Using Reduced Order Models to Predict Modal Coupling in Jointed Structures

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ABSTRACT

Finite element models can be used to capture the nonlinear behavior exhibited by bolted joints in structures. The nonlinearity leads to a shift in the frequency and damping of modes of vibration as a function of vibration amplitude. However, to model this accurately, a high-resolution mesh is often required at the interfaces which can cause the model to be prohibitively expensive to run. As a result, reduced order models (ROMs) are used to decrease the size of the model, while preserving the global behavior. ROMs are often calibrated to reproduce the behavior of individual modes, including the amplitude dependent damping and frequency backbone curves. Recent studies have shown that these nonlinear modes can couple when simultaneously excited such that the excitation of a particular mode may influence the frequency and damping of another. The objective of this research is to evaluate the ability of various ROM strategies for jointed structures to capture this modal coupling. The new Multi-mode Quasi-Static Modal Analysis approach is used to predict the modal coupling on various ROMs that are meant to describe a full-fidelity finite element model of a 2D cantilever beam. The full-fidelity model is taken as the truth model and time integration is used to further validate the predictions. The ROMs considered include an industry standard "spidering" ROM with physical Iwan elements in place of the joints and a System-Level Characteristic Constraint (S-CC) ROM, where the S-CC modes are constrained with Iwan elements.

Keywords: Joints, Modal Coupling, Reduced Order Models, Spidering, QSMA

INTRODUCTION

Large finite element models comprised of multiple piece parts are typically connected by joints. As a result, this introduces uncertainty in modeling the physics involved with frictional contact and the effective stiffness and damping of the joint. Consequently, frequency and damping have been observed to change with excitation amplitude. To form a predictive model of this phenomenon, significant mesh refinement is required at the interfaces, often being computationally prohibitive [1]. As a result, in industry most dynamic models have a relatively coarse mesh near the joints and model reduction strategies are used to reduce the model to a small number of interface nodes, which may then be connected with joint models whose parameters may be tuned to match the linear/nonlinear characteristics of the full order model. While many methods exist, this work will examine the Hurty/Craig-Bampton (HCB) reduction in conjunction with "spidering" and the System level Characteristic Constraint (S-CC) methods [2], [3]. The former relies on using stiff constraints to reduce an interface to a single node, whereas the latter uses the deformation shapes of the interface modes as the basis of reduction. For a review and usage of these techniques, refer to [4], [5] and [6], [7], respectively. In recent studies, both methods in conjunction with Quasi-Static Modal Analysis (QSMA) have been shown to accurately model the nonlinear amplitude dependent frequency and damping behavior of a single mode.

While modeling a single mode provides insight into the joint nonlinearity of the model, the coupling phenomena between modes is neglected. Recent studies have shown that there exists a coupling between two modes whose frequencies are not

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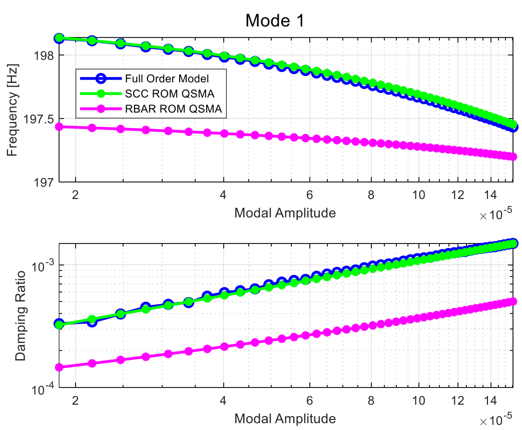
integer related, such that the effective frequency and damping of one mode is influenced by the other mode [8], [9]. To model this, dynamic transient solutions are effective, although the computational costs remain prohibitive. Singh et al. [9] introduced an extension to the quasi-static framework that was computationally efficient to form a conservative bound on the coupling between modes. While effective, both methods can be prohibitive in larger models. For reference, a model with ~350,000 elements requires 8 hours for a coupled quasi-static solution, or days for a dynamic transient solution on 12 cores. Although not shown in this paper, the multi-mode quasi-static analysis will be referenced in the presentation.

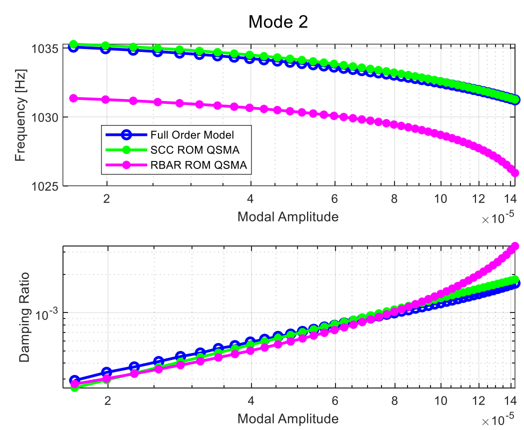
Utilizing the model reduction techniques and the novel coupled quasi-static framework, this work seeks to determine the effectiveness of ROMs to capture the coupling between modes. This is demonstrated on a 2D cantilever beam model that has been previously studied with both reduction techniques and on single uncoupled modes [7]. The ROMs are first evaluated based on their ability to capture the true amplitude dependent frequency and damping for a single mode. The solution for that mode is then perturbed by exciting another mode simultaneously and applied to both the full FEM and reduced models and the results are compared to see how well each ROM captures mode coupling. This work shows that the proposed ROM approaches show a perturbation about a single mode excitation, although further work is necessary to determine whether the ROMs capture the real coupling that exists in the truth model.

