Application of Black-Box NIXO to Experimental Measurements

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Abstract

Nonlinear identification methods seek to create a mathematical representation of a mechanical system, which can then be used to: (*i*) predict the structure's motion or (*ii*) design, redesign or optimize the structure. In a prior work the Nonlinear Identification through eXtended Outputs (NIXO) algorithm was found to work well if the model form is known a priori. Moreover, the black-box NIXO-based algorithm was successful for the data generated numerically. However, when it comes to actual experimental measurements, the black-box identification procedure has proven more challenging. This work builds on the previous efforts seeking to create a black-box NIXO and to demonstrate it on experimental measurements. The identification attempt is performed on a 3D-printed flat beam and the results are validated against experimental measurements collected during sweep sine vibration testing.

Keywords: Nonlinear system identification, Nonlinear parameter estimation, Black-box methods, Nonlinear Normal Modes, NIXO methods

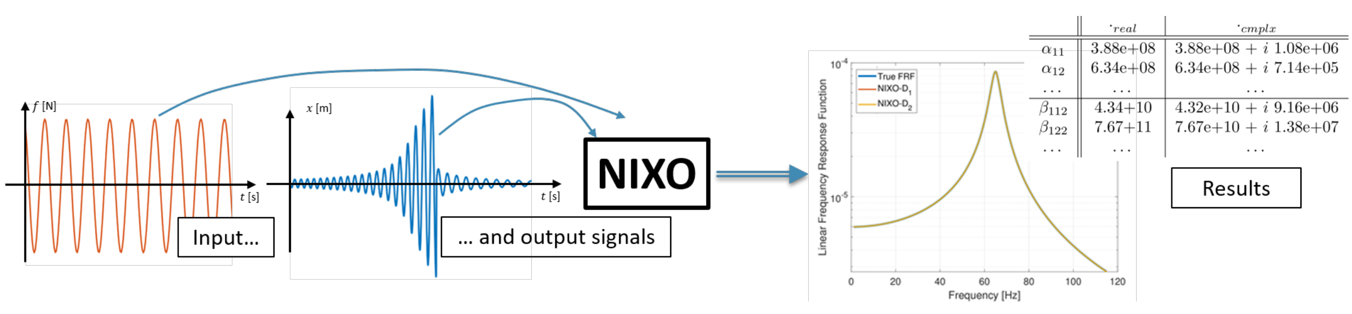
Introduction

The NIXO-based black-box system identification procedure was first introduced in [1, 2]. The authors showed there a successful application of the algorithm to the case studies with signals generated numerically. The objective of this publication is to test this black-box technique with the experimental measurements. Before we share the details from the experimental case studies performed, we would like to provide a brief overview of the black-box NIXO procedure. It consists of three steps:

1. Assuming the (most) general form of the nonlinear equation of motion (EOM) describing the mechanical system (e.g. EOM consisting of every quadratic and cubic term, see Eq. (1)).

$$\ddot{q}_k + 2\zeta_k\omega_k\dot{q}_k + \omega^2 q_k + \alpha_{11}^k q_1^2 + \alpha_{12}^k q_1 q_2 + \cdots + \beta_{111}^k q_1^3 + \beta_{112}^k q_1^2 q_2 + \cdots = \mathbf{\Phi}_k^T \mathbf{f}(t) \tag{1}$$

2. Providing the measured input and output signals to the D_1- and/or D_2-NIXO algorithms with the nonlinear EOM assumed in the previous step. The algorithms return estimates of the frequency response of the underlying linear system as well as the parameters describing the mechanical system's nonlinearities, as illustrated in Fig. 1.



	·real	·cmplx
α_{11}	3.88e+08	3.88e+08 + i 1.08e+06
α_{12}	6.34e+08	6.34e+08 + i 7.14e+05
...	...	...
β_{112}	4.34+10	4.32e+10 + i 9.16e+06
β_{122}	7.67+11	7.67e+10 + i 1.38e+07
...	...	...

