Predicting Nonlinearity in the TMD Benchmark Structure Using QSMA and SICE

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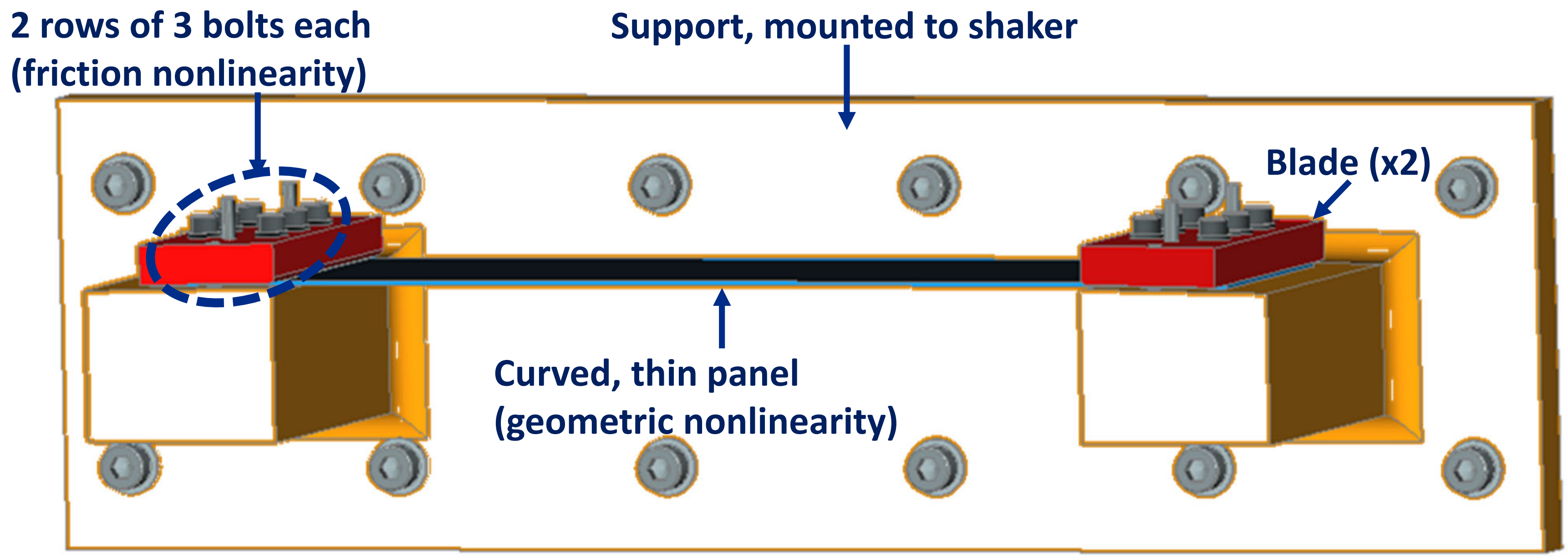
TMD challenge

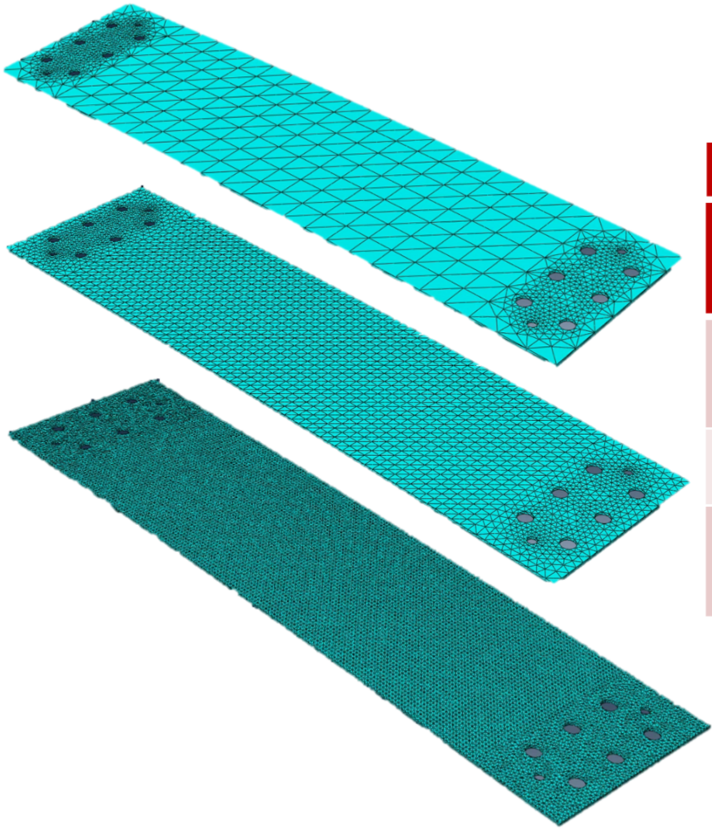


Aim is to predict:

- Linear frequencies of the first five modes
- Amplitude-dependent freq. and damping of the first mode

Modeling and Linear analysis

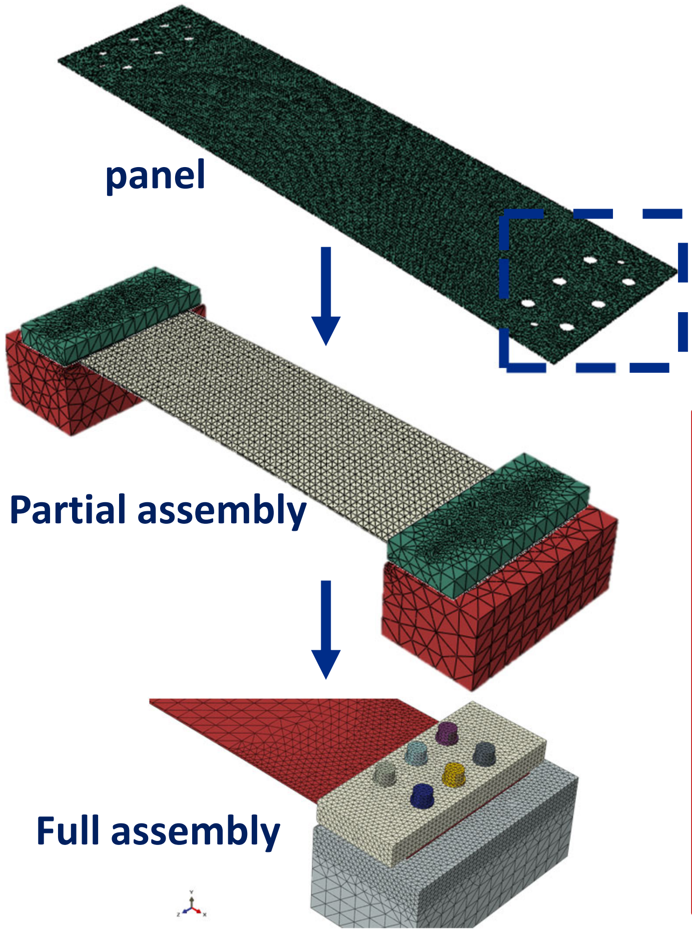
3D model of panel – checking mesh convergence

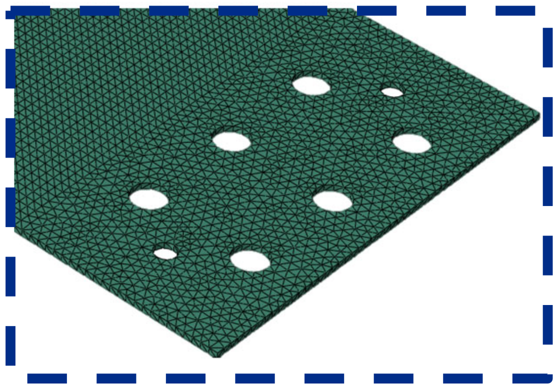


Solid Tetrahedral Elements		
Mesh (mm)	**Freq. (Hz)**	**% Δ**
15	123.26	1.3%
5	121.6	-0.1%
2	121.73	0%

- A simple 3D model of just the panel was considered, varying the mesh size to identify when the element size does not significantly affect the linear frequencies.
- Using solid tetrahedral elements, reducing the global mesh size from 5mm to 2mm only changes frequency of mode 1 by 0.08% - therefore 5mm was considered sufficient.
  - Hence, the uncertainty in the first natural frequency due to the mesh is estimated to be a few tenths of a percent.
  - Uncertainty <1-2% for higher frequency modes.

3D FE models





Global mesh size = 5mm

Material Properties:

Blades, Supports, and Bolts

- $E = 210$ Gpa
- $\nu = 0.3$
- $\rho = 7800\ \mathrm{kg/m^3}$

Panel

- $E = 200$ Gpa
- $\nu = 0.3$
- $\rho = 7900\ \mathrm{kg/m^3}$

Co-efficient of friction $\mu = 0.6$

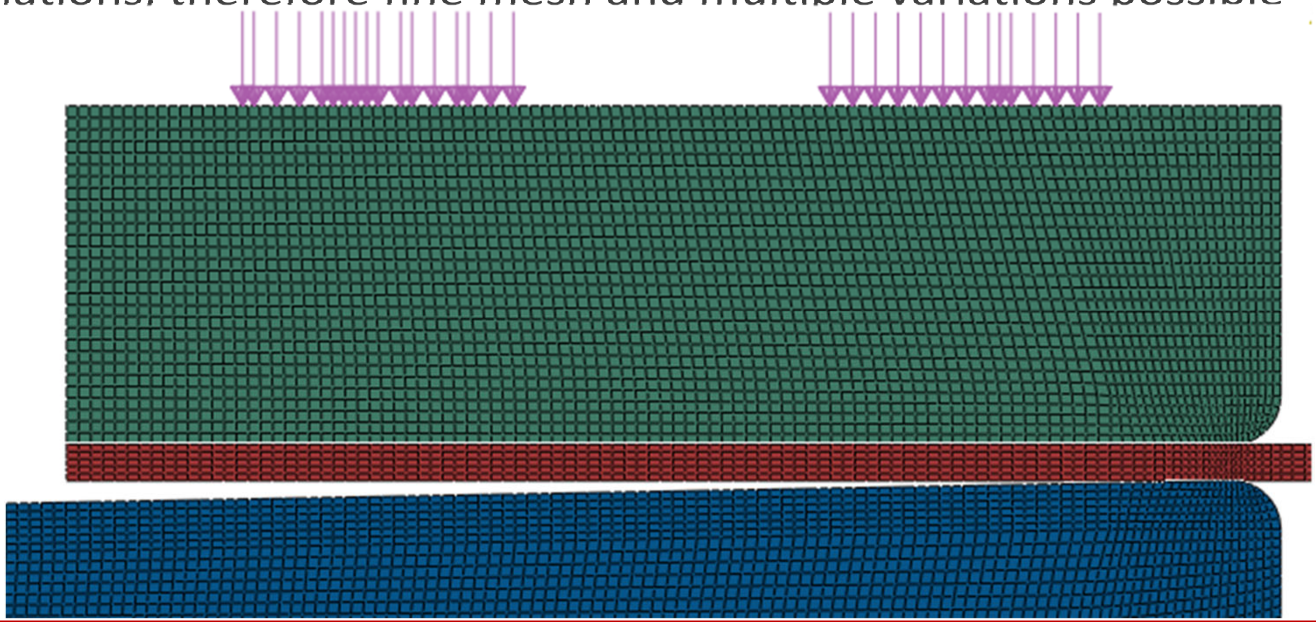
Complexity ↑

Computational cost ↑

Need a simpler model to study contact behavior

2D FE model

- Preload applied as a pressure; bolt geometry neglected
- Stiffness due to cantilever nature of support neglected
- Faster simulations, therefore fine mesh and multiple variations possible



