Experimental Modal Substructuring to Extract Fixed-Base Modes from a Substructure Attached to a Flexible Fixture

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Abstract:

It is well known that fixed boundary conditions are often difficult if not impossible to simulate experimentally, but they are important to consider in many applications. Mayes and Bridgers ["Extracting Fixed Base Modal Models from Vibration Tests on Flexible Tables," IMAC XXVII, Orlando, Florda, 2009.] recently presented a method whereby one can determine the fixed base modes of a structure from measurements on the system and the fixture to which it is attached. They used modal constraints in conjunction with an admittance approach to perform the substructuring computation, demonstrating that the first fixed-base mode could be estimated accurately using the coupling procedure, even though the free fixture had a mode at almost the same frequency. This work builds on that by Mayes and Bridgers, but employs modal substructuring instead of frequency-response based methods to estimate the fixed-interface modes. The method is validated by applying it to a experimental measurements from a simple test system meant to mimic a flexible satellite on a shaker table. A finite element model of the subcomponents was also created and the method is applied to its modes in order to separate the effects of measurement errors and modal truncation. Excellent predictions are obtained for many modes of the fixed-base structure, so long as modal truncation is minimized, verifying that this modal substructuring approach can be used to estimate fixed-base modes of a structure without having to measure the connection point displacements and rotations and even with the limitations inherent to real measurements.

1. Introduction

Testing and model validation campaigns often include both a low-vibration level modal test, used to extract the modal parameters of the system, and subsequent shaker testing at higher amplitudes to evaluate the durability of the system. Modal parameters extracted from the former are correlated with FEA models which are then used to predict the life of the structure in a specified environment. The latter tests are meant to verify the FEA predictions either by verifying that the test article survives the vibration environment or that the strains measured at critical points are the same as those predicted by the model. There are a number of reasons for desiring to combine those two tests. First, the high amplitude shaker tests better describe the system in the environment of interest, so it would be preferable to extract modal parameters from those tests in case the structure has any nonlinearity that would change its effective stiffness or damping with excitation level. This assures that the appropriate model is used for the environment of interest. Second, each test increases the time and cost required to develop the system, so significant savings might be realized if one of the tests can be shortened or eliminated. Finally, this approach may help minimize uncertainty in the boundary conditions that are applied in shaker testing, and it may be preferable to use the fixed-base modal parameters for model updating rather than the free-free modal parameters. Blair discussed some of these issues in the context of model validation for space shuttle payloads in [1]. However, it is well known that fixed boundary conditions can be exceedingly difficult to achieve in practice. Any real boundary condition has some flexibility, and it very often has an important effect on the system's modal parameters. If the article of interest is made from stiff engineering materials then often no material exists that can provide an adequately rigid boundary condition out to sufficiently high frequencies.

A few previous works have explored methods for predicting fixed-base modal parameters from test measurements, as outlined by Mayes and Bridgers [2]. Using classical frequency-based substructuring (FBS) [3] or impedance coupling [4, 5] methods, one can, in principle, estimate the fixed-base properties of a test article if all of the displacements, rotations, forces and moments between the test article and the supporting structure are measured. This is rarely possible and often leads to ill-conditioning in the coupling equations even if the necessary moments and forces can be measured. Mayes and Bridgers presented a method that avoids the need to measure the interface forces and displacements and which seems to minimize this ill conditioning [2]. They used FBS to couple the modal motions of a shaker table to ground using Allen & Mayes' modal constraint approach [6-8]. An experiment verified that their approach was capable of extracting the first fixed-base mode of a cantilever beam from measurements on a simplified shake table.

This work expands upon Mayes and Bridgers' work in two ways. First, this work employs modal substructuring rather than FBS, so one only needs to manipulate the modal parameters of the structure rather than all of the FRFs. Second, this work considers a large number of modes of both the fixture alone and the fixture+substructure, showing that the approach also works well for higher modes and exploring how high of a test bandwidth is needed to estimate fixed-base modes over a desired frequency range. The proposed substructuring approach is illustrated schematically in Figure 1, where a structure of interest is attached to a flexible fixture (plate). The modes of the substructure of interest are found when it is attached to the flexible fixture, and then modal constraints are used to constrain the motions of the fixture to ground. Modal constraints are constraints applied to modal motions, which are estimated using a modal filter [9], rather than to the motions of physical points or surfaces.

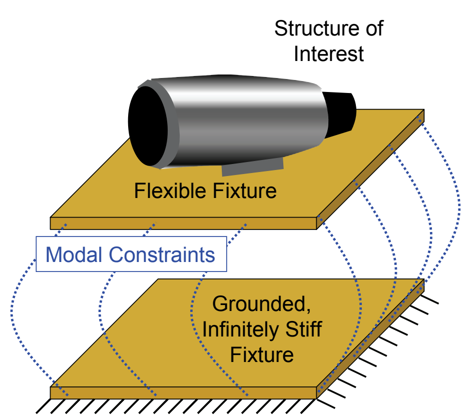

Figure 1: Schematic of proposed modal substructuring approach, which estimates the fixed-base modes of the structure of interest from the measured modes of the structure+fixture.

