

# Modeling Bolted Joints in the S4 Beam at Various Preloads with Discrete Iwan Elements

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## Abstract

Prior works [i.e. Lacayo & Allen, MSSP, 2019] have used discrete four-parameter Iwan elements to capture the localized energy dissipation and loss of stiffness that is experienced near bolted interfaces. Those works have validated that approach by inserting these joint models into a structural model and then verifying that it accurately captures how the natural frequency and damping of one mode of the structure vary with vibration amplitude. Those studies were limited to only a single level of preload in the joint. If this were to be used as an empirical means of modeling structures, one would need to know how the Iwan parameters of the joints vary with preload and to verify that the method is effective in a wider range of scenarios. This work explores this issue in more detail, seeking to ascertain whether a library of Iwan elements can be identified that capture a structure with two bolts when those bolts have a range of different preloads. Measurements were acquired from the S4 Beam [Singh et al., IMAC, 2018] at various preloads and with a few permutations of the preloads. A finite element model was then created with rigid bar spiders that reduce the joint to a single pair of nodes, and Iwan elements were inserted between those to model the slip in the joint. The model was tuned to capture each set of measurements, to understand how the Iwan elements vary with bolt preload. The results presented show how the Iwan parameters evolve as preload is increased, and also how the frequency and damping versus amplitude evolve over a wide range of preload and response amplitude.

**Keywords:** quasi-static, model updating, contact nonlinearity

## 1. Introduction

Mechanical joints, with the friction, intermittent contact and uncertainties that they introduce, are thought to be the source of most of the damping in built up structures [1], [2], as well as the most frequent source of nonlinearity. Hence, in many applications it can be important to have models that can accurately capture their damping and nonlinearity, while keeping computational costs as low as possible.

The most common approach to modeling joints in industry is to simplify the area near the joint and then use a “spider” or multi-point constraint (MPC) over an area to reduce the joint to a single node. Then, two surfaces are joined by inserting an element, typically a linear spring, between the joint nodes. The stiffnesses of those springs can then be adjusted during model updating to bring the model into agreement with measurements. A good example of this was recently presented by Winkel et al., who performed a study comparing the standard industry/NASA approach to other alternatives [3]. When a structure contains nonlinear joints, this same approach has been used to good success (see, e.g. [4]), with the linear springs joining the structures replaced with nonlinear elements such as Coulomb sliders or Jenkins elements.

In 2016, Lacayo, Allen & Brake [5], [6] presented some extensions that sped up this approach, and used it to update a Hurty/Craig-Bampton (HCB) model of a structure in which the joints were reduced to five pairs of nodes that were joined with five Iwan elements [7]. They were able to obtain excellent results for the Brake-Reuss beam [8], showing that, after updating, the model reproduced the damping and natural frequency of the first and third bending modes as a function of vibration amplitude. Those were the two modes that were most strongly affected by the bolted interfaces, and the results spanned a considerable range of amplitude, although all measurements were taken in the micro-slip regime [9]. Singh et al. [10] later applied a similar approach to update an HCB model of the S4 Beam<sup>1</sup> [11] to match measurements over a range of amplitude, obtaining excellent results as long as the spiders used to reduce the joint interfaces were of the rigid-bar type; if the joints were too flexible, then it seemed to be impossible to update the model such that sufficient energy would be dissipated by the joint, and the model would under-predict the damping quite severely. In either case, the studies showed that this modeling approach was able to capture the power-law damping that the structures were observed to exhibit, a feature in which the log of the damping increases linearly with the log of the vibration amplitude. The Iwan element is unique in this regard; other joint models are not able to capture this behavior [12] over a wide range of vibration amplitude.

While these prior studies show that this modeling approach does have promise, the method has still been exercised on relatively few structures, and never for a set of measurements in which the joint was driven to macro-slip and beyond. Additionally, the properties of bolted connections are known to depend on the preload in the joint, and so one might question whether this approach could work over a range of preloads. This study presents a set of measurements on the S4 Beam in which the preload varies from 133 N (30 lbf), in which the bolts were barely finger tight, to 8896 N (2000 lbf), in which the bolts were near their yield point. A similar model to that used in [10] is updated to seek to capture the behavior of the structure in each case, and the resulting Iwan parameters are examined to show how they evolve as the preload increases. This can be seen as the first step in an effort to identify a set of Iwan parameters that could populate a handbook or database to which an engineer could turn when modeling a structure, looking up a model for a specific joint based on its geometry, construction and preload.