ANALYSIS

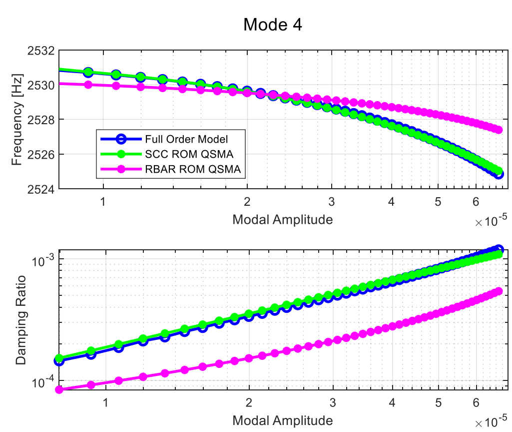
Prior to examining the coupling between modes in the ROMs, each ROM was tuned to capture the nonlinearity in the first three bending modes of the 2D cantilever beam. For the RBAR model, linear springs are added in the normal and rotational directions, whereas an Iwan element is added in the axial direction to model the slip at the interface [10]. The linear stiffnesses are obtained from the work by Singh et al. [7]. Figure 1 shows the calibrated ROMs to the nonlinear frequency and damping of the three modes of interest.



(a)



(b)

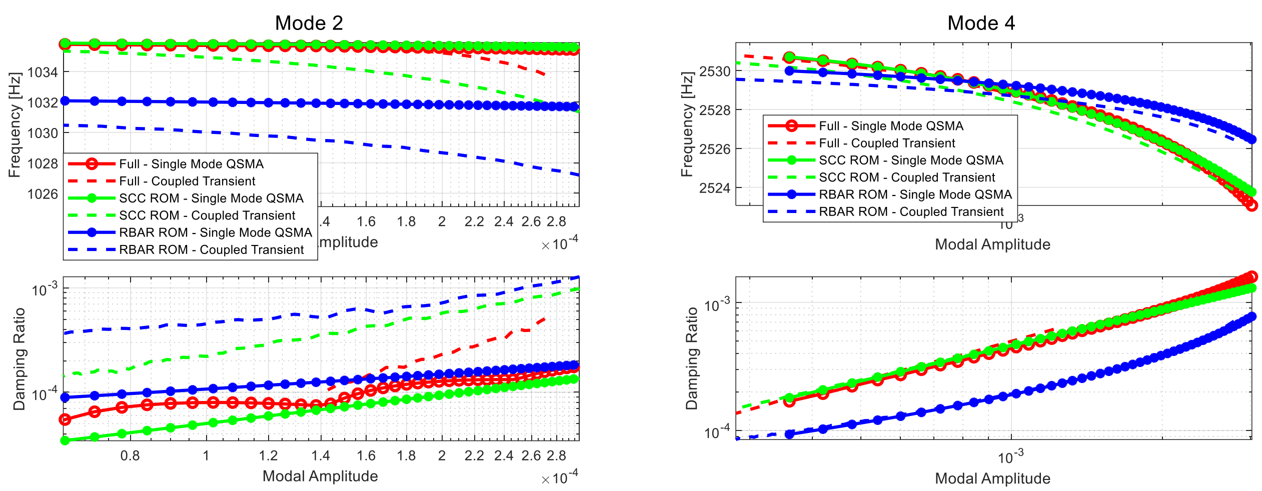


(c)

Figure 1: Single mode calibration for the (a) first, (b) second, and (c) third bending modes of the cantilever beam structure

The results show that the S-CC reduced model captures the behavior of each mode very well, both frequency and damping. In contrast, the RBAR ROM was able to capture each natural frequency well (note that the scale on the frequency axis above is quite fine), but not as well as the S-CC ROM. Furthermore, the RBAR ROM is only able to accurately capture the damping of Mode 2, and even there it shows some inaccuracy. When compared to the full model nonlinear simulation, this results in RMS frequency errors less than 0.03% for both ROMs, and RMS damping errors between 4-7% and 10-61% for the S-CC and RBAR ROM respectively. While the RBAR ROM does not have excellent correlation as compared to the S-CC ROM, both ROMs provide a baseline to compare to the multi-mode excited ROMs.