The rest of this paper is organized as follows. Section 2 presents the proposed modal substructuring technique, which estimates fixed base modal parameters from measurements on a flexible fixture. Section 3 presents the hardware used to evaluate the proposed methodology, a stiff rectangular plate with a beam attached perpendicular to the plate, and discusses experiments used to find the modes of both the fixture (plate) and the fixture+structure (plate+beam). In Section 4, the technique is applied with finite element derived modes for both subcomponents, in order to assess the proposed substructuring technique when the measurements are perfect. Section 5 summarizes the conclusions.

2. Theoretical Development

Suppose that one performs a modal test [4] to obtain N modes of vibration of a structure of interest at a certain set of measurement points, when it is attached to a flexible fixture. One then has estimates for the following modal parameters,

$$\omega_r \quad , \quad \zeta_r \quad , \quad \begin{bmatrix} \phi_f \\ \phi_s \end{bmatrix} \tag{1}$$

where ω_r are natural frequencies of the structure, ζ_r its damping ratios, ϕ_f and ϕ_s denote the mass-normalized mode shapes at the measurement points on the fixture and substructure of interest, respectively, and $r=1...N$. The fixture is a dynamic system itself, although it is meant to approximate a rigid boundary condition. For example, the shake tables discussed in [2] can be thought of as stiff, translating fixtures.

One can also obtain the modal parameters of the fixture alone (without the structure of interest attached), either through test or analysis, so its modal parameters are denoted

$$\omega^{\text{fixt}}{}_r \quad , \quad \zeta^{\text{fixt}}{}_r \quad , \quad \phi^{\text{fixt}}{}_f \, , \tag{2}$$

The natural frequencies and damping ratios of the fixture are not needed for this approach, only the mass normalized mode shapes of the fixture, $\phi^{\text{fixt}}{}_f$.

If the fixture were truly rigid and perfectly fixed to ground, then ϕ_f would be zero for all of the measured modes of the fixture+structure. In practice this will not be the case, but one can approximate the measured fixture motion, y^{fixt}, as follows in terms of N^{fixt} modes of the fixture,

$$y^{\text{fixt}} \approx \phi^{\text{fixt}}{}_f q^{\text{fixt}} \, , \tag{3}$$

where q^{fixt}, denotes the modal coordinates of the fixture. One can then estimate the participation of each of the fixture modes in the measured response of the fixture+structure by multiplying the upper partition of the mode shapes in eq. (1) by the pseudo-inverse of the fixture mode shapes (i.e. using a modal filter [9]),

$$q^{\text{fixt}} \approx \left(\phi^{\text{fixt}}{}_f\right)^+ \phi_f \, , \tag{4}$$

where $()^+$ denotes the Moore-Penrose pseudo-inverse of the matrix. The estimate of the fixture modal amplitudes is only meaningful if one has at least as many measurement points as modes of interest and if the measurement locations on the fixture are chosen such that $\phi^{\text{fixt}}{}_f$ has full column rank. Our desire is to estimate the modal parameters of the substructure when attached to a base that is truly fixed. This work proposes to do that by applying the constraints,

$$q^{\text{fixt}} = 0 \, , \tag{5}$$

to the modes of the fixture+structure using the Ritz method [6, 7, 10]. In terms of the modal coordinates of the fixture+subsystem, q, the constraint equations are

$$\left[\left(\phi^{\text{fixt}}{}_f\right)^+ \phi_f\right] q = 0 \, , \tag{6}$$

where the term in brackets is an N^{fixt} by N matrix of constraint equations. This matrix is denoted $[a]$ in the text by Ginsberg [10], or $\mathbf{B}$ in the review by De Klerk, Rixen and Voormeeren [3]. The procedure described in either of those works can be used to enforce these constraints and hence to estimate the modes of the fixture+subsystem with the fixture motion nullified. The "ritzscomb" Matlab routine, which is freely available from the first author, was used to perform the calculations for this work.

It is important to note that the constraints above only enforce zero motion at the fixture measurement points if the number of measurement points equals the number of fixture modes. In practice one should use more measurement points on the fixture than there are active modes in order to be able to average out noise or measurement errors on the mode shapes. Hence, the motions of the physical measurement points may not be exactly zero. There may also be residual motion in the fixture that is physical, since one is seeking to constrain an infinite dimensional system with a finite number of constraints. Fortunately, one can readily observe the fixture motions after applying the constraints to see whether the constraints were effective in enforcing a rigid boundary condition. This is illustrated in the following sections and provides a valuable way to check whether enough modes were used in eq. (6).