Fig. 1: System identification process with NIXO. The algorithms use the input and output time series to estimate the linear and nonlinear parts of the equation of motion.

3. Grouping the nonlinear parameters into the dominant and irrelevant sets, where the division is based on the values of two indicators: Δ_* and Δ_{**}. The inequalities that should be satisfied (simultaneously) by these two parameters are presented in Eq. (2). For the detailed description of the Δ-indicators please refer to [1, 2].

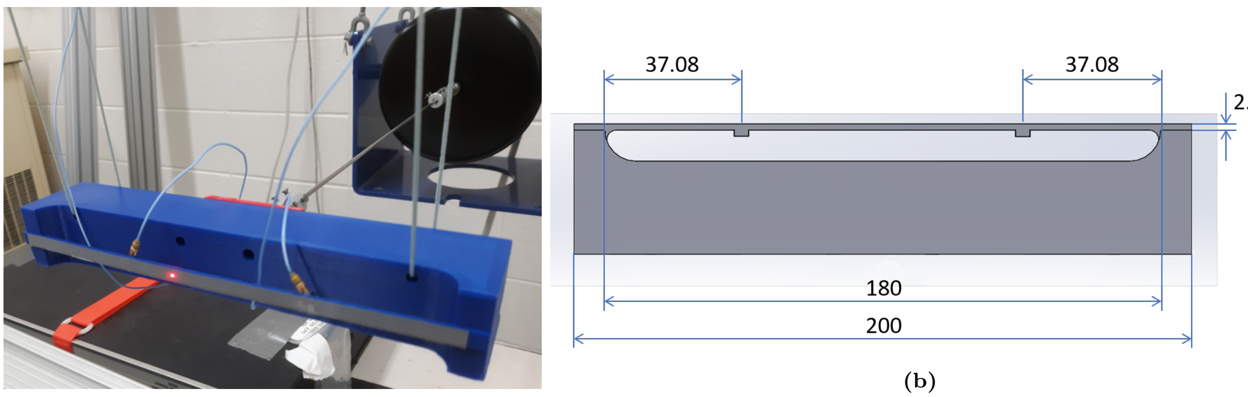
$$\begin{cases} \Delta_* < 5\% \\ \Delta_{**} > 95\% \end{cases} \tag{2}$$

Experimental set-up and signals measured

The experimental set-up considered here is shown in Fig. 2a. The structure is excited using a Modal Exciter 100 lbf Model 2100E11 powered by a 2050E05 Linear Power Amplifier. The shaker is connected with the backing structure of the beam using a metal stinger. The experimental data is collected with the Polytec software, where the oscillations of:

- the point at the beam's center is measured with the PSV-400 Scanning Vibrometer
- the two points distant by 37.08 mm from each of the beam's ends (see Fig. 2b) are measured with the PCB352C23 accelerometers.

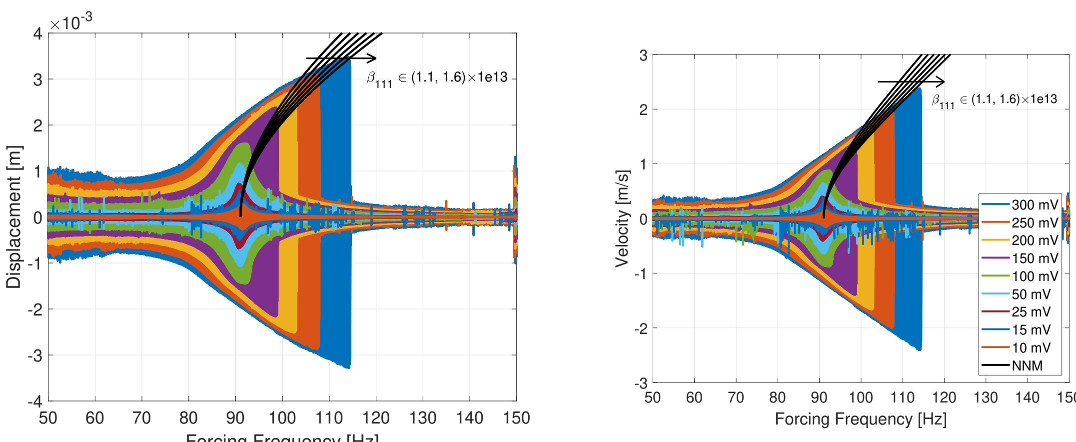
The input signals used to excite the structure are multiple 204.8-second-long linear sweep sines of various amplitudes and frequencies increasing from 50 to 150 Hz. The time series of these signals are presented in Fig. 3.