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<sup>1</sup> The name S4 or S<sup>4</sup> Beam stood for the “Sandia Symmetric Steel Sandwich” Beam, although in many subsequent studies it has simply been called the “Sandwich Beam” or “C-Beam.” The name also happens to denote the last names of the primary authors of the first study, i.e. “Sandia-Singh-Scapolan-Saito.”

## 2. Theory

Measurements of the S4 beam were taken in the same manner described in [13]. After setup of the beam, the bolts were tightened to varying preloads, measured using a load cell inserted between the bolt head and the beam. Initially, the case of 500 and 2000 lbf preload were measured, followed by 30 lbf, and then the 200 lbf, 1000 lbf, and 1500 lbf cases in relatively quick succession. The drive points were chosen to specifically excite modes 2 and 6, as these modes experience the most nonlinear damping caused by the bolted joints, as well as an uncoupling to the other modes. This is desirable during these tests as the nonlinear properties are being studied on the individual modes. Only mode 2 was used in the results presented here. The data gathered during these measurements was then used for parameter verification and tuning.

A reduced order finite-element model (ROM) of the S4 beam was then created using the Hurty/Craig-Bampton method [14], [15]. One of the requirements of using this method is that the only source of nonlinearity in the structure comes from the interface of the joints, which the model represents. After this reduction is performed, the model is then further simplified by creating spider joints to reduce the contact area. A spider joint ties the nodes in a predefined contact area down to a single node. While Singh [10] explored the use of both rigid (i.e. RBAR) and averaging type (i.e. RBE3) spiders, he found the former to perform better and so only rigid spiders were used in this work. To model the nonlinearity, the two spider nodes in contact are then connected using an Iwan element, replacing the linear spring used in linear analysis. For a more in-depth explanation of the reduction techniques used in this model, refer to [16].

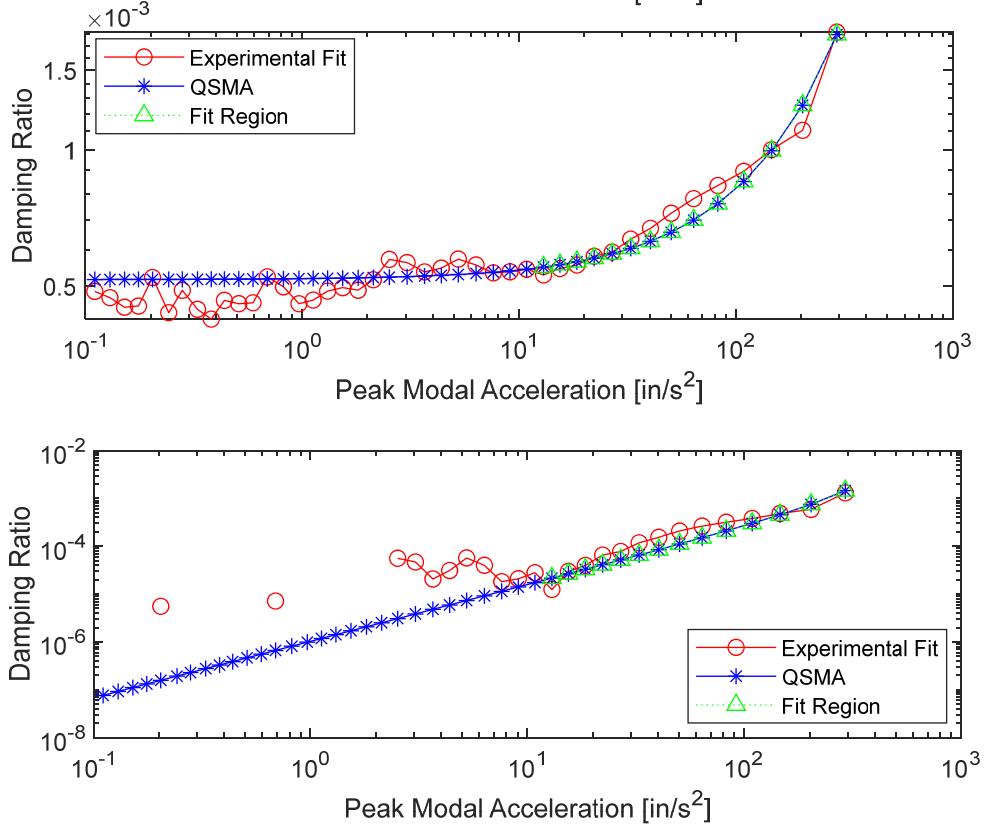
An Iwan element is a simple multi-spring model that is able to replicate the power-law energy dissipation seen in nonlinear joint dynamics. This element consists of many spring-slider sub-elements, or Jenkin elements [17], in parallel. Each of these sliders has a different slip force, but the same spring stiffness. There are four parameters that control the Iwan element:  $K_T$ ,  $F_s$ ,  $\chi$ , and  $\beta$ .  $K_T$  is the spring stiffness of the Jenkin elements before the element has slipped,  $F_s$  is the force necessary to cause macroslip,  $\chi$  is the exponent that describes the slope of the energy dissipation curve, and  $\beta$  is related to the relative strength of micro- and macro-slip. The power-law energy dissipation is controlled by  $K_T$  and  $\chi$ , where  $F_s$  and  $\beta$  control the point when the transition to macroslip occurs. For more details on the Iwan element, refer to [7].