3. Experimental Application

A simple system, consisting of a 12.35-in. long steel beam attached to a 12 by 10-in. steel plate was constructed to evaluate the proposed approach. The 1-in. by 0.5-in. cross-section beam is the substructure of interest, and the 0.625-in. plate is stiff and so approximates a rigid boundary condition. A schematic of the system is shown in Figure 2, with the left picture looking down the beam onto the plate (the beam extends from the origin in the negative x-direction), and the right figure (green) looking parallel to the surface of the plate showing the beam standing up. The plate was designed such that many of its natural frequencies would be equal to those of the fixed-base beam. This assured that the modes of the plate and beam would interact creating an interesting case study. The measurement grid used in the experiments is also shown, along with the locations used for three uni-axial accelerometers. The red circles show the locations of the accelerometers used to test the plate+beam, the blue show those used when the plate was tested alone to find $\phi^{fixt}{}_{f}$. The accelerometers placed on the plate were actually placed on the underside of the plate so that the structure could be excited from above at each of the points shown.

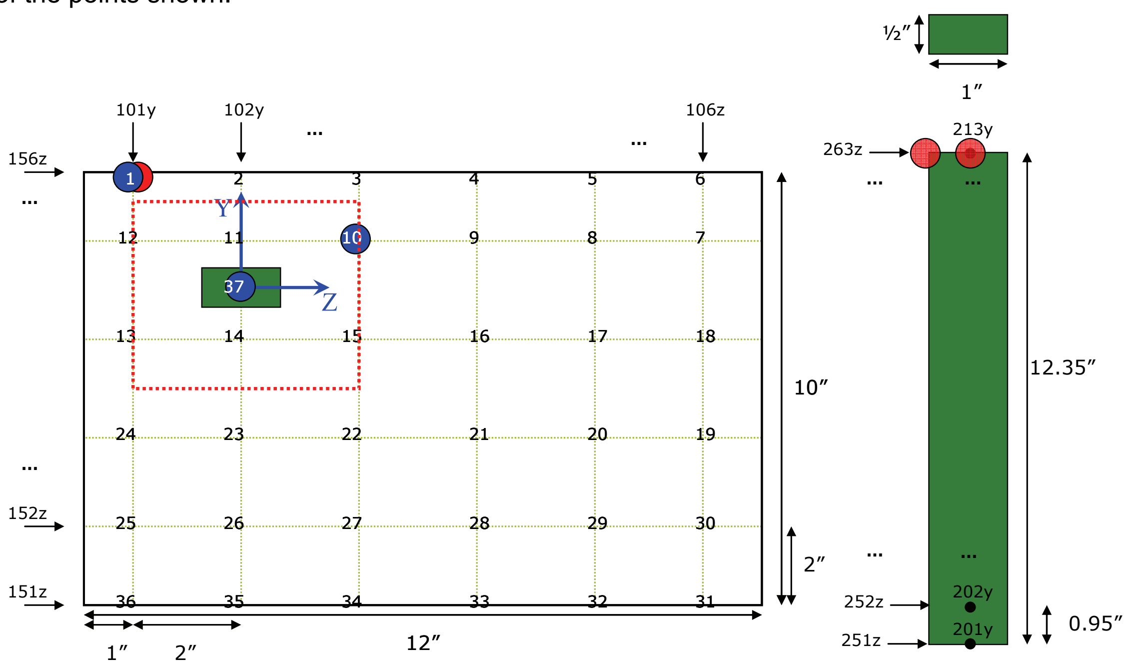

Figure 2: Schematic of plate-beam system used to estimate the fixed-base modes of a steel beam.

The assembly was placed on an inflated rubber tube to simulate free-boundary conditions, which were realized quite effectively since the first vibration mode of the system was about ten times higher than the highest rigid body mode of the system on the inner tube, which conforms to the guidelines in [11]. Figure 3 shows pictures of the setup. The beam is attached to the plate with two hex screws. Some initial measurements were processed with the zeroed-fast Fourier transform nonlinearity detection method described in [12], which revealed that the natural frequencies of the system did decrease significantly with increasing excitation amplitude. To remedy this, the system was disassembled and reassembled with adhesive between the beam and plate and with the bolts very tight. After doing this, the method in [12] no longer revealed significant nonlinearity.