Linear modal analysis – results and discussion

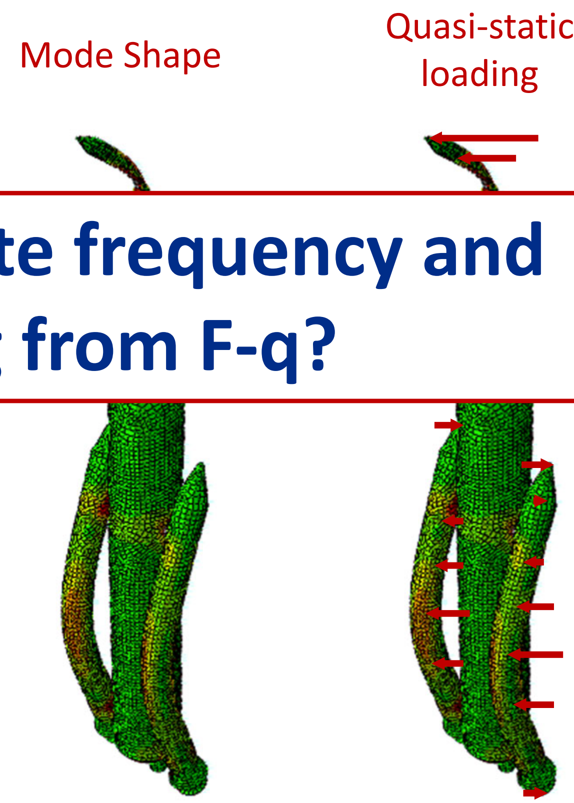
Mode	3D model	2D model	Final estimate	Uncertainty %
1 (first bending)	115.73	118.12	115.73	2.07
2 (first torsion)	204.36	--	204.36	
3 (second bending)	234.75	253.55	234.75	8.01
4 (second torsion)	405.51	--	405.51	
5 (third bending)	463.46	486.29	463.46	4.93

- 3D model includes bolt geometry details but does not include the stress in the panel due to curvature
- 2D model includes stress due to curvature but the bolts are modeled as a simple line pressure load
- The 3D model was also analyzed with fixed boundary conditions at the base of the support to replicate the 2D model boundary conditions. The change in frequency estimates was negligible.
- Therefore, at this point, our best estimate of the natural frequencies result from the 3D model and the uncertainty that arises due to the discrepancies in the two models is equal to the % difference between the two frequency estimates.
  - This neglects one very important potential source of uncertainty – any deviations of the panel from the nominally flat geometry. Additionally, uncertainty of material properties is yet to be considered.

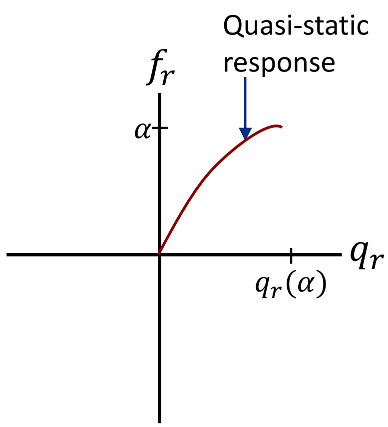
Nonlinear analysis approach

Using QSMA to estimate restoring force

- Quasi-static Modal Analysis (QSMA) [1]
  - Static load applied to the structure in the shape of the mode of interest
  - Corresponding d… nonlinearities act… FE model
  - Done over the amplitude range of interest to obtain the restoring force backbone curve.



How to calculate frequency and damping from F-q?



SICE can predict frequency from F-q for geometric nonlinearity

Park and Allen [1] showed that a QSMA-based SDOF ROM accurately captures strong nonlinear behavior of geometrically nonlinear structures.

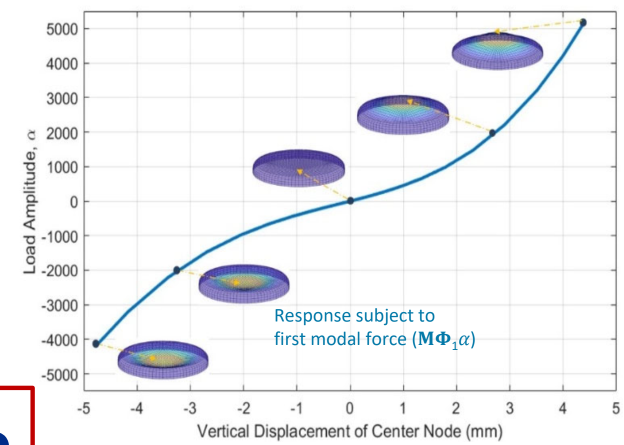
- Quasi-static response: $\mathbf{M}\ddot{\mathbf{z}} + \mathbf{C}\dot{\mathbf{z}} + \mathbf{K}\mathbf{z} + \mathbf{f}_{nl}(\mathbf{z}, \dot{\mathbf{z}}) = \mathbf{f}(t)$ (with $\mathbf{M}\ddot{\mathbf{z}}$ and $\mathbf{C}\dot{\mathbf{z}}$ struck through)
- Projected respons
- Find nonlinear co

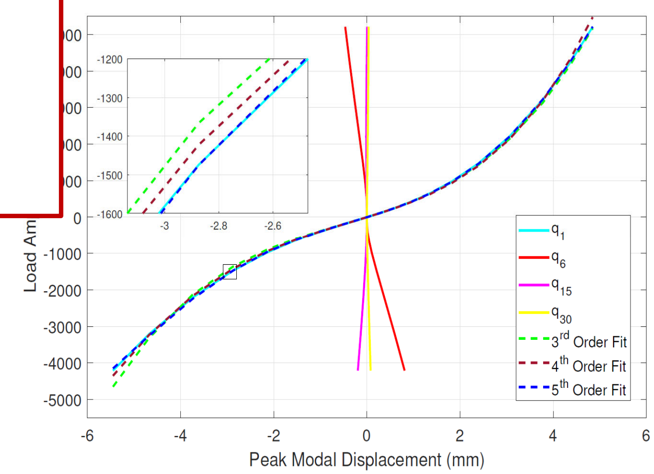
Geometric nonlinearity can be easily characterized using QSMA + SICE

$$\theta_{r(q_r)} = \sum_{i=2} k_i\, q_r^i = k_2 q_r^2 + k_3 q_r^3 + \cdots + k_l q_r^l$$

SDOF ROM from QSMA : $\ddot{q}_r + \boxed{\omega_r^2 q_r + \sum_{i=2}^{l} k_i\, q_r^i} = 0$

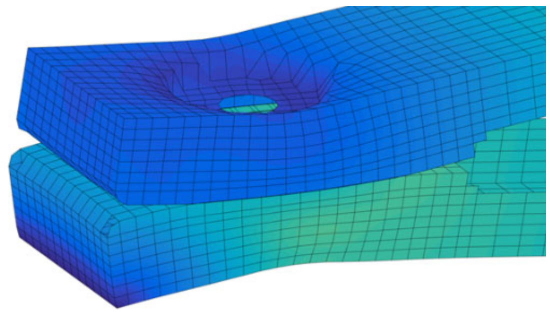
"SICE ROM"

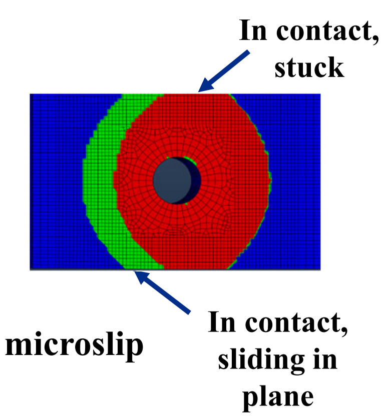


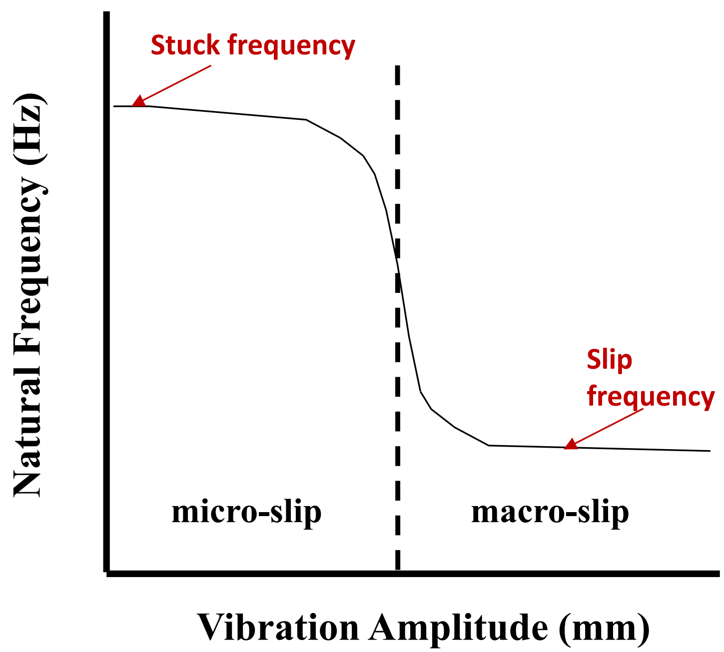


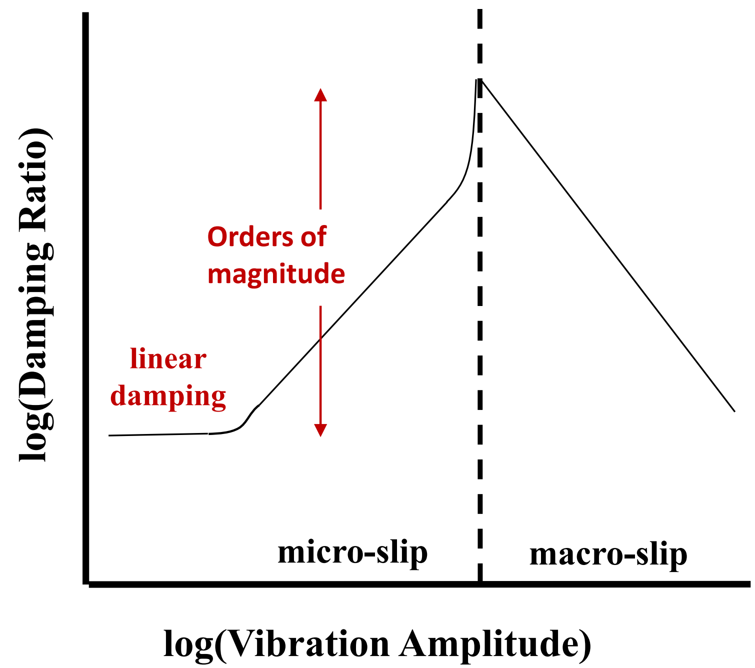
[1]K. PARK AND M. S. ALLEN, "QUASI-STATIC MODAL ANALYSIS FOR REDUCED ORDER MODELING OF GEOMETRICALLY NONLINEAR STRUCTURES," JOURNAL OF SOUND AND VIBRATION, VOL. 502, P. 116076, JUN. 2021.