Before Iwan elements replace the springs at contact, a linear modal analysis is run to estimate the spring values for all the degrees of freedom for the system. The joints at either end of the beam have six degrees of freedom, three displacements and three rotations. Thus, twelve spring constants need to be estimated to bring the natural frequencies of the model into agreement with those that were measured. After these values were obtained using linear modal analysis coupled with an optimization routine (see [16] for details), the Iwan element replaces the linear spring and the  $K_T$  values for each degree of freedom are applied to their corresponding Iwan elements. After these stiffness values are applied to each element, a Monte Carlo simulation is used to estimate the other three parameters of the Iwan elements, which are assumed to be the same on both ends of the beam. This Monte Carlo simulation is a parametric optimizer that searches a predefined parameter space for the smallest root mean square error in natural frequency and damping ratio versus vibration amplitude. Additionally, a new parameter,  $\gamma$ , is introduced to slightly scale the  $K_T$  values, in case those found in the linear updating step cannot produce the required nonlinear behavior; however, because any change in  $\gamma$  reduces the agreement between the linear natural frequencies of the model and those measured experimentally,  $\gamma$  should remain

near unity. The parameter estimations from the Monte Carlo simulation are then further refined by hand to make the model match the data as closely as possible.

One deviation was taken here from the method described in prior studies; the material damping of the S4 beam has previously been subtracted from the measured data before fitting it with the model. This is because the model only characterizes the damping introduced by the bolted joints. However, it can be difficult to determine the correct amount of material damping to remove when the measured data is noisy, and subtracting the wrong amount of damping impacts the ability of the model to fit the data well. Additionally, subtracting the material damping often results in damping values below zero, which do not show up well on a log plot. In this study the material damping was not removed from the data but instead added to the QSMA model to avoid these difficulties. Figure 1 compares the two approaches when there is a 10 percent difference in the material damping between the model and the measured data. With the material damping added to the model, it is easier to see the discrepancy and correct it.