Table 1 lists all of the equipment used to perform a roving hammer modal test of the structure. Three hits were applied to each of the 36 points on the plate, 6 on the y-side, 6 on the z-side, 12 on the y-side of the beam, and 12 also on the z-side of the beam giving 72 total measurement points. Initial analysis revealed that the phase of certain FRFs sometimes showed large and unreasonable delays, apparently due to a fault in the data acquisition software, resulting in distortions to the measurements, so those points were re-tested. Once a reasonable set of FRFs had been obtained, the modes were identified using the AMI algorithm [13-15] and the mode shapes and natural frequencies were imported into Matlab to perform the substructuring analysis. A similar set of tests was also performed on the plate without the beam attached to estimate $\phi^{fixt}{}_{f}$.

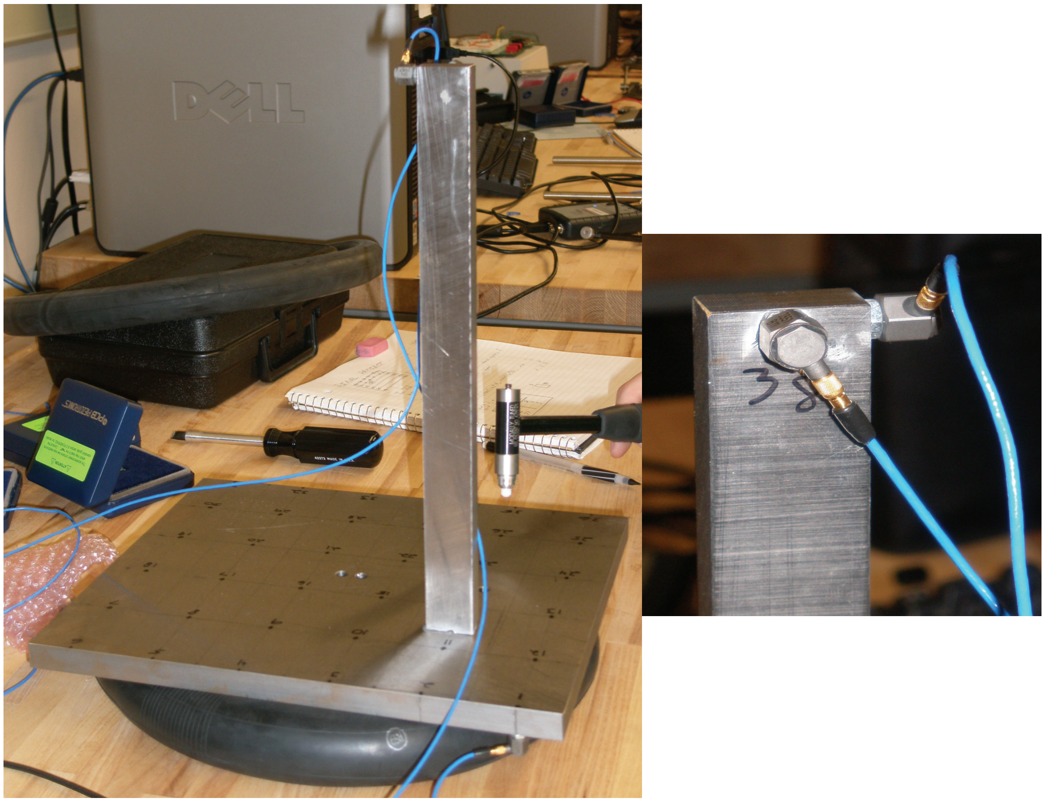

Figure 3: Photographs of the experimental setup. The left picture shows the entire setup with the plate on top of an inflated tube, and the beam mounted on the plate. The right picture shows the two accelerometers located on the tip of the beam.

Equipment	Specifications
Data Acquisition	LDS Dactron Photon II, Model #5880283
Software	RT Pro Photon 6.33
Accelerometers	PCB Piezoelectronics, Model #J351B11
Hammer	PCB Piezoelectronics – Modally Tuned Model #086C01

Table 1: List of equipment used, manufacturer and model.

Table 2 lists the natural frequencies found for the plate and plate+beam. The experiments were designed to extract all modes below 3kHz and the results show that those and quite a few more were extracted. Comparing the natural frequencies before and after adding the beam, we see that the beam causes the first mode of the plate to split, the familiar vibration absorber effect, and similarly due to the interaction of the 2z and 3y modes of the beam with the 3rd and 4th elastic modes of the plate. This is illustrated in Figures 4 and 5, which shows the 3rd and 4th mode shapes of the plate+beam. The surface plot shows the deflection of the plate and the two line plots show the bending of the beam in both the y- and z- directions. The beam bending has an opposite sense in the 4th mode as in the 3rd mode, demonstrating that the beam is acting as a vibration absorber for the plate. In either mode shape the motion of the beam resembles the 2nd analytical mode shape for a cantilever in the y-direction, so one would expect this pair of modes to merge to a single bending mode of the beam once the constraints at the base of the beam are enforced. The beam's displacement in the z-direction is small and presumably dominated by noise or errors in the alignment of the hammer blows.

