurements shown were not used to identify the SNRM-based model, so this constitutes a validation of the method.

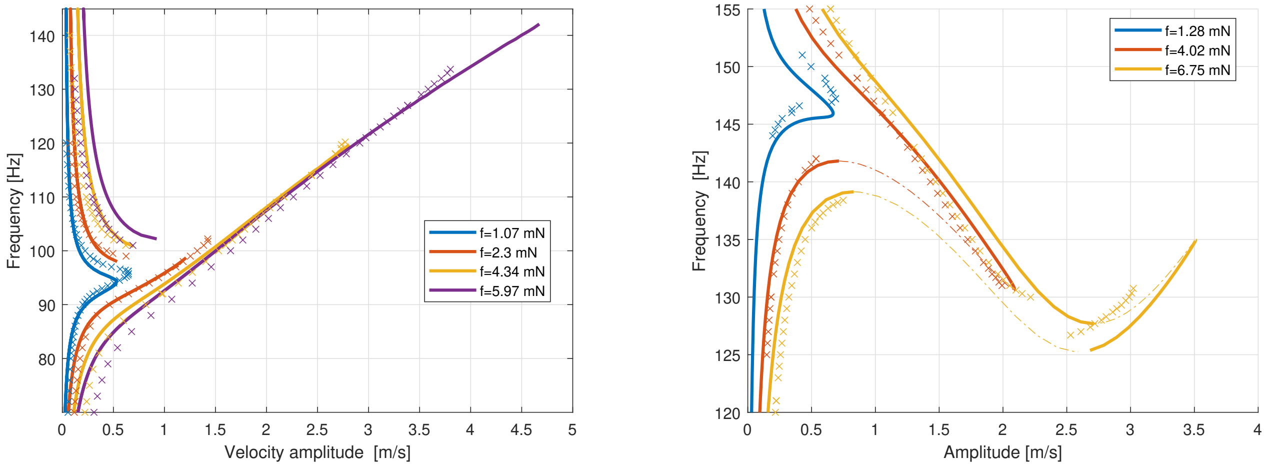
3.2 METHOD PROPOSED BY BREUNUNG ET AL.

Applying the method [1; 2] to the measured steady state response of the flat beam yields the equation of motion

$$\ddot{q} + c\dot{q} + \nu_1 \dot{q}^2 q + \nu_2 \dot{q}^2 q^3 + kq + k_5 q^5 = f\cos(\Omega t), \tag{4}$$

whereas the governing equation obtained for the curved beam reads

$$\ddot{q} + c\dot{q} + \nu_1 \dot{q}^2 q + kq + k_3 q^3 + k_5 q^5 + k_7 q^7 = f\cos(\Omega t). \tag{5}$$



(a) **(b)**

Fig. 3: Results obtained using the method developed by Breunung et al. [1; 2] for the **(a)** flat and **(b)** curved beams.

Aside from nonlinear stiffness terms, both Eqs. (4) and (5) contain mixed terms depending on the velocity and position. When modeling this system in [8; 9; 10; 11], Kwarta and Allen did not think to include terms such as this, and that may have contributed to the less than ideal results that they obtained. With the procedure in [1; 2] these terms were automatically identified as being important. Equations (4) and (5) were simulated to obtain the forced response curves shown in Fig. 3, which illustrates that both models capture the measured data accurately enough.

4 CONCLUSIONS AND FUTURE WORK

The most significant difference between the SNRM and Breunung methods, is that the former approximates the system as linear and amplitude dependent, whereas the latter identifies a nonlinear model, or a nonlinear differential equation governing the system. Using the inverse SNRM-based approach, one can readily derive the steady-state response using Eq. (2), although it is less clear how this approach might be used to derive the response to an arbitrary input, and one would expect the algorithm to lose accuracy as the model is only presumed to be accurate when the nonlinear mode in question is dominant and all other modes are weak.

It is also interesting to compare the Breunung method to the Nonlinear Identification through eXtended Outputs (NIXO) algorithm presented in [10; 11]. In the former work [10], the white-box version of NIXO was introduced. This version of the algorithm is applied in cases where the nonlinear equation of motion form is known explicitly and has proven to be an improvement over NIFO [12; 13] and similar techniques. The latter work [11] presents the black-box version of NIXO, along with its applications to both numerical and experimental case studies.

The black-box NIXO algorithm identifies a nonlinear differential equation for the system of interest by assuming the general form of the EOM and subsequently eliminating irrelevant higher-order polynomial terms. Therefore, it bears more similarity to the Breunung method than the inverse SNRM-based approach presented here. However, although the two methods, the black-box version of NIXO and the Breunung method, share mathematical similarities and can be directly compared, the black-box NIXO was initially developed with the assumption of broadband measurements being available. In all studies to date, it has been applied exclusively to swept-sine measurements. Furthermore, while the black-box NIXO algorithm has demonstrated its effectiveness in numerical case studies, as yet it has proven relatively challenging to obtain an acceptable model for the experimental measurements from the flat and curved beams presented here. The identified models were found to contain errors in the coefficients of the nonlinear terms, which became evident when comparing the measured NNMs to those from the model. In contrast, both the inverse SNRM-based and Breunung techniques have proven highly effective and relatively straightforward when applied to steady-state vibration measurements as presented in this study. As mentioned previously, the NIXO algorithm did not contain the coupling term $q^2\dot{q}$, which was identified as important by the Breunung method in both cases, so it would be interesting to include that in the NIXO approach in [11].

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