(a)

(b)

Fig. 2: (a) Photograph of the experimental setup. **(b)** Cross-section of the SolidWorks model of the flat beam; small rectangles illustrate locations of the accelerometers attached to the beam.



(a)

(b)

Fig. 3: Time response of the beam measured at its center: **(a)** displacement (obtained by integrating velocity) and **(b)** velocity. The signals measured with the two acceleromets are analogous. The NNM curves computed for the equation of motion with the nonlinear part consisting of only the $\beta_{111}q_1^3$ term are overlayed on the time signals ($\beta_{111} \in (1.1, 1.6) \times 10^{13} \frac{1}{\mathrm{kg\ m^2\ s^2}}$).

System Identification Results

The motion of the beam is modeled with the first two symmetric modes, i.e. modes 1 and 3. Moreover, the authors decided to represent the nonlinear part of EOM with the most general form of the polynomial consisting of the quadratic and cubic terms. Hence, the resulting number of the nonlinear terms that occur in each modal equation is 7, see Eq. (3).

$$\ddot{q}_k + 2\zeta_k\omega_k\dot{q}_k + \omega^2 q_k + \alpha_{11}^1 q_1^2 + \alpha_{12}^k q_1 q_2 + \alpha_{22}^1 q_2^2 + \beta_{111}^k q_1^3 + \beta_{112}^k q_1^2 q_2 + \beta_{122}^k q_1 q_2^2 + \beta_{222}^k q_2^3 = \mathbf{\Phi}_k^T \mathbf{f}(t), \tag{3}$$

where: ω_k ζ_k, $\mathbf{\Phi}_k$ are, respectively, the linear natural frequency, damping ratio and mode shape of the k-th mode; the quantities $q_k(t)$ and $\mathbf{f}(t)$ are the time representations of the k-th modal coordinate and the force distribution; α's and β's are, respectively, the quadratic and cubic coefficients, and $k \in \{1, 2\}$. In this work we focus on identification of the nonlinear mode 1 only.

Different pairs of signals were provided to the NIXO algorithms in order to identify the structure; this approach worked well for the data generated numerically. The estimates of the underlying linear as well as the nonlinear parts of the system are presented in, respectively, Fig. 4 and Tab. 1.

Tab. 1: Estimated values of the nonlinear coefficients that satisfy the accuracy criteria specified in Eq. (2). The parameters that do not meet the criteria are not shown in this table.

β_{111} estimate $\left[\frac{1}{\text{kg m}^2\ \text{s}^2}\right]$		min	max	average	st. dev. [%]
Black-box	D1 NIXO	2.44e13	3.19e13	2.81e13	9.16
	D2 NIXO	2.10e13	2.47e13	2.36e13	5.07
White-box	D1 NIXO	1.14e13	1.66e13	1.37e13	13.58
	D2 NIXO	1.07e13	1.27e13	1.15e13	5.62