Nat. Freqs (Hz) of Elastic Modes				
Mode	**Plate Alone (Fixture)**	**Mode**	**Plate + Beam**	**Plate+Beam Modes:**
1	670.5	**1**	130.7	Beam 1y
2	893.7	**2**	224.0	Beam 1z
3	1344.0	**3**	628.3	Beam 2y + Plate
4	1620.3	**4**	693.8	Beam 2y + Plate
5	1850.4	**5**	902.4	Plate
6	2538.0	**6**	1254.4	Beam 2z + Plate
7	3107.8	**7**	1350.2	Beam 2z + Plate
8	3143.6	**8**	1657.8	Plate
9	3591.1	**9**	1770.7	Beam 3y + Plate
10	4082.0	**10**	1797.8	Beam 3y + Plate
11	4814.5	**11**	1906.1	Beam 3y + Plate
12	4837.7	**12**	2311.9	Plate
13	5220.6	**13**	2995.7	Plate
14	5561.4	**14**	3107.7	Plate
15	6765.1	**15**	3233.0	Plate
16	7093.4	**16**	3424.5	Beam 3z + Plate
17	7147.8	**17**	3522.8	Beam 4y + Plate
18	7179.0	**18**	3845.5	Plate
19	7808.4			
20	8288.7			
21	8414.8			

Table 2: Experimentally measured natural frequencies (Hz) of the Plate and Plate+Beam.

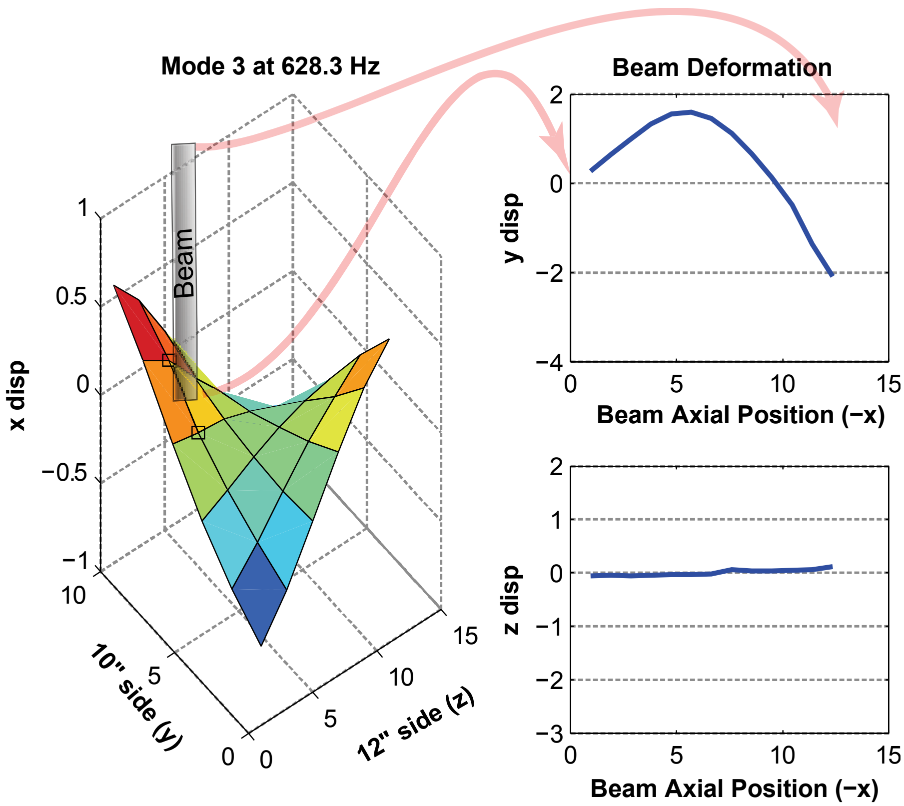

Figure 4: Mode shape of 3rd elastic mode of the plate+beam. The left plot shows the deformation of the plate, while the panes on the right show the deflection of the beam in the y- and z- directions. The beam is fixed to the plate midway between the two square markers, as illustrated schematically on the top.

