Simultaneous Direct Time Fitting of a Multi-Mode Response to Determine the Instantaneous Frequency and Damping

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ABSTRACT

Traditional methods to fit a multi-harmonic time response use filters to isolate a single modal response to determine the amplitude dependent natural frequency and damping ratio. However, to avoid spurious end effects, filters require that the response fully decays. As a result, the Hilbert Transform often requires trimming extracted frequency and damping data as well as parameter tuning to minimize these end effects. To preserve the response and minimize end effects, a novel method was presented by Goyder and Lancereau (ASME IDETC 2017) where time responses were fit in the time domain through a two-step optimization procedure. First, the instantaneous frequency and damping are extracted from small time blocks of response, and then the amplitude and phase are found through least squares. This work presents an expansion to that method by directly fitting the modes of a multi-harmonic response simultaneously using a windowed nonlinear least square fitting method on small time blocks of the response. In addition, this method was used to determine the effectiveness of three variations: (1) forward fitting a response from start of a response to the end, (2) reverse fitting from the end of the response to the start, and (3) fitting and recreating the response in the frequency domain. Several computational and experimental responses are used to demonstrate the effectiveness of the simultaneous direct time fitting method.

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INTRODUCTION

Traditional methods using the Fast Fourier Transform (FFT) have seen widespread success for stationary signals to determine the modal frequency and damping characteristics of a response. However, with nonstationary signals, the FFT inherently provides a single solution for the frequency and damping that is smeared across the time domain response. A number of nonlinear system identification methods have been proposed [1], but there has been limited success using them with hysteretic systems such as structures with friction in mechanical joints. The Hilbert Transform (HT) [2] and the restoring force methods (RFS) [3] have proven effective but are limited due to end effects and the requirement that the signal be mono-harmonic. Goyder and Lancereau [4] presented a method that fits a damped sine to short intervals of SDOF response where within each interval, the frequency and damping is assumed to be constant. However, signals are rarely monoharmonic, thus requiring some form of modal decomposition to output SDOF oscillators for these nonlinear fitting methods. This multi-step process can yield spurious end effects in the frequency and damping solutions of the time response and may also miss additional nonlinear phenomena by decomposing the signal into SDOF responses. This work presents an extension to the time fitting method by Goyder and Lancereau by directly fitting a multi-harmonic response using a windowed nonlinear least square method with a summation of sinusoids on small intervals of the time response.

METHODOLOGY

The method introduced by Goyder, fits a damped sine to the modally filtered SDOF response to obtain the modal parameters as a function of amplitude. Equation 1 shows the extension of Goyder's work to fit N modes of a multi-harmonic response where $y_{fit}(t)$ is the fit of the response of the segment, ζ and ω_n are the damping and natural frequency, A and B are amplitudes and i represents the ith mode.

$$y_{fit}(t) \;=\; \sum_i^N e^{-2\zeta_i\omega_{n,i}t}\left[\mathrm{A_i}\cos\!\left(\sqrt{1-\zeta_i^2}\,\omega_{n,i}t\right)-\mathrm{B_i}\sin\!\left(\sqrt{1-\zeta_i^2}\,\omega_{n,i}t\right)\right] \tag{1}$$

The frequency and damping can be written in terms of auxiliary variables $\alpha = \sqrt{1-\zeta^2}\omega_n t$ and $\beta = 2\zeta\omega_n t$. Values for frequency and damping may be found by solving these equations to get

$$\omega_n \;=\sqrt{\alpha^2\;+\beta^2} \tag{2}$$

$$\zeta \;=\frac{\beta}{\sqrt{\alpha^2+\beta^2}} \tag{3}$$

The modal parameters can be found in two stages; first, the values for α and β are iteratively found to minimize the phase error between fit and response segments, and then the amplitude error is minimized using a least squares process for each segment. This is then repeated for each segment of the time history. Furthermore, each response segment is overlapped with the previous and following time segments to smooth over any erroneous change in modal characteristics of each segment due to noise. The workflow for this method is shown in Fig. 1 corresponding to the following steps:

1. Segment time history into intervals with assumed constant frequency and damping
2. Fit a multi-harmonic damped sine to the segment
3. Return the fit for each mode present in the response
4. Generate the amplitude dependent damping and frequency curves for each mode

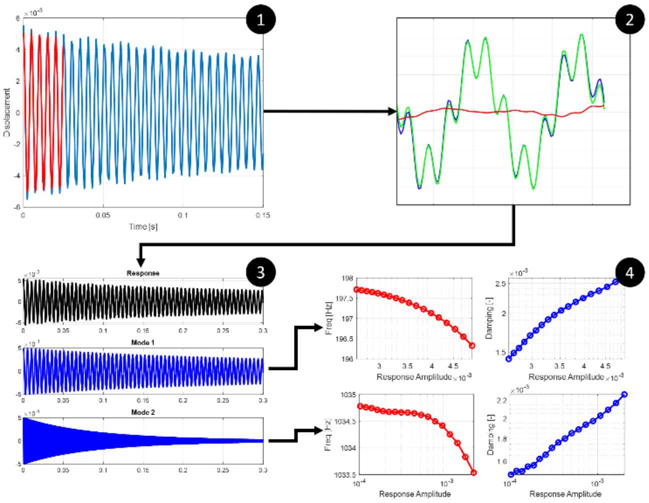

Figure 1: Workflow for direct time-fitting of multi-harmonic signals with (1) time response of fit region, (2) Zoom in on data (blue), fit (green) and difference (red), (3) Repeat the process for each mode, (4) Natural frequency and damping as a function of amplitude found for each mode.

ANALYSIS

The data used for this work is from experimental testing of the S4Beam (C-Beam) by Wall et al. [5]. Figure 2 shows the measured time response and the FFT of the response, where the modes of interest are the first two modes of the system which are both weakly nonlinear.

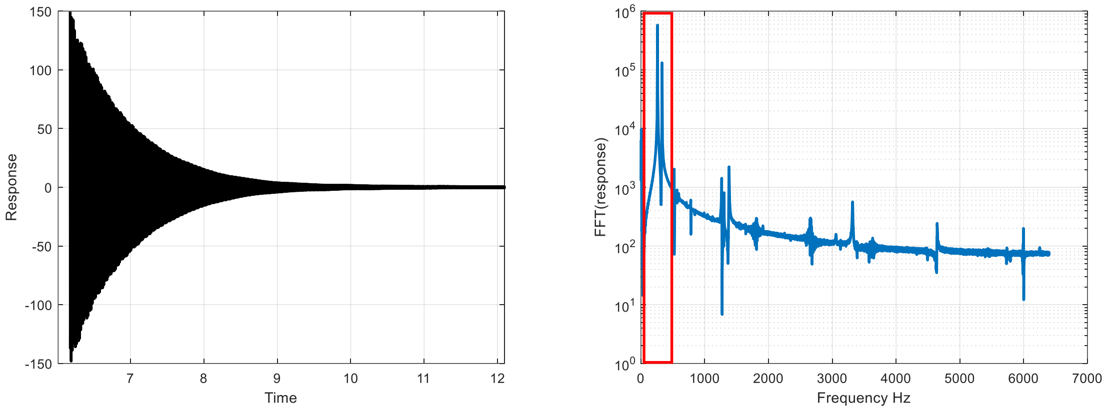

Figure 2: (Left) Time response and (Right) FFT of the time response with the modes of interest highlighted in the red box

The response was fit with two harmonics corresponding to the first two modes with 90% overlap between adjacent time segments. Each segment consisted of 1/10 of the signal, and with overlap, there were a total of 91 segments analyzed in approximately 1.4 seconds. For conciseness, the response was fit using forward fitting, i.e., fitting the response with increasing time. Reverse fitting and frequency domain fitting will be presented at IMAC XLI. The resulting fit and error are shown in Fig. 3 with the resulting frequency and damping responses in Fig 4.