Friction between bolted surfaces causes the frequency and damping to change with vibration amplitude.









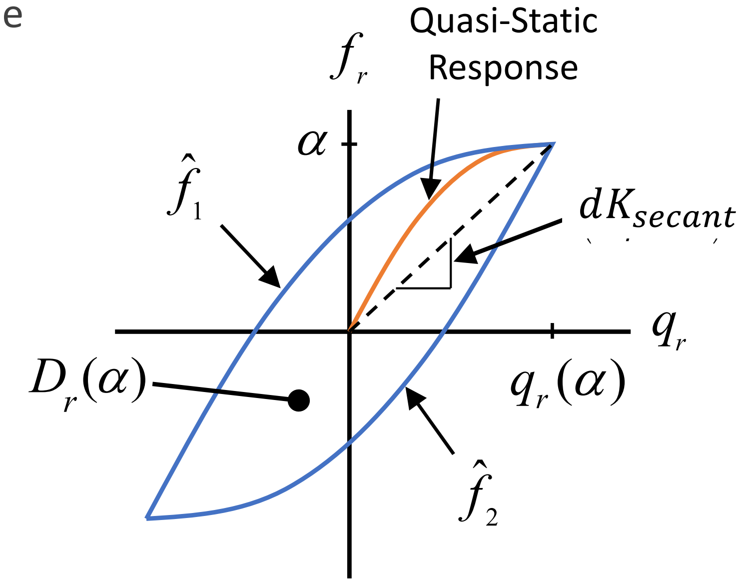
Masing's rules can be used to estimate nonlinear effects of friction

- Masing's rules applied to get hysteresis loop at each load amplitude
- Change in stiffness calculated using secant [2]

$$dK_{secant} \cong \frac{\alpha_j}{q_r(\alpha_j)}$$

- Damping calculated using the area inside the loop [2]
  - Dissipation (D) equals area of the hysteresis loop

$$\xi(\alpha_j) \cong \frac{D(\alpha_j)}{2\pi\alpha_j q_r(\alpha_j)}$$



[1] D. J. SEGALMAN AND M. J. STARR, "INVERSION OF MASING MODELS VIA CONTINUOUS IWAN SYSTEMS," INTERNATIONAL JOURNAL OF NON-LINEAR MECHANICS, VOL. 43, NO. 1, PP. 74–80, JAN. 2008

[2] ROBERT M. LACAYO AND MATTHEW S. ALLEN. UPDATING STRUCTURAL MODELS CONTAINING NONLINEAR IWAN JOINTS USING QUASI-STATIC MODAL ANALYSIS. MECHANICAL SYSTEMS AND SIGNAL PROCESSING, MARCH 2019.

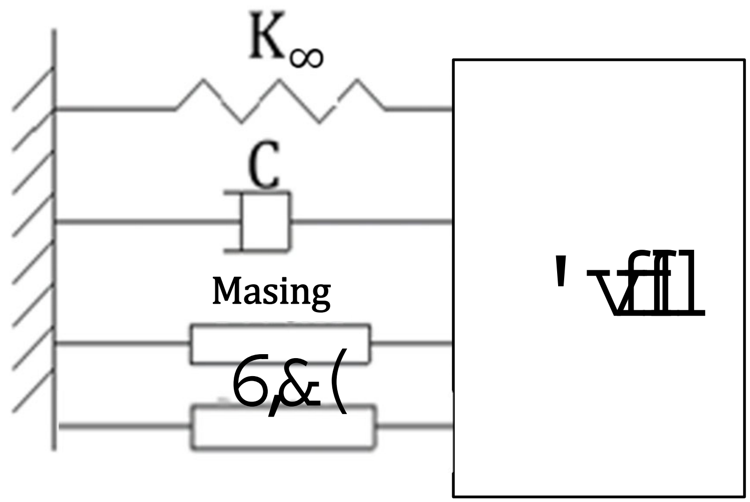
Proposed idea: combining the two ROMs

- Parallel arrangement of non-parametric Iwan model and the SICE
- EoM: $\ddot{q}_{\mathrm{r}} + 2\xi\omega_{\infty}\dot{q}_{\mathrm{r}} + \underbrace{F_{nl,joint}(q_{\mathrm{r}}, t, \phi)}_{\text{Masing model}} + \underbrace{\omega_{\infty}^{2}q_{\mathrm{r}} + \theta_{\mathrm{r}}(q_{\mathrm{r}})}_{\text{SICE ROM}} = F_{ext}(t)$

  where $\theta_{\mathrm{r}}(q_{\mathrm{r}}) = k_2 q_r^2 + k_3 q_r^3 + \cdots$

- Use of ROMs expected to be computationally efficient

How do we isolate the two nonlinearities?



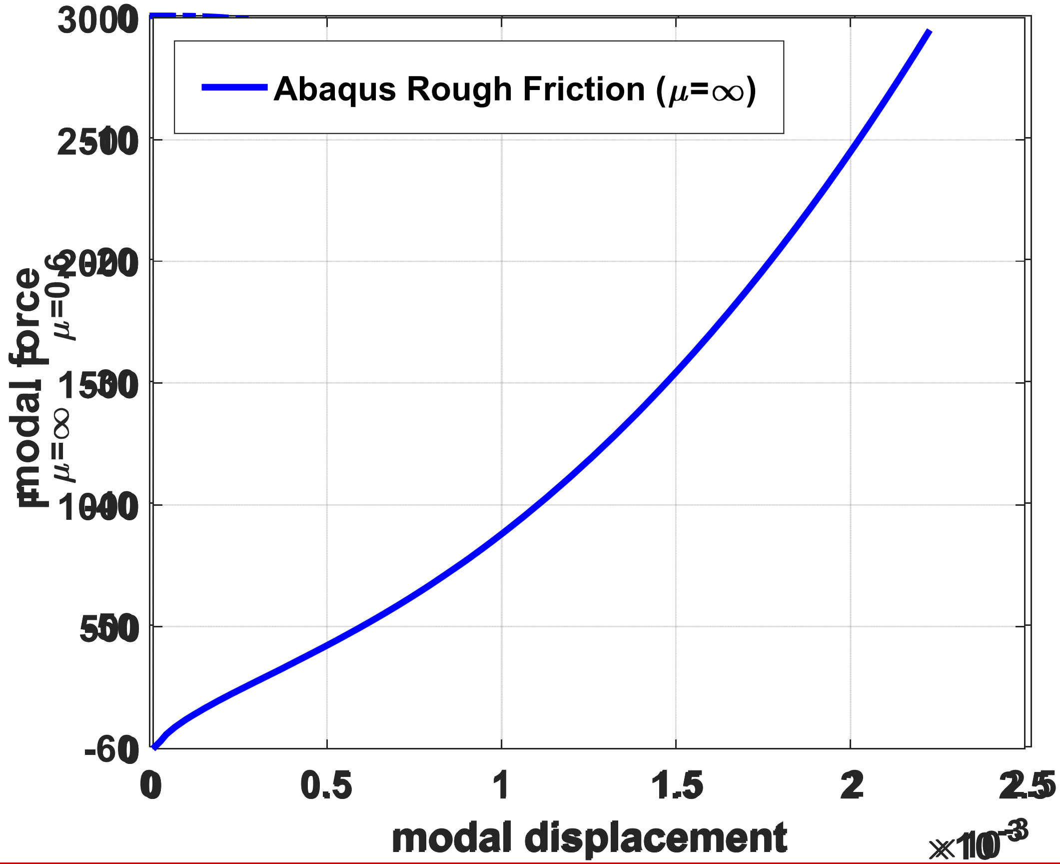
Rough friction ($\boldsymbol{\mu = \infty}$) vs $\boldsymbol{\mu = 0.6}$

1. First performing QSMA with Rough Friction setting in Abaqus – no slip at the interfaces
   - Only geometric nonlinearity
   - ~~$\ddot{q}_{\mathrm{r}} + 2\xi\omega_{\infty}\dot{q}_{\mathrm{r}} + F_{nl,joint}(q_{\mathrm{r}}, t, \phi)$~~ $+ \omega_{\infty}^{2} q_{\mathrm{r}} + \theta_{\mathrm{r}}(q_{\mathrm{r}}) = \alpha$

2. Changing contact condition to include slip ($\mu = 0.6$)
   - Both geometric and friction nonlinearity

Assuming there is no interaction between the two nonlinearities,

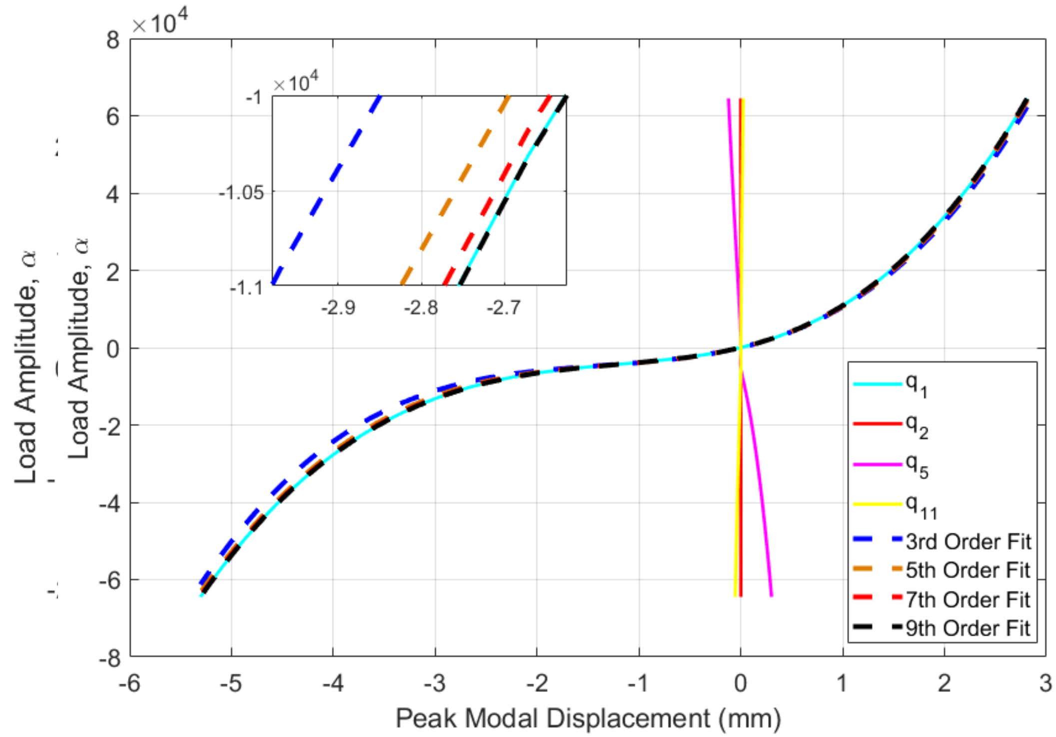
- Subtracting (1) from (2) gives $F_{nl,joint}$
- Negative slope since friction leads to decrease in frequency