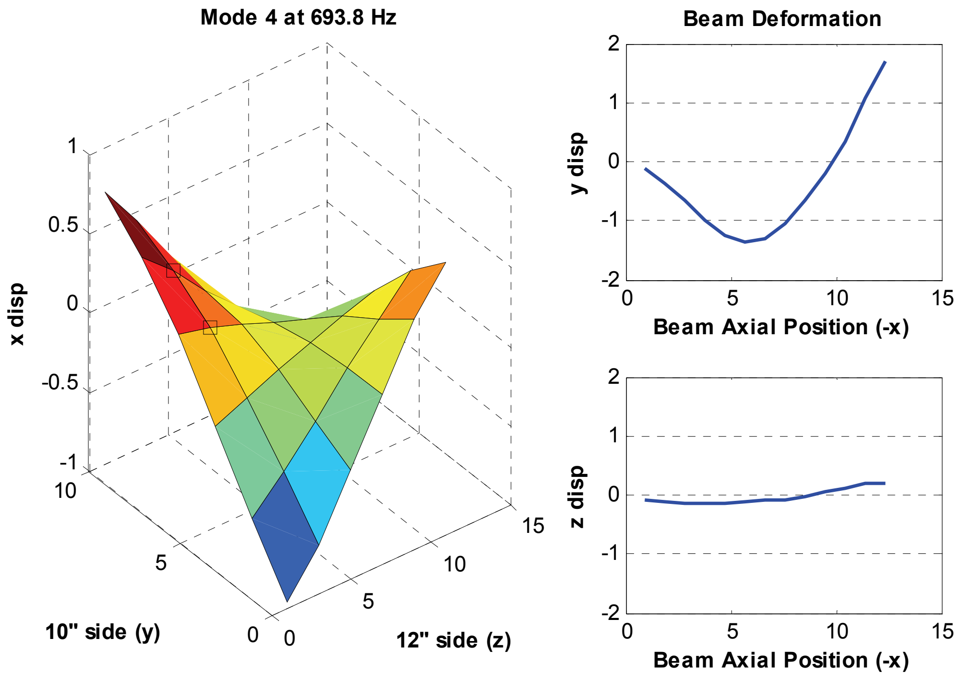

*Figure 5: Mode shape of 4^{th} elastic mode of the plate+beam (see description for Figure 4**Error! Reference source not found.**). These mode shapes show that the 2^{nd} mode of the beam interacts with the 1^{st} mode of the plate, so the dynamics of the plate must be accounted for to estimate the fixed-base modes of the beam.*

The experimentally measured plate and plate+beam modes were used in the procedure described in Section 2 to estimate the fixed-base modes of the beam. That procedure requires an estimate of the rigid body modes of the system, and rather than measure those, a finite element (FEA) model (the same one described in Section 4) was used to estimate them. Hence, CMS was performed by combining six FEA rigid body modes with 18 experimentally measured plate+beam modes, and then creating $\phi^{fixt}{}_{f}$ with six FEA rigid body modes for the plate alone and six measured plate modes for a total of 12 constraints in eq. (6). The density of the FEA model was adjusted to reproduce the measured mass of the system. (The FEA model was also validated by comparing it with the experimental results, as described in Section 4, but that is not particularly relevant to the results presented here.) Table 3 shows the modal substructuring (CMS) predictions as well as the analytical natural frequencies of a cantilever beam with the same properties as the actual beam. The first five CMS predicted natural frequencies agree fairly well with the analytical ones, but beyond the fifth the predicted natural frequencies are inaccurate. The usual rule of thumb for CMS predictions is that coupled system predictions are typically valid over a bandwidth, BW, if modes for each substructure are used out to 1.5*BW or 2*BW. Here, modes out to 3000 Hz were used in the CMS predictions, so one would expect the results to be accurate out to 1500 or 2000 Hz. The errors in the CMS predicted natural frequencies are below 10% for the first five modes, which suggests that this rule of thumb may be valid for this CMS procedure. One also observes that the modes that involve z-direction motion have larger percent errors. The z-direction is the stiffer of the two bending directions of the beam.

CMS Prediction with 12 MCFS Constraints	Analytical Fixed-Base Beam Freq. (Hz)	Percent Error in CMS Prediction
104.1	107.35	-3.0%
195.2	214.70	-9.1%
651.4	672.8	-3.2%
1230.5	1345.5	-8.6%
1777.1	1883.4	-5.6%
1823.0	3691.4	-50.6%
2369.4	3766.8	-37.1%
2923.8	6102.2	-52.1%
3045.3	7382.9	-58.8%
3352.8	9115.6	-63.2%
3503.8	12204.3	-71.3%
3620.2	12731.7	-71.6%

Table 3: Natural frequencies of plate+beam and the estimated fixed-base natural frequencies of the beam found using the proposed procedure with 12 modes of the plate and 24 plate+beam modes. The natural frequencies of the Plate+Beam before substructuring are shown in Table 2.