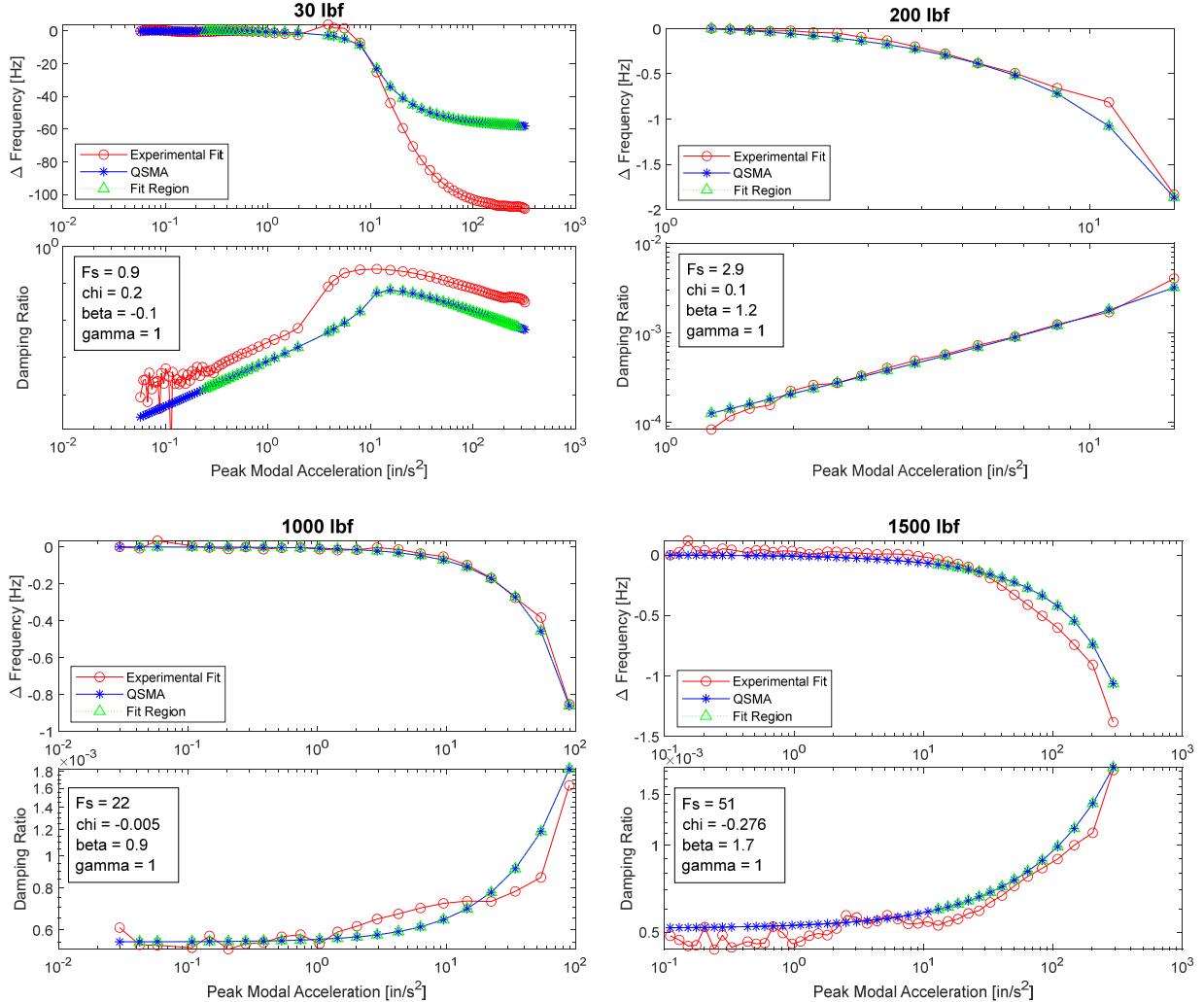
A downside to this approach is that it is more difficult to see the power law behavior of the damping curve; for example the power-law region is clearly seen on the right of the bottom pane of Figure 1, while it is not clear whether power-law behavior exists on the top pane. However, after the optimization one could always subtract the material damping from the data and the model to see the power law behavior.



**Figure 1.** Measured damping versus amplitude for 1500 lb preload case, compared to the damping predicted by QSMA for the reduced order model. The top plot shows the result when the measured damping is used and material damping is added to the QSMA simulations. The bottom plot shows the result when the material damping is subtracted from the measurement.