To evaluate the coupling between modes, the dynamic response was found for a case where the structure was initially deformed into the shapes of Modes 2 and 4 simultaneously where Mode 4 has 10x the peak amplitude of Mode 2, and then the structure was released and its free response was found. The dynamic responses were processed to compute the effective natural frequency and damping of each mode in this case where the modes are excited simultaneously. Based on the study in [7], it is expected that these two modes would couple more with one another than with the first mode. Figure 2 shows the frequency and damping predicted by single-mode QSMA for each model, compared with the dynamic responses. The dynamic response of the full order FEM was computed in Abaqus in ~36 hours on 12 cores whereas the coupled S-CC ROM simulation was found using the Newmark-Beta method in 0.2 minutes on a single core. The following figures show the full model and ROM single mode quasi-static responses (as shown in Fig. 1) and the coupled dynamic responses. The difference between the dynamic response and the single-mode QSMA solution gives a measure of how much coupling each model captures.

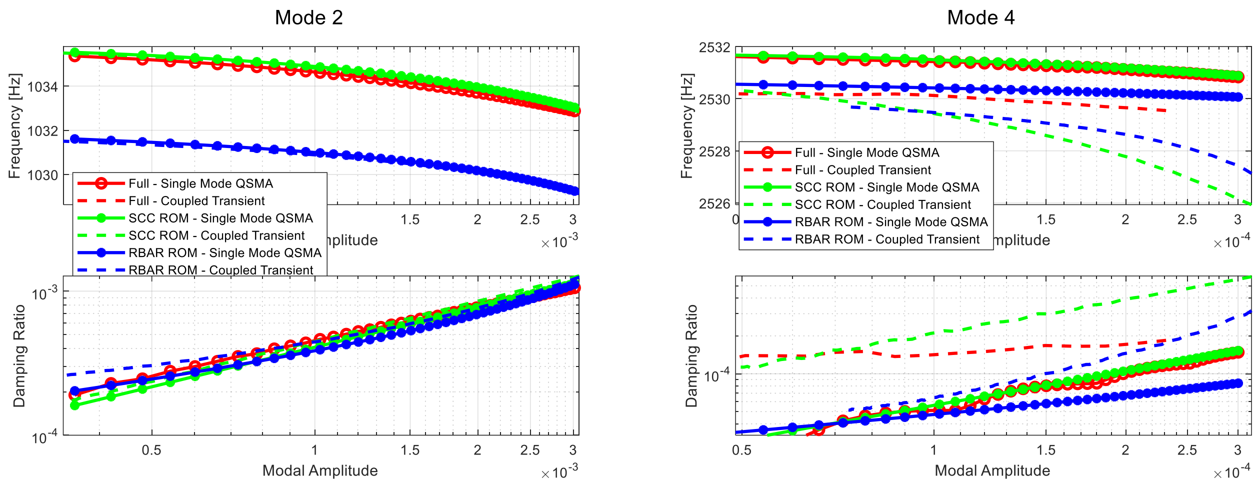


(a) (b)

Figure 2: Dynamic responses with multi-mode excitation where Mode 4 has 10x the peak excitation amplitude of Mode 2. (a) Mode 2, (b) Mode 4. The results are compared to single-mode QSMA predictions for reference.

The results show that all of the ROMs correctly predict that the effect of Mode 2 on Mode 4 will be negligible (i.e. see Fig. 2b), and they all predict that Mode 2 will be influenced by Mode 4 (Fig. 2a). However, the ROMs over-predict the effect of the coupling on Mode 2. According to the truth model, Mode 4 should have a small effect on Mode 2 even though Mode 4 is ten times stronger.

The study was repeated for the case where Mode 2 has 10x the peak excitation amplitude of Mode 4, and the results are shown in Fig. 3. The S-CC ROM is able to predict the coupled effect on the damping of Mode 4 closer to the full model, whereas the RBAR ROM underpredicts the coupling. Both ROMs overpredict the frequency response of Mode 4. Similar to Fig. 2, both ROMs predict little to no effect of the sub-dominant mode (Mode 4) on the dominant Mode (Mode 2).



(a) (b)

Figure 2: Dynamic responses with multi-mode excitation where Mode 2 has 10x the peak excitation amplitude of Mode 4. (a) Mode 2, (b) Mode 4. The results are compared to single-mode QSMA predictions for reference.

CONCLUSION

This work examined the use of ROMs to explore the modal coupling phenomenon. While ROMs have been shown to be effective in capturing the response of a single mode, they offer insight as a conservative estimate for the coupling. In the cases where a mode was unaffected by another mode (Fig. 2b, Fig. 3a), the ROMs were successful in that both predicted that the coupled response would align closely with the single mode response. In the case studies to date, the truth model shows relatively weak modal coupling, and the ROMs both seem to over-predict that to varying degrees. Future work will seek to extend these ROMs to larger structures that have different types of motion, e.g., shearing, sliding, etc. Furthermore, additional work is required to understand the degree of coupling ROMs are capable of simulating.

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