As mentioned in Section 2, it is advisable to observe the motion of the fixture after applying the constraints in eq. (6) to check whether enough constraints were used to force the fixture motion to zero. This was done and the norm of the motion over the plate was found to be between 0.6% and 4.3% for the first six modes estimated by CMS, suggesting that the constraints were quite effective. The shape of the plate after applying the constraints had no recognizable pattern, suggesting that it was due to noise in the measured mode shapes, so they are not shown. The mode shapes over the beam are shown in Figure 6. Each plot overlays the estimated y-bending shape, z-bending shape and the analytical shape of a cantilever with the same properties as the experimental. The shapes agree very well. There is a scale difference between the first bending modes and the analytical, but otherwise the shapes are quite similar. The second modes also agree closely, but the shapes near the root of the beam suggest that the rotation there may not be exactly zero. Some of the rotation at the connection point may be physical, since the beam is connected to the plate with a real joint that does have finite stiffness.

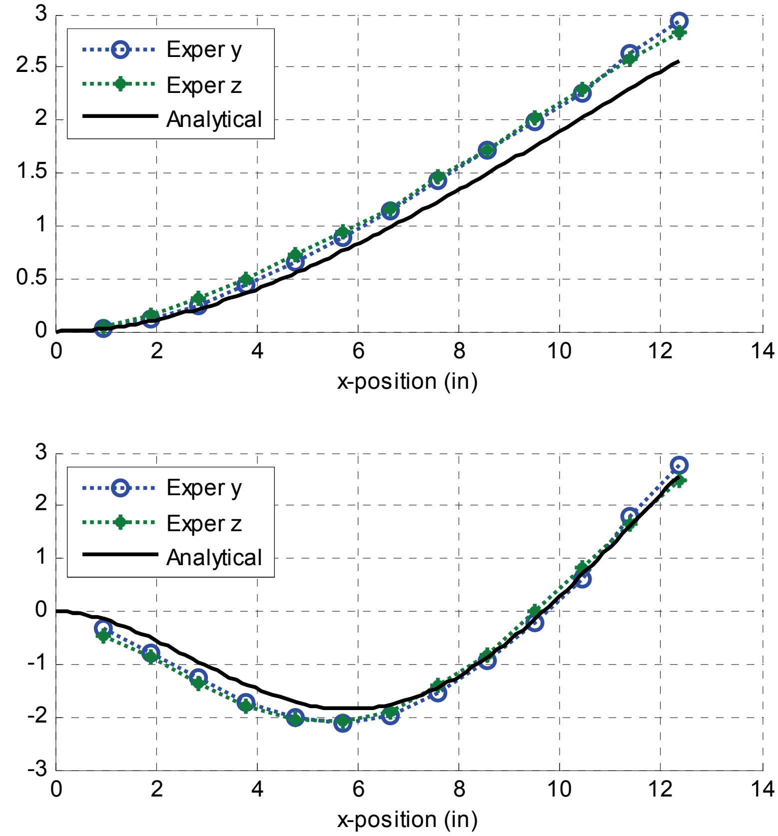

Figure 6: Mode shapes 1 and 2 of the beam in both the y- and z- directions found by CMS procedure compared with analytical mode shapes of an Euler-Bernoulli cantileer.

The CMS procedure was repeated using different numbers of plate modes, and hence different numbers of constraints. There were larger errors in the predicted natural frequencies for modes 3 and 4 if fewer than eight constraints were used, while mode 5 was not accurately predicted unless at least 12 constraints were used. However, there was virtually no improvement in the natural frequencies if the number of constraints was increased from 12 to 16. If more than 16 were used then one begins to have only a few modes left in the system (i.e. 24 – 18 = 6), so the results begin to degrade.

4. Finite Element Modeling of Substructuring Analysis for Plate+Beam

A finite element model was created to estimate the rigid body mode shapes, as mentioned previously, and also to determine whether the discrepancies between the CMS and analytical modes observed in Section 3 were due to noise in the experimental results or whether modal truncation was the culprit. The FE model was constructed with simple shell and beam elements, as summarized in Table 4 below. It was validated by comparing the FEM modes and natural frequencies with those obtained experimentally, the results of which are shown in Table 7 in the Appendix.

Model	Number of Nodes	Mesh
Beam	21	Meshed with 0.95-in long 3-node beam elements
Plate	405	Meshed with 1-in x 1-in 8-node shell elements.
Plate + Beam	418	Combination of the meshes described above.

Table 4: Details regarding finite element models for each system. Each node has six degrees of freedom.

Table 7 in the Appendix shows that the correlation between the experiment and FEA model is excellent, suggesting that the FE model is an accurate representation of the real system. The Modal Assurance Criterion (MAC) [4] between the experimentally measured mode shapes and the FEA mode shapes are all above 0.92, indicating good correlation. This is a simple system and easy to model, so one would expect the FEA model to be highly accurate and to show how well the proposed CMS procedure would work with near perfect measurements.