Nonlinear analysis results

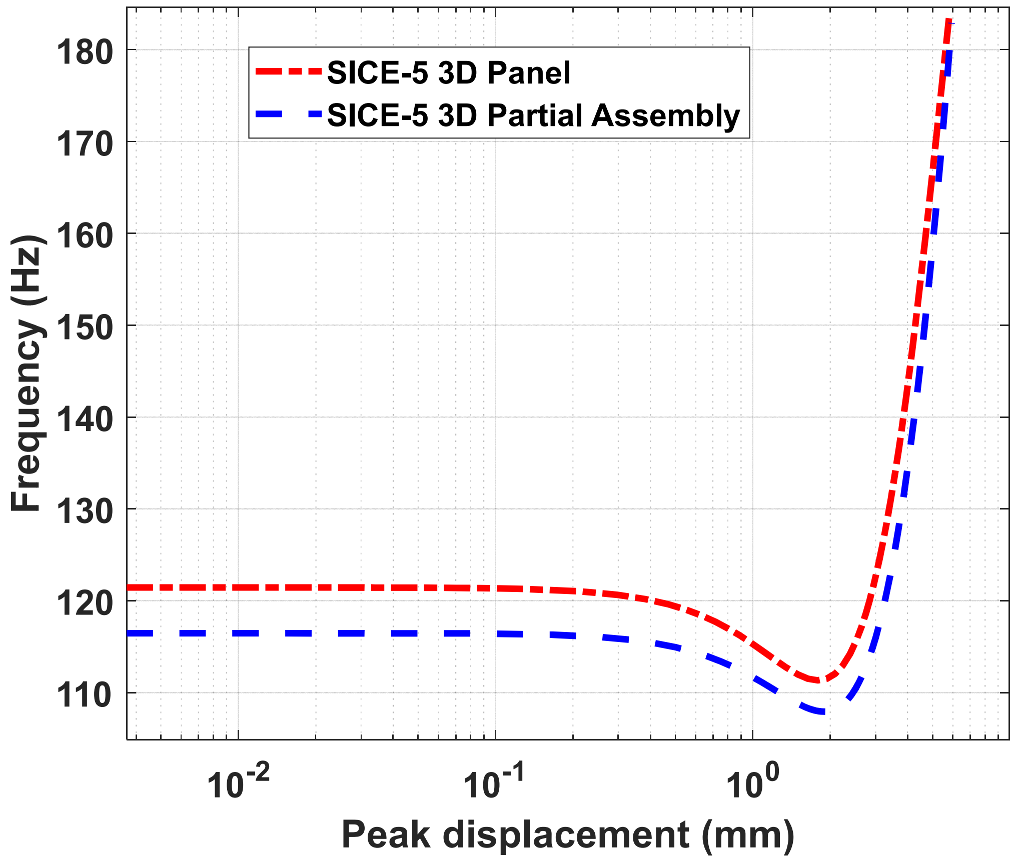
Using the 3D models to study geometric nonlinearity

- QSMA+SICE was tried on both the shell panel and the partial assembly
  - Only a small difference could be found (slight softening)
  - The boundary conditions are rigid enough and have negligible effect on the nonlinear response
- Relatively weak modal coupling between mode 1 & 5 (6%)
- SICE-3 already well captures the static nonlinear behavior
- Accuracy gets higher as order increases. Will be using SICE-5 for further analysis



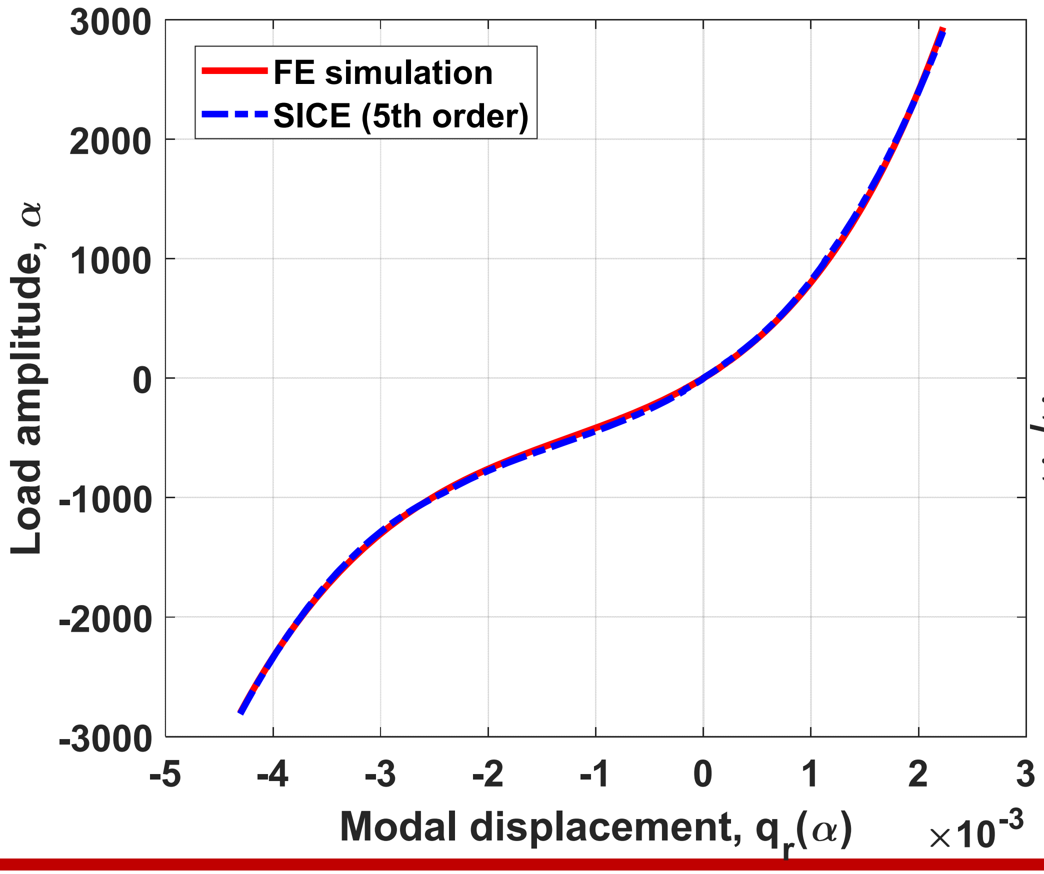
Natural frequency estimated from the SICE ROM

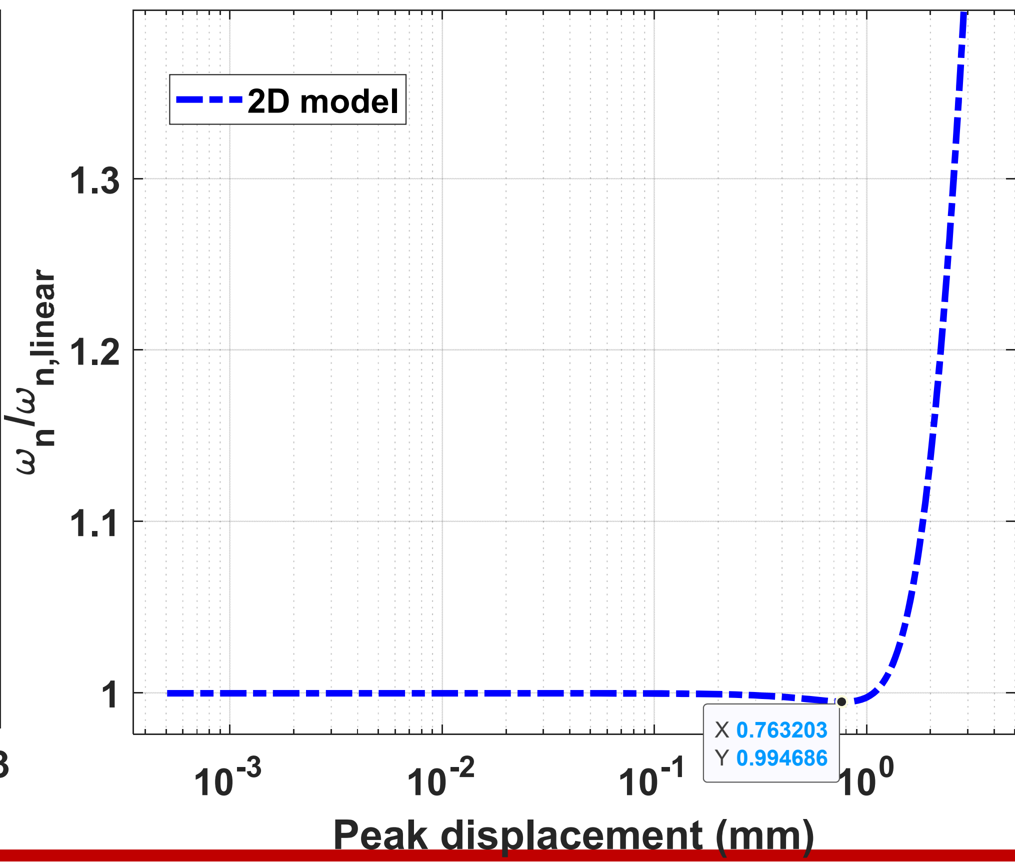
- Amplitude-dependent frequency of SICE ROMs are computed by the shooting and pseudo-arclength continuation method[1]
- SICE approach quickly captures the main backbone curve and easily estimates the geometrically nonlinear modal behavior of the curved structure
- Softening followed by hardening behavior appears; this is expected due to the observed softening during unloading



[1]M. PEETERS, R. VIGUIÉ, G. SÉRANDOUR, G. KERSCHEN, AND J.-C. GOLINVAL, "NONLINEAR NORMAL MODES, PART II: TOWARD A PRACTICAL COMPUTATION USING NUMERICAL CONTINUATION TECHNIQUES," MECHANICAL SYSTEMS AND SIGNAL PROCESSING, VOL. 23, NO. 1, PP. 195–216, JAN. 2009.