### 3. Results

Figure 2 shows a comparison between the measurements for all preload cases and each optimized model. The QSMA model was consistently able to match the experimental data very well for all the preloads measured except for the 30 lbf case. While many attempts were made, none was able to obtain a better match to the damping for the 30 lbf case without increasing the frequency error even more.



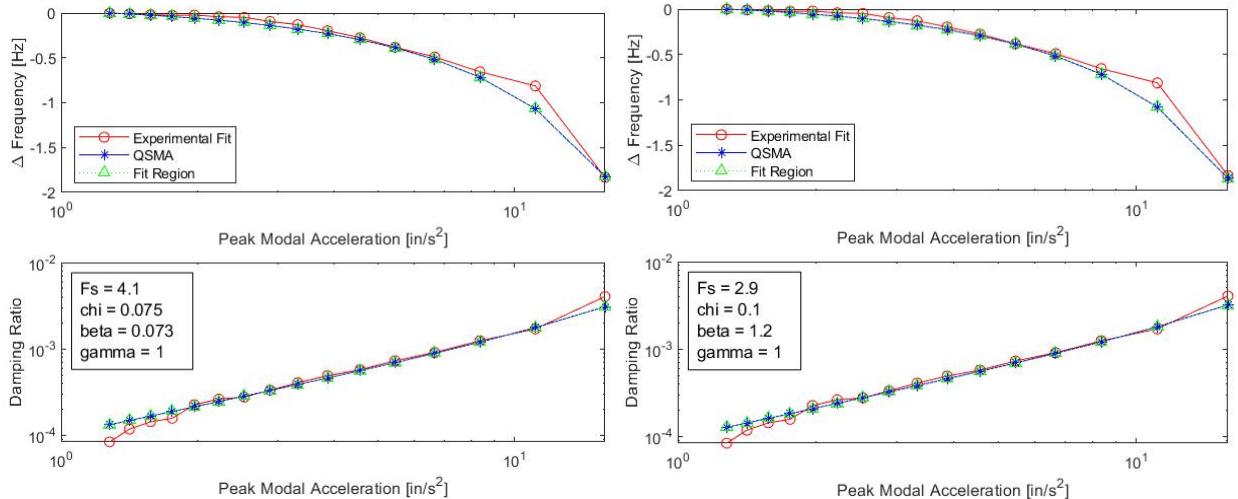
**Figure 2.** Measured natural frequency and damping as a function of vibration amplitude for four preload cases, compared to the fit obtained with the reduced order model (denoted QSMA) after optimization. The points used in the optimization are denoted “Fit Region.”

This 30 lbf preload case was the only one that included measurements of the macroslip regime. For the results above the focus was on capturing the micro-slip regime. Further trials revealed that it was impossible for this model to obtain excellent agreement in both the micro- and macro-slip regimes. One could match both the frequency and damping very well in either the microslip region or the macroslip region, depending on the chosen Iwan parameters, but not both

simultaneously. The reasons for this are still unclear. The bolts are extremely loose in this case, only barely finger tight, and perhaps this changes the mechanics of the joint such that the modeling assumptions are no longer valid. For example, the model used in this work only uses Iwan elements in the axial direction; the joints are prevented from opening/closing due to a stiff linear spring. Perhaps nonlinearity must be considered in both axes for this low preload.

The other cases did not include measurements of macro-slip, so it is unclear whether the model provides a good fit throughout both microslip and macroslip in those cases. Ideally, future studies should obtain macroslip measurements at the other preloads, so one can see whether this modeling approach is applicable to macroslip; for the present we focus on the micro-slip regime only.

Focusing on the micro-slip data, some patterns begin to emerge as one compares the Iwan parameters that give the best fit for each of the four preloads. As discussed in past works, the Iwan elements are coupled such that changing one parameter affects the other parameters in a complicated way. It can be difficult to know which set of parameters is “correct”, especially when the data is incomplete (such as only having the microslip region). For example, the two different sets of Iwan elements shown in Figure 3 for the 1500 lbf preload case yield an identical-looking fit, and it is not immediately clear which fit is better. This is particularly challenging as the preload increases and the nonlinearity becomes weaker and hence less clearly defined.