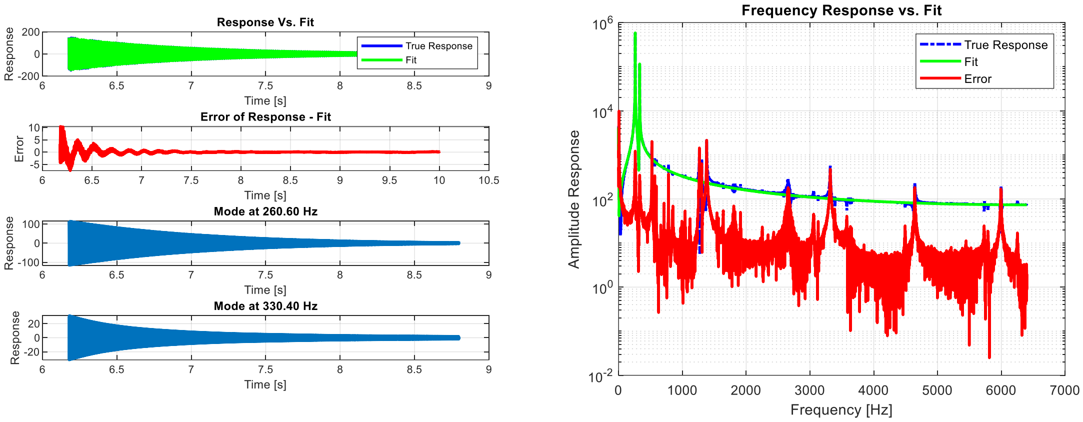

Figure 3: (Left) Fit time response vs. true response and (Right) FFT of the fit time response, error, and the true response

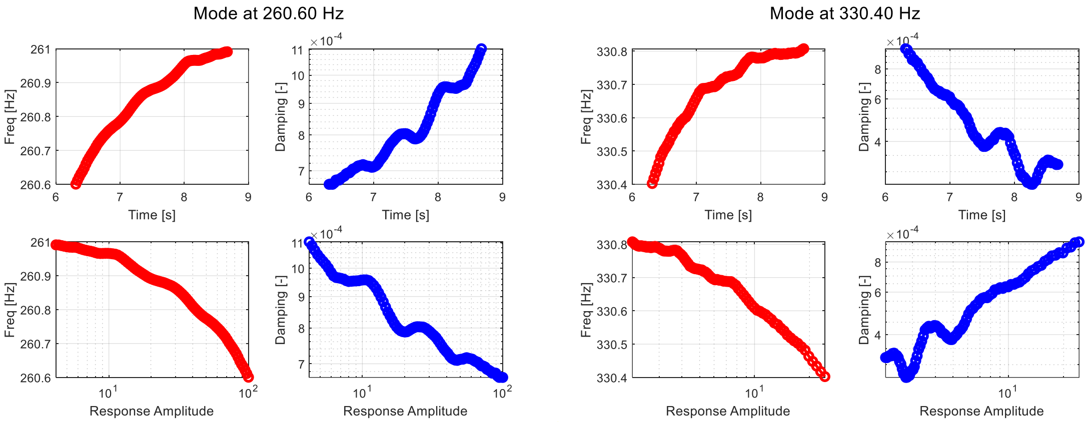

Figure 4: (Left) Modal characteristics of Mode 1 (Right) Modal characteristics of Mode 2

CONCLUSION

A novel direct time fitting approach was presented here to extract the instantaneous frequency and damping content from experimental responses. First the time response is split into segments where the frequency and damping are assumed constant. Then each segment is fit with a sum of sinusoids at different harmonics. Finally, the full response is synthesized using the fit of each segment. This methodology was shown on data from the S4Beam using forward fitting which resulted in a synthesized response for both modes of interest without requiring modal decomposition methods. However, with modes with large differences in frequency, using a single segment size and overlap may not always work. In that case, the response at the higher frequency will have more oscillations wherein the frequency and damping at the segments may not remain constant. Future work will seek to automate the initial sizing of the segments and the overlap size for various modes to mitigate this concern.

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