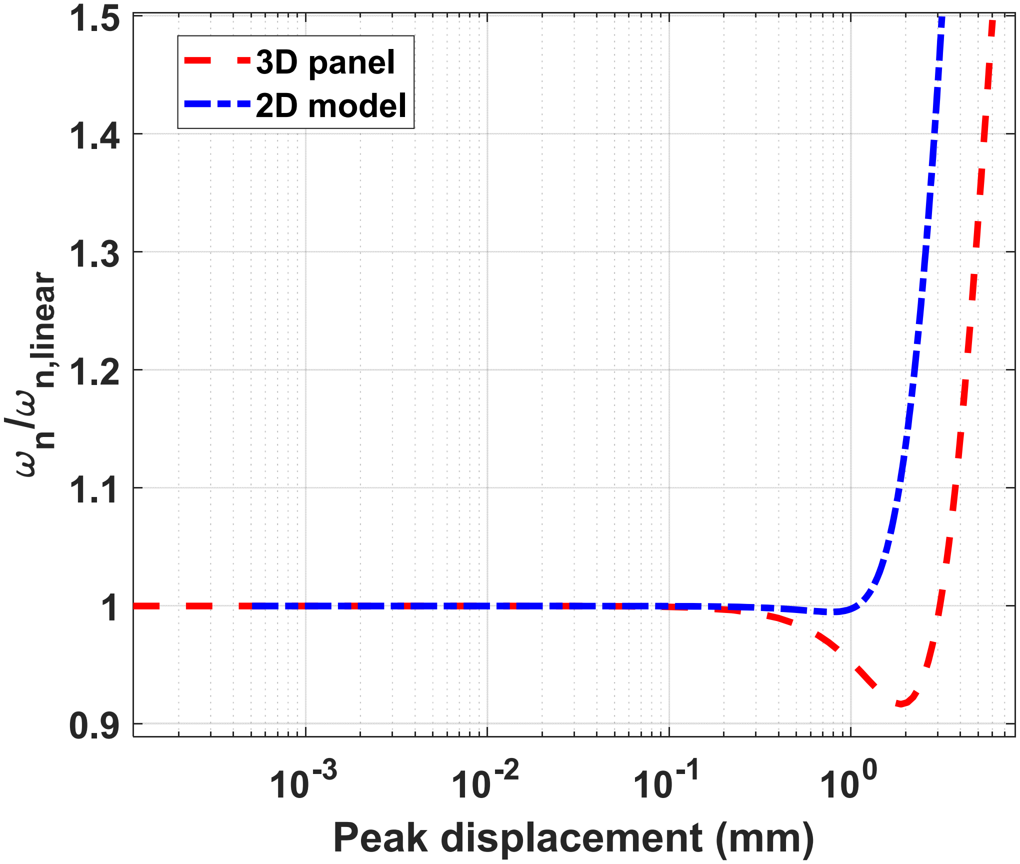
SICE Results for 2D model ($\boldsymbol{\mu} = \infty$)



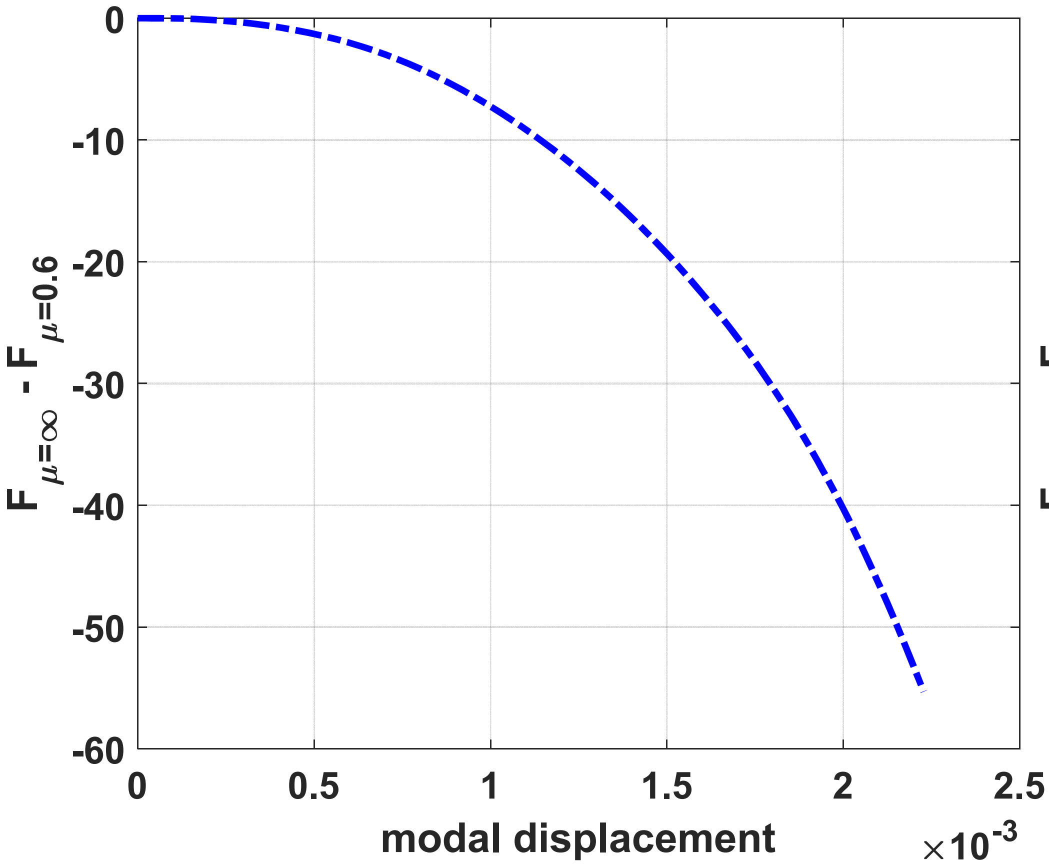


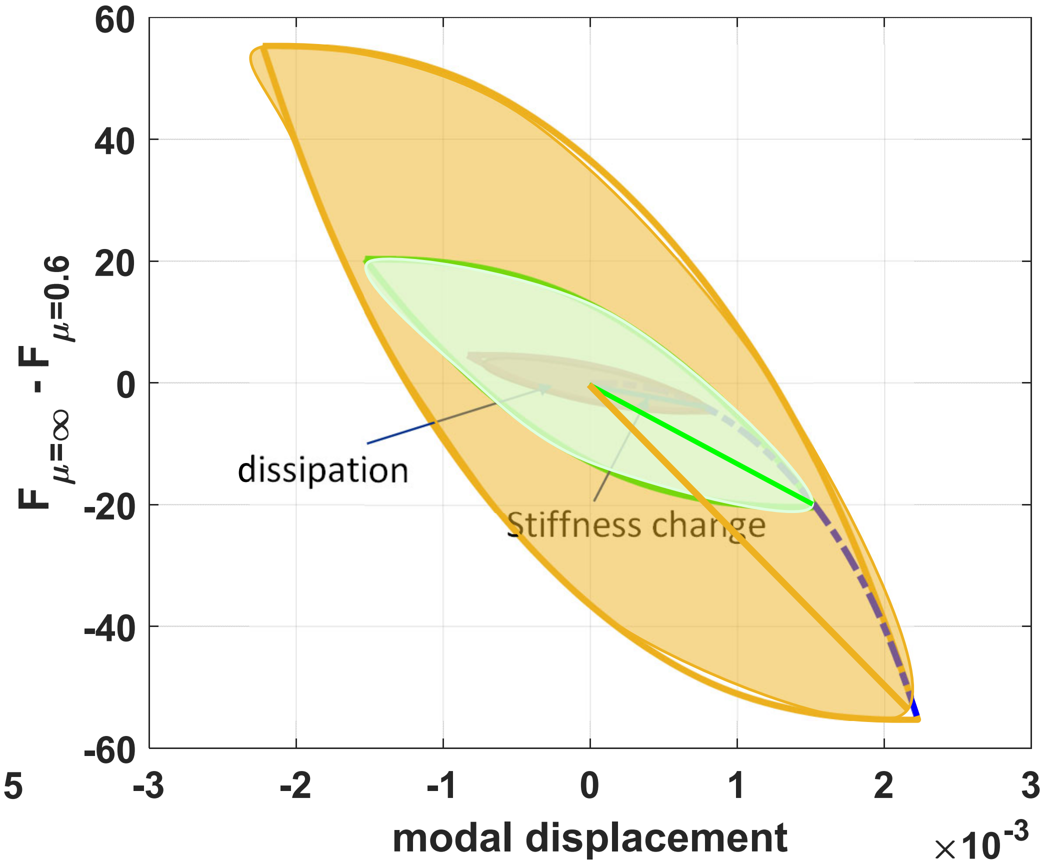
Comparing the 2D model with the 3D panel results

- 3D panel shows more softening at low-amplitudes
- The main difference between these models → curvature
  - 3D panel starts with a curved geometry
  - In 2D model, the panel is initially flat and curves due to the preload
- This needs to be further investigated