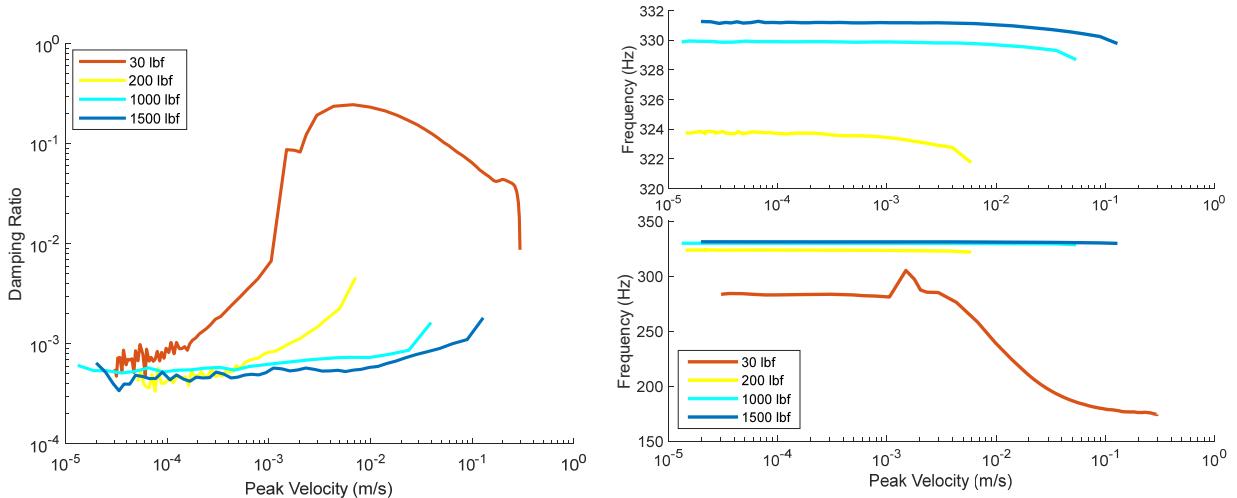
**Figure 3.** Two optimized models for the 1500 lbf preload case, showing that there can be multiple sets of Iwan parameters that reproduce the measurements well.

The parameters  $\gamma$  and  $F_s$  seem to be especially related, such that adjusting either one can change the damping and frequency curves in a similar way. Because  $\gamma$  is just a scaling factor for  $K_T$ , it was fixed at one for each of these results in order to keep  $F_s$  more consistent. With  $\gamma$  held at unity, a clear trend emerges where  $F_s$  increases with preload (see Table 1). This pattern makes physical sense, as can be seen when comparing the damping curves for all data sets (Figure 4). As preload increases, the transition to macroslip begins at a higher peak velocity, which indicates

a higher vibration amplitude and greater forces in the bolted connections. Thus, the force required for macroslip to occur must be greater as preload increases.

**Table 1.** Iwan parameters obtained for each preload while holding  $\gamma = 1$ .

	30 lbf	200 lbf	1000 lbf	1500 lbf
$F_s$	0.90	2.90	22.0	51.0
$\chi$	0.20	0.10	-0.005	-0.276
$\beta$	-0.10	1.20	0.90	1.70
$\gamma$	1.0	1.0	1.0	1.0