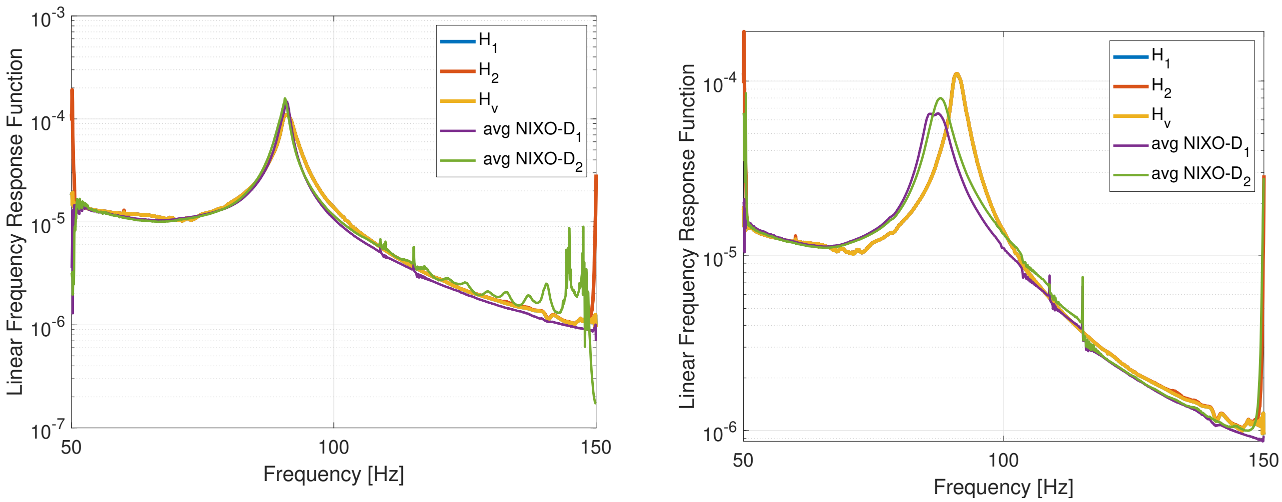


Fig. 4: Underlying linear system estimated by the NIXO algorithm in the **(left)** black-box and **(right)** white-box identification attempts. The FRFs labeled H_1, H_2 and H_v were found by applying the named estimator to low-amplitude, white noise excited measurements.

The NIXO-based black-box algorithm identified $\beta_{111}q_1^3$ as the only dominant term in the nonlinear equation of motion. However, the values of β_{111} that were identified are approximately 2-3 times larger than expected. Table 1 shows that the estimates of β_{111} belong to set $(2.10, 3.19) \times 10^{13} \left[\frac{1}{\text{kg m}^2\ \text{s}^2}\right]$, while the accurate value of the coefficient most likely belong to $(1.10, 1.60) \times 10^{13} \left[\frac{1}{\text{kg m}^2\ \text{s}^2}\right]$ (see Fig. 3). On the other hand, the linear frequency response function is found fairly accurately by both D_1- and D_2-NIXO based techniques.

However, in the prior black-box identification we sought to identify all of the system parameters and their values in a single step. If we instead repeat the NIXO identification, while only seeking to identify those terms that black-box NIXO found to be important, we obtain the results labeled "white-box" in Tab. 1 and Fig. 4. Using this approach, the estimated values of β_{111} are satisfactory since they belong to $(1.07, 1.66) \times 10^{13} \left[\frac{1}{\text{kg m}^2\ \text{s}^2}\right]$. However, the resultant linear FRF is less accurate than the one obtained in the black-box identification.

Conclusion and Future Work

This brief publication presents the application of the black-box NIXO-based methods to the experimental data. The results presented here are preliminary, yet still show that the algorithms can identify the underlying linear system as well as point out which nonlinear terms should be kept in the equation of motion. However, the estimated values of the nonlinear coefficients are frequently found to be inaccurate when using this approach. However, if we extend the 3-step-long procedure (described in the beginning of this work) by adding a fourth step in which NIXO only seeks to identify the values of the dominant parameters, we obtain accurate estimates of the nonlinear parameters. In future work, the authors will apply these algorithms to other structures to see if the proposed approach also works well for other nonlinear systems.

References

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