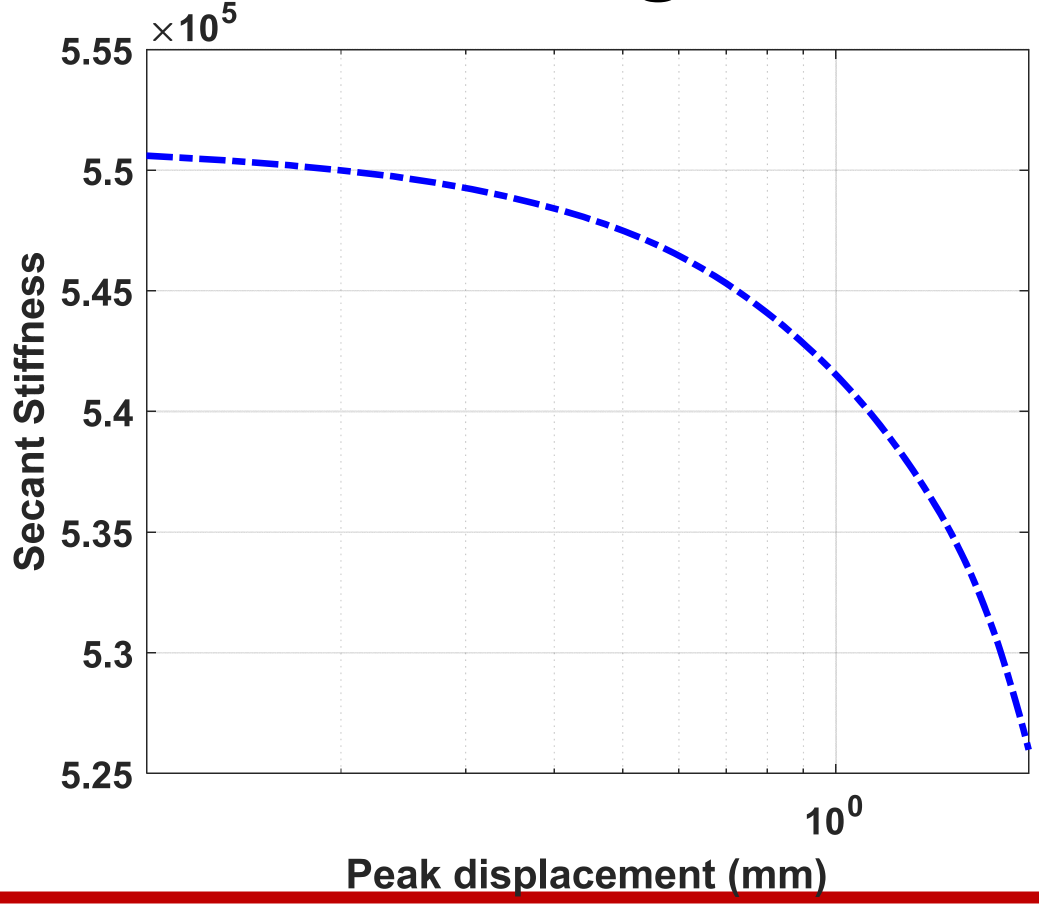


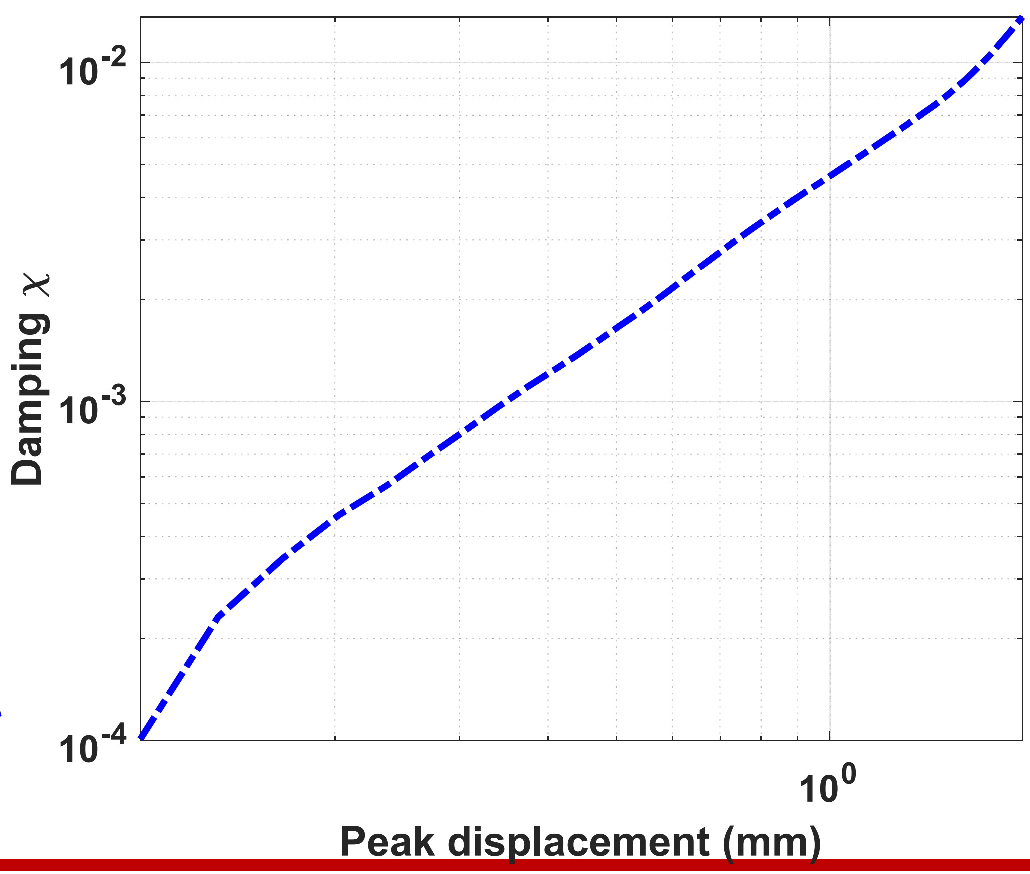
Using Masing's rules for friction ($\boldsymbol{\mu = 0.6}$)





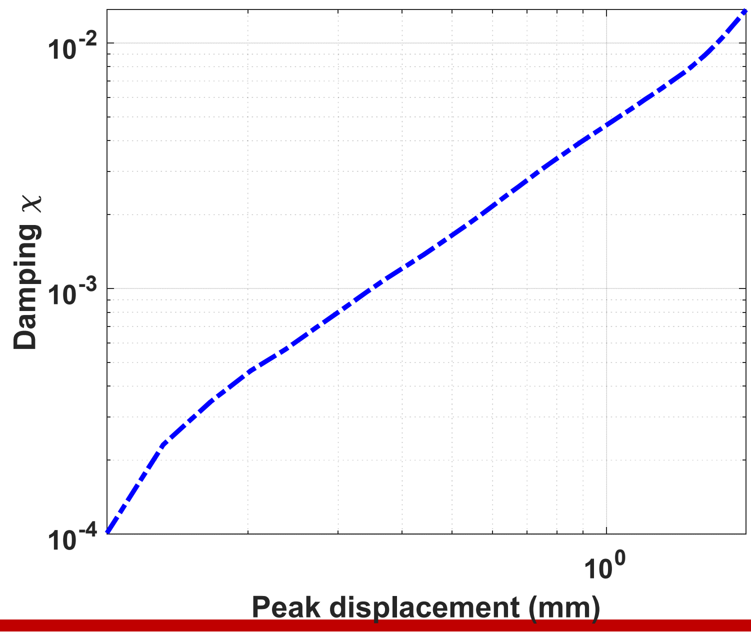
Stiffness change and damping due to friction

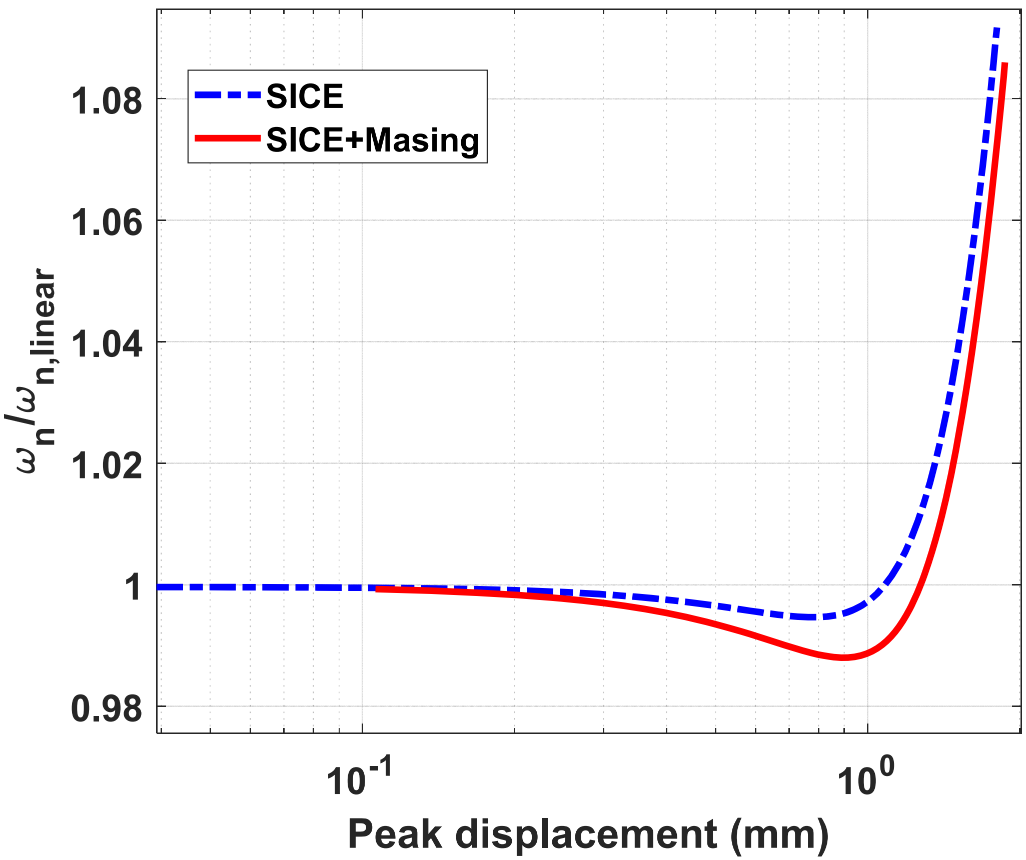




Combining the two models

- To combine the stiffness:
  - $\omega_n(X) = \sqrt{(\omega_{n,geo}(X))^2 + dK_{secant}}$
- Damping only due to friction
  - $\xi = \xi_{joint}$





Computational cost

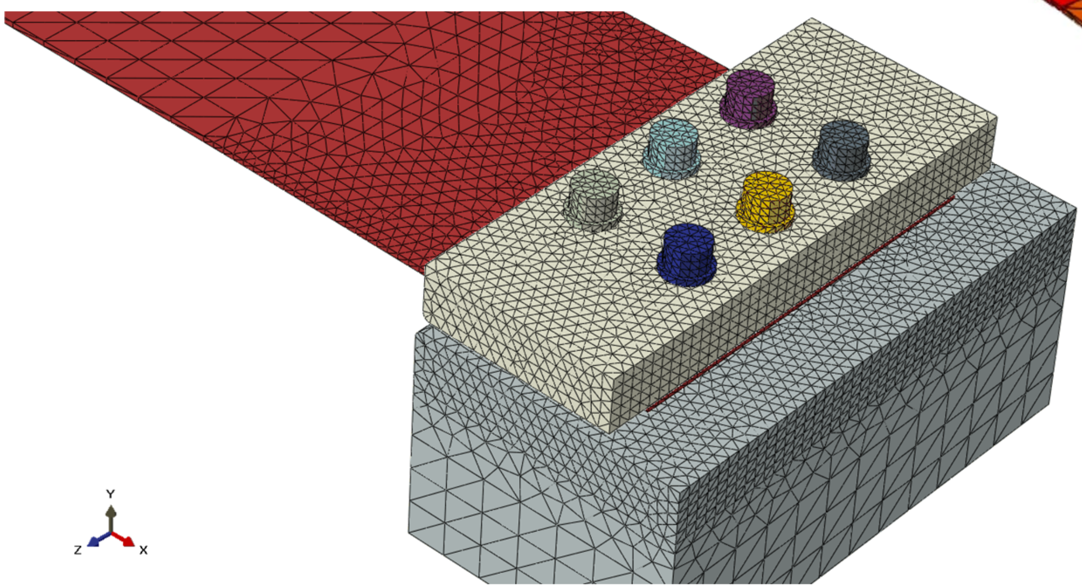
	Preload + Modal analysis	Quasi-static analysis	Processor
2D model	7781 s	735 s	Intel(R) Core(TM) i7 CPU 950 @ 3.07GHz
3D model	46,708 s (12.974 h)	TBD	Intel(R) Core(TM) i7-7700 CPU @ 3.6GHz