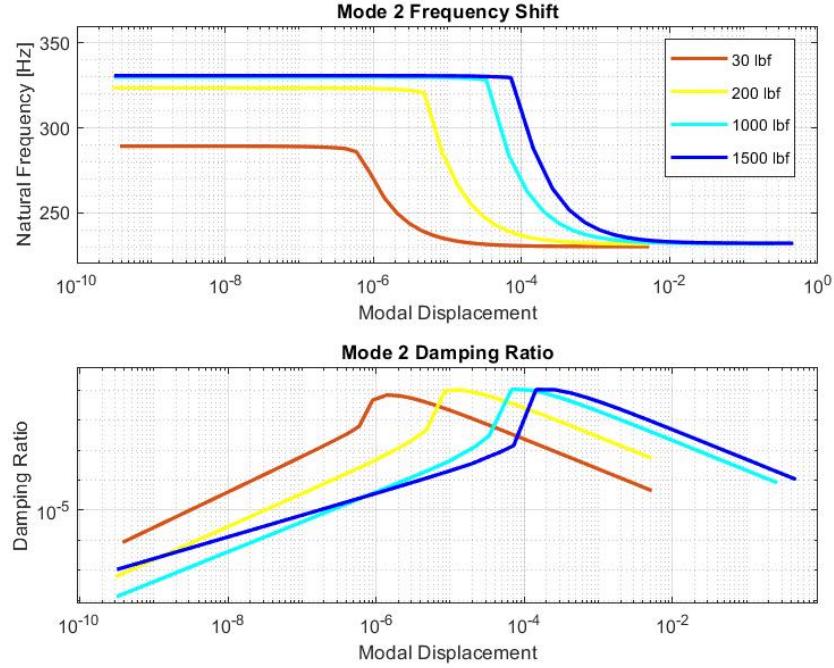


**Figure 4.** Measured natural frequency and damping versus vibration amplitude for all preloads

Patterns in  $\chi$  and  $\beta$  are not as clear (see Table 1). The power-law exponent  $\chi$  has a possible decreasing trend, but more evidence is needed to verify this. Most of the data sets do not contain a large enough range of amplitude to capture this clearly, and as was discussed previously the amount of material damping that is assumed affects the estimate of  $\chi$ . The parameter  $\beta$  has no recognizable pattern; this parameter mainly controls the transition from microslip to macroslip, which is not fully captured in this data. Having data in the macroslip region would provide more insight into how  $\beta$  changes with preload.

Although the experimental data does not include the macroslip regime, the “best-fit” models in Figure 2 can be extended to show the macroslip region and then compare between the models. This comparison, in Figure 5, reveals that the damping and frequency curves for each preload are almost identical, but with the transition to macroslip happening at different vibration amplitudes. One exception is the slope of the damping curve in the microslip region for the 1500 lb preload model. This could be due to an error in the  $\chi$  value used for that data set, due to the uncertainty mentioned previously. However, this possibility was investigated and when  $\chi$  was kept closer to the value used for the other data sets, it was not possible to identify a set of Iwan parameters that

produced an acceptable fit to that data set. Hence, it appears that the power-law exponent  $\chi$  may indeed decrease with increasing preload.



**Figure 5.** Natural frequency and damping versus vibration amplitude for the optimized, reduced order models obtained at each preload, with the amplitude range extended so that the macrolip region can be seen for each case.

It should be noted that two other data sets were collected a few months prior to these, at 500-lb preload and at 2000-lb preload. Unfortunately, as the analysis progressed it was discovered that these data sets exhibited a significant shift in damping that didn't follow the trends shown above. This could be a result of the joint changing, perhaps due to being assembled differently or because the surfaces had worn, although the surfaces were inspected and no wear marks were visible. It was thought that these data sets were most likely not post processed correctly, and because there wasn't sufficient time to resolve the issue before publication they were left out of the paper.

#### 4. Conclusion

This study verifies that discrete 4-parameter Iwan elements can be used to successfully model the nonlinear frequency and damping of the S4 beam at a wide range preloads, from 200 lbf to 1500 lbf. It further shows that, although the four Iwan parameters are highly coupled and have a complicated effect on each other, some clear trends can be observed in how the parameters evolve with bolt preload. More work needs to be done to fully understand how bolt preload affects each of the four parameters. Taking measurements at preloads in between those presented here will help the observed trends become clearer. Even then, the results presented here constitute a first attempt at identifying an Iwan model that is a function of preload. For example, if the preload is denoted  $P$  then it appears that one can identify the parameters as functions of

preload, such as  $F_s = F_s(P)$  or  $\chi = \chi(P)$ , and a first estimate of these functions could be made by interpolating the results in Table 1. While further work would be needed to obtain high confidence in the relationships obtained so far, the results are promising.

While one of the data sets investigated contained strong macro-slip of the joint, the reduced model was not able to capture that data well. It is unclear whether this is a deficiency of the Iwan model itself or due to some other modeling assumption. This should be studied in more detail before using the Iwan element in applications where macroslip is of interest.

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