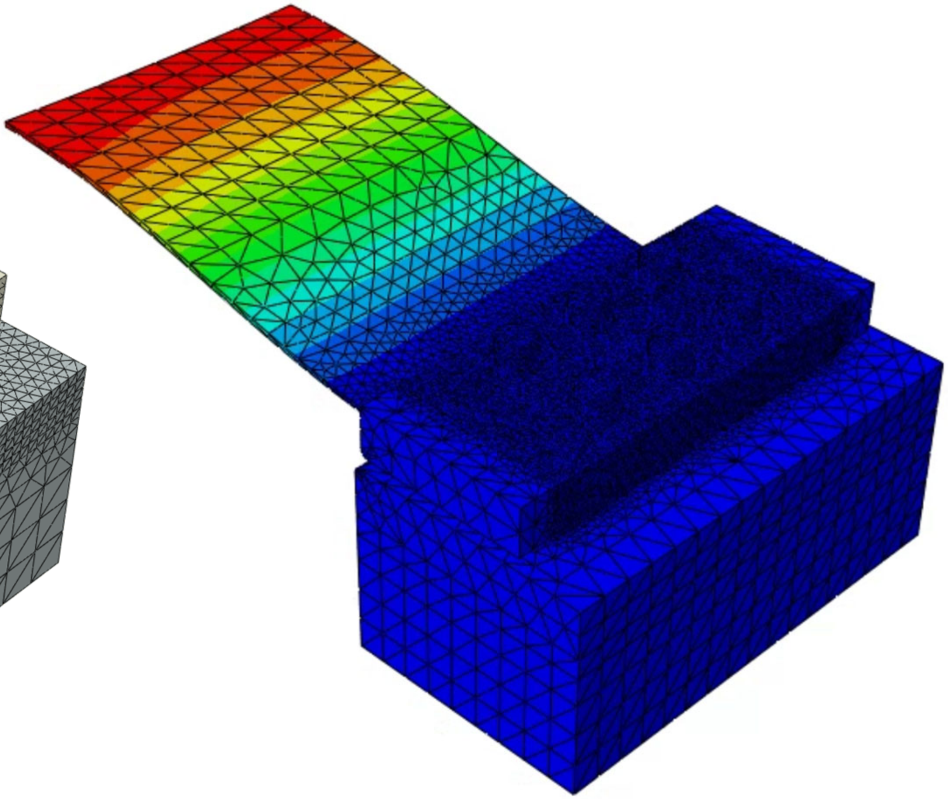
- Time above is the total CPU time taken by Abaqus
- The quasi-static analysis time reported above corresponds to a static analysis step where the model was excited in the shape of mode 1 such that the center of the panel is deflected to 4.5 mm (i.e. 3 times its thickness).
- The 3D model ran on two CPUs, so, the total computation time (wall clock time) was approx. 7 hours.
- Additionally, NNM computation of SICE ROM took about 25s with Intel core i7-7700K 4.2GHz with 64 GB RAM

Future work

- Additional factors need to be considered to adequately address the uncertainties
- (How) can we verify the proposed reduced-order modeling approach?
  - Using ABAQUS dissipation estimate
  - Simulating a complete loading cycle to compare
  - Dynamic simulations (highly computationally expensive)
  - 3D model
  - Experimental results

Future work – same approach for 3D model





Conclusions

- A 2D FE model can be implemented to significantly speed up nonlinear computations
- QSMA+SICE can be used to predict the amplitude-dependent frequency behavior due to geometric nonlinearity
- Masing's rules can be used to fit joint nonlinearity
- Proposed approach: nonlinearity represented as a parallel arrangement of a SICE ROM and a Masing model

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Appendix

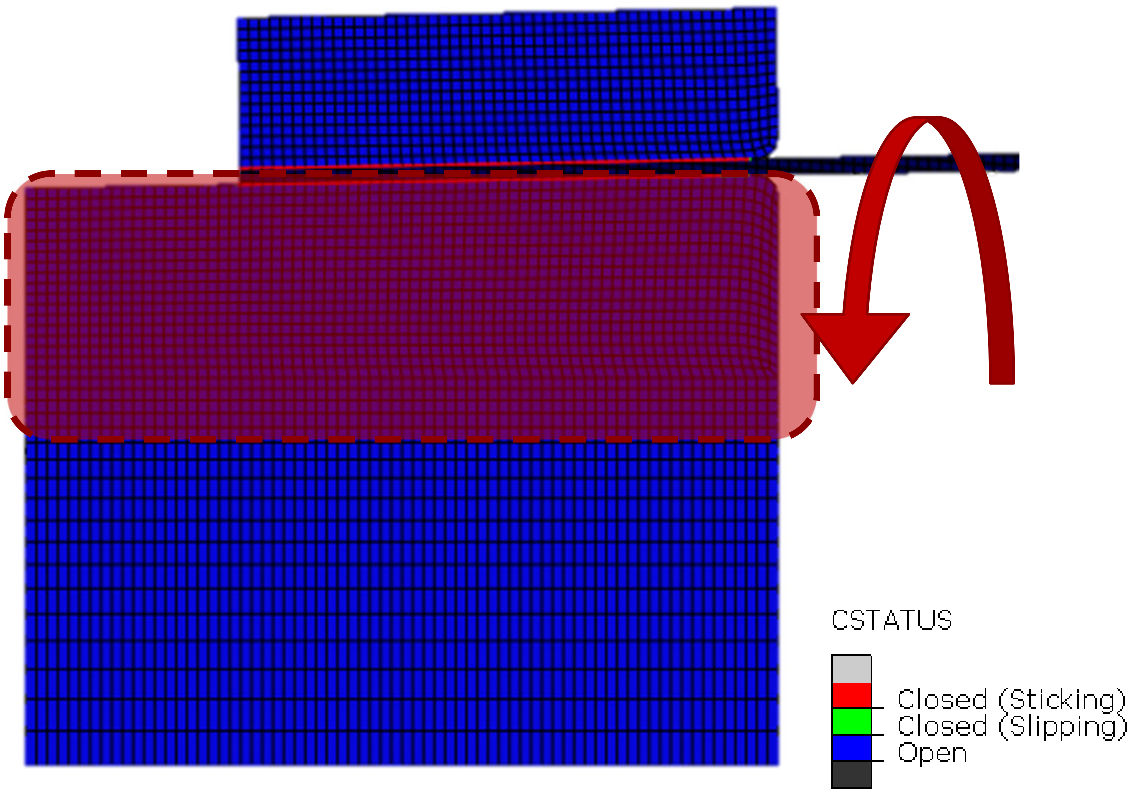
Check for Nonlinear Coupling: Harmonics

Freq (Hz)	Harmonic	Harm. Freq
116.37	-	-
206.79	3	349
239.55	3	349
412.78	3	349
472.34	3	349
681.5	5	581
780.79	5	581
894.88	7	814
1016.9	7	814
1099.6	9	1047
1169.5	9	1047

- For coupling: harmonic + mode shape compatibility
- Most likely modal interaction between B1 and B3.
  - If Mode 1 to stiffen from 116 to 157 Hz.
- Interaction possible between B1 and B4 if Mode 1 stiffens to 156 Hz, but these shapes are likely incompatible.
- Could asymmetry cause an interaction between B1 and T2?
  - Requires Mode 1: 116 →137.5 Hz

Preliminary analysis will neglect modal coupling.

Using the 2D model to study joint nonlinearity



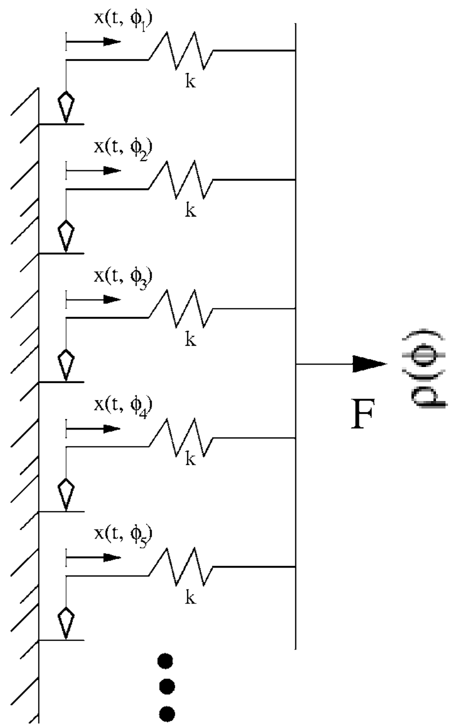
Non-parametric Iwan model can predict the frequency and damping behavior of joints

- The Iwan model effectively captures the modal behavior due to joints.
- Distribution function $\rho(\phi)$: Measure of density of sliders having strength ϕ
- Shetty and Allen[1] showed that a non-parametric Iwan model can be derived from force-displacement data obtained quasi-statically.

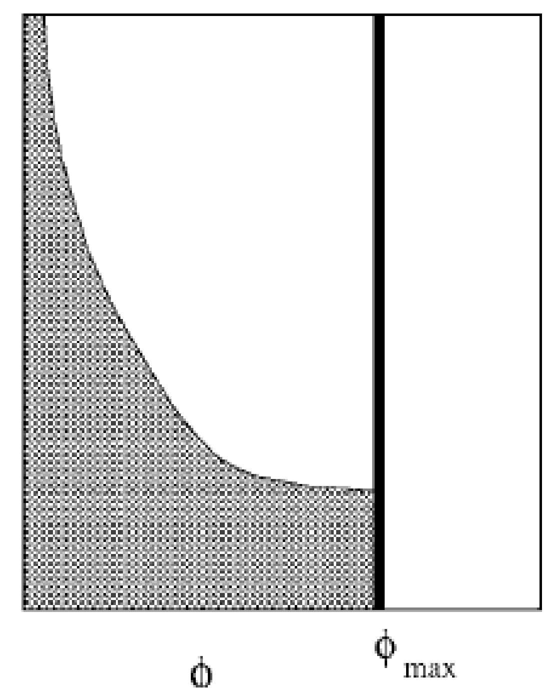
$$\boldsymbol{\rho}(\boldsymbol{\phi}) = -\left.\frac{\partial^2 F_{nl,joint}}{\partial q^2}\right|_{q=\phi}$$

If we know $F(q)$ then we can derive the distribution function $\rho(\phi)$

- Frequency and damping can then be obtained by simulating impulse response and post-processing using [3]



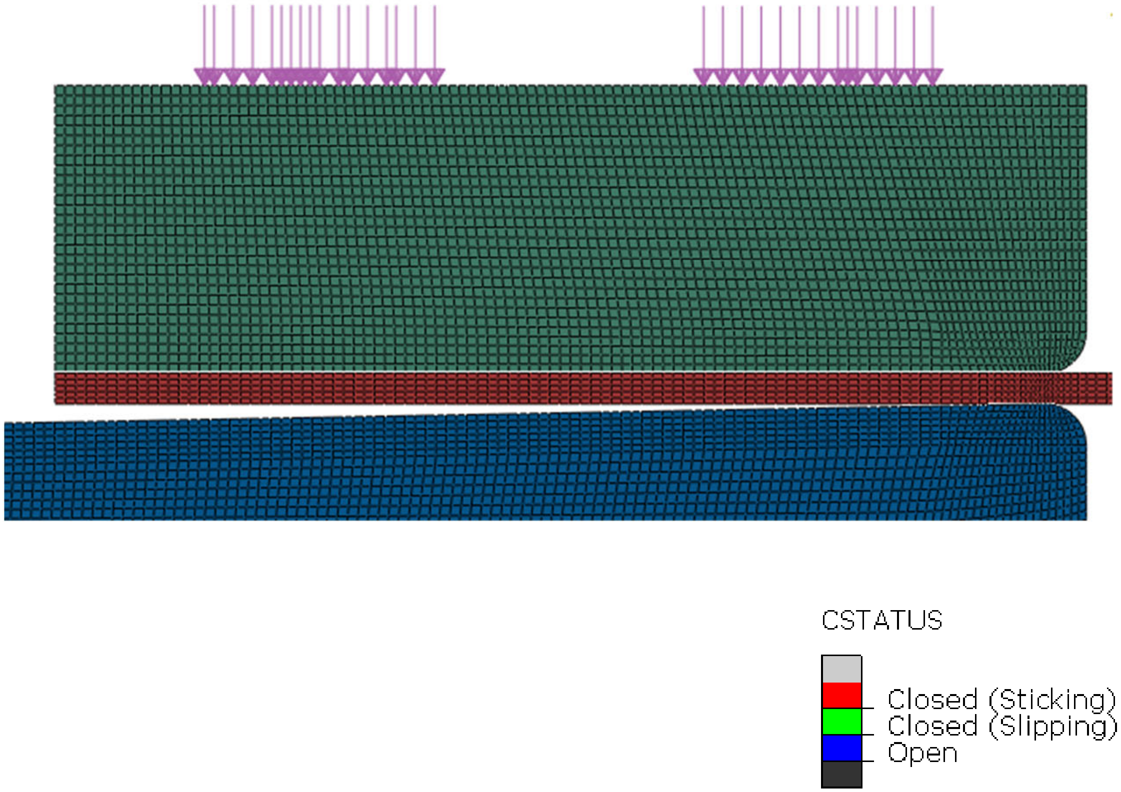
Parallel-series Iwan model

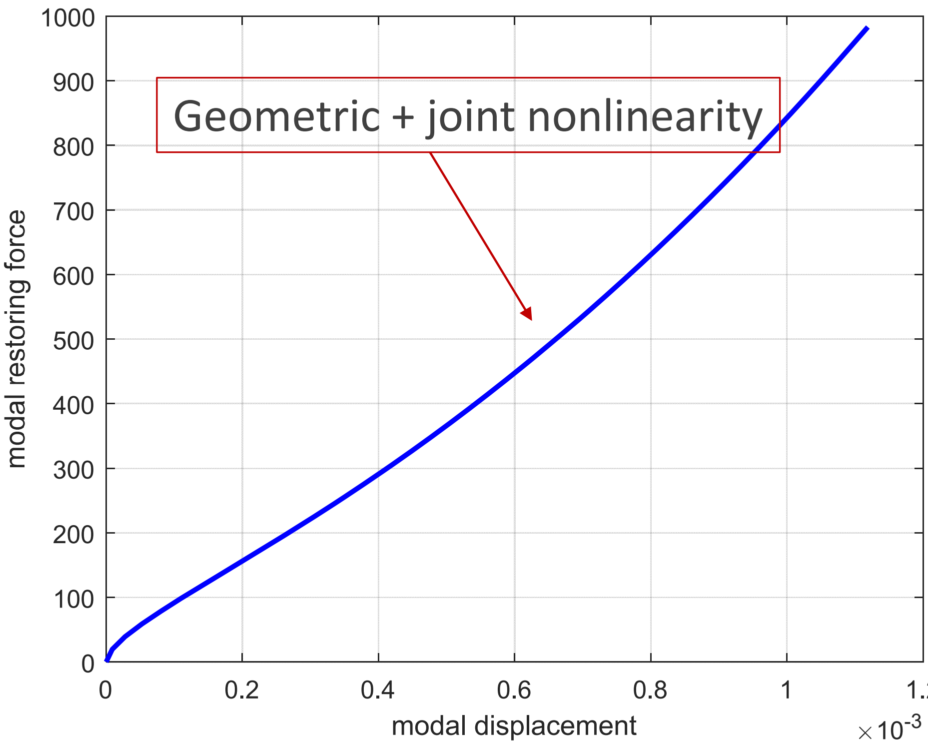


Example of distribution function (4-parameter Iwan model [2])

[1]D. SHETTY AND M. S. ALLEN, "A GENERAL IWAN ELEMENT DERIVED FROM QUASI-STATIC FORCE-DISPLACEMENT DATA," ,"40TH IMAC, ORLANDO, FLORIDA, 2021.
[2]D. J. SEGALMAN, "A FOUR-PARAMETER IWAN MODEL FOR LAP-TYPE JOINTS," *JOURNAL OF APPLIED MECHANICS*, VOL. 72, NO. 5, 2005,.
[3]B. MOLDENHAUER, A. SINGH, M. ALLEN, AND D. ROETTGEN, "EXTENSIONS TO A METHOD FOR CHARACTERIZING INSTANTANEOUS FREQUENCY AND DAMPING OF NONLINEAR SYSTEMS,"40TH IMAC, ORLANDO, FLORIDA, 2021.

Finer mesh needed to better capture slip





QSMA can be used to obtain the nonlinear restoring force

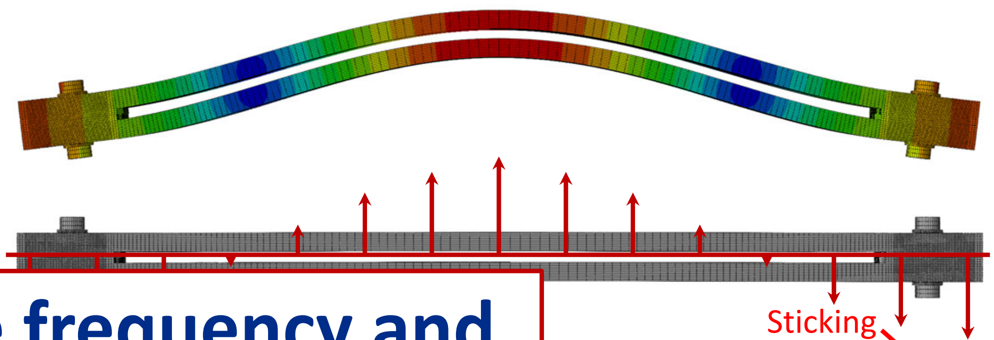
Apply a static LOAD in the shape of the mode of interest.

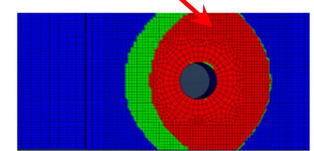
- Simulates inertial loading during vibration
- Deform the stru
  according to th

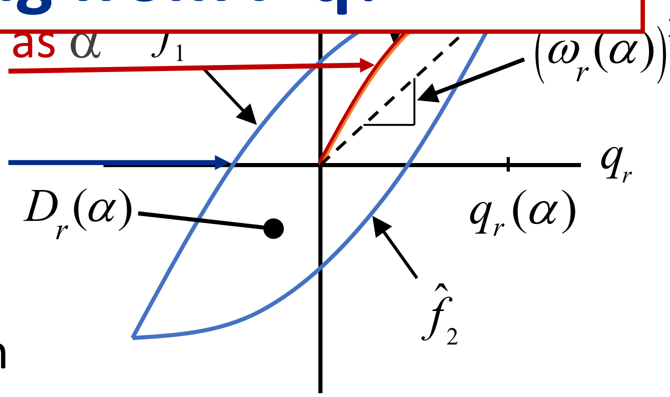
  $\mathbf{f} = \mathbf{M}$

- Solve for modal response $q_r = \boldsymbol{\psi}_r^T \mathbf{M}\mathbf{x}$ as α ramps to some peak force level.
- For joint nonlinearity→ Masing's rules used to obtain hysteresis loop.
  This case→ likely inaccurate.
- Simulating the hysteresis loop for each amplitude value would be expensive

How to calculate frequency and damping from F-q?







[1] ROBERT M. LACAYO AND MATTHEW S. ALLEN. UPDATING STRUCTURAL MODELS CONTAINING NONLINEAR IWAN JOINTS USING QUASI-STATIC MODAL ANALYSIS. MECHANICAL SYSTEMS AND SIGNAL PROCESSING, MARCH 2019.

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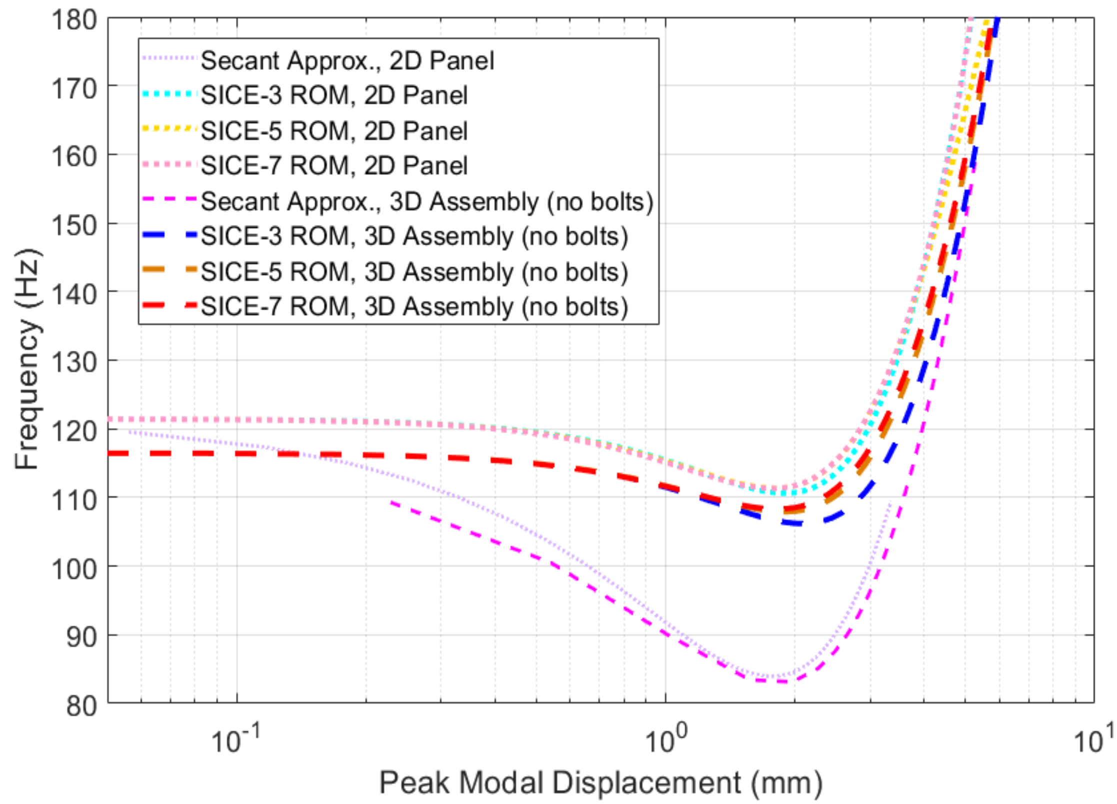
- Softening followed by hardening behavior appears when the vertical defection is from 0.5 to 1 panel thickness
- Backbone curve computation of SICE-3 ROM took about 25s with Intel core i7-7700K 4.2GHz with 64 GB RAM



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