The FEA models for the plate and plate+beam were used in conjunction with the proposed procedure to estimate the fixed-base modes of the beam. Only the mode shapes at the measurement points defined in Figure 2 were used to facilitate the comparison with the experimental results that were presented previously. The results, shown in Table 5, are qualitatively very similar to those found experimentally (Table 3). As observed in the experimental results, only the first five modes are well predicted by CMS, and the even natural frequencies (z-direction bending) are less accurate than the odd ones. Surprisingly, the 2nd and 4th natural frequencies found here using the FEA modes are less accurate than those found using the experimental modes. The mode shapes predicted by CMS, shown in Figure 7, match almost perfectly with the analytical ones, although if one looks closely some small residual rotation is visible at the base of the cantilever.

FEA Plate + Beam Frequency (Hz)	CMS Prediction with 12 MCFS Constraints	Analytical Fixed-Base Beam Freq. (Hz)	Percent Error in CMS Prediction
126.34	103.25	107.35	-3.8%
207.2	183.8	214.7	-14.4%
630.96	647.53	672.77	-3.8%
688.82	1180.5	1345.5	-12.3%
899.33	1810.4	1883.4	-3.9%
1207.9	2431	3691.4	-34.1%
1349.2	2930.6	3766.8	-22.2%
1641.3	3072.3	6102.2	-49.7%
1786.1	3345.6	7382.9	-54.7%
1895.3	3389.2	9115.6	-62.8%
2316.9	3536.2	12204	-71.0%
2996.3	4173.8	12732	-67.2%

Table 5: CMS predictions of fixed-base natural frequencies for cantilever beam using FEA modes of the plate and plate+beam.

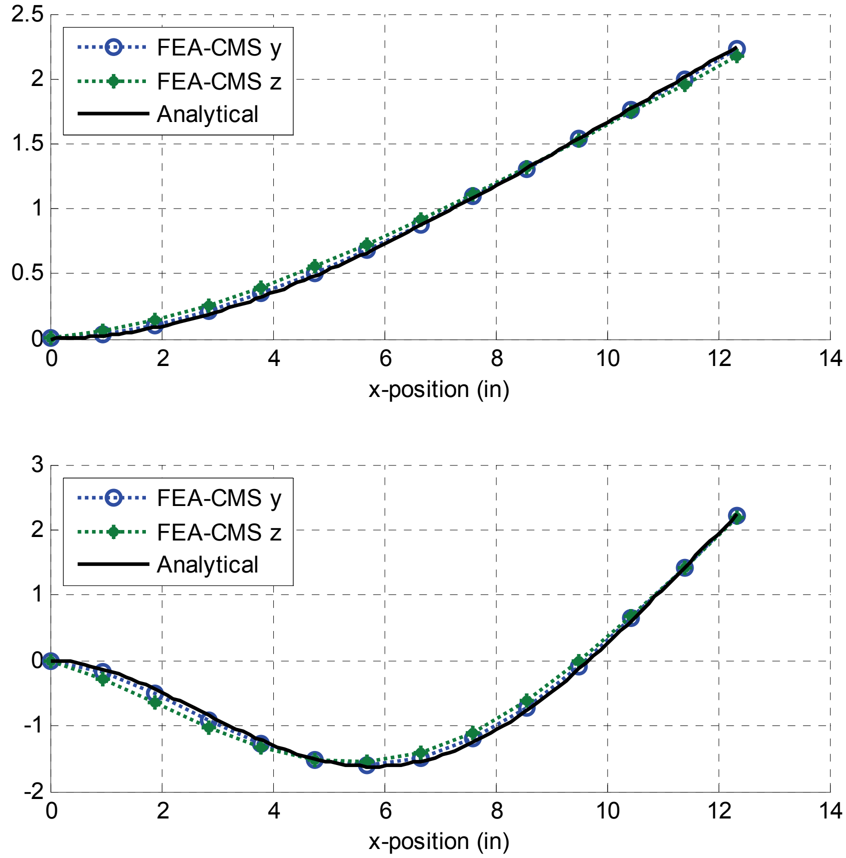

Figure 7: Mode shapes of fixed-base beam predicted by CMS using FEA-derived mode shapes, compared with the analytical mode shapes.

As mentioned in Section 2, it is advisable to check the motion of the fixture to assure that the constraints were adequate to reduce its motion to a negligible amount. This was investigated by plotting the mode shapes of the plate after applying the constraints in eq. (6). Those plots do show a marked rotation of the plate in the bending direction of the beam for each of the beam's bending modes, as illustrated for mode 4 in Figure 8. The plate motion is less than 0.03 $kg^{-0.5}$. Before the constraints were applied the maximum displacement in the plate was typically about 1.0 $kg^{-0.5}$ for each of the modes of the plate. The plate motion shows a nonzero rotation in the bending direction of the beam (rotation in the z-direction, about the y-axis), which would reduce the effective stiffness of the beam somewhat. However, the rotation is less than a degree, so one would expect it to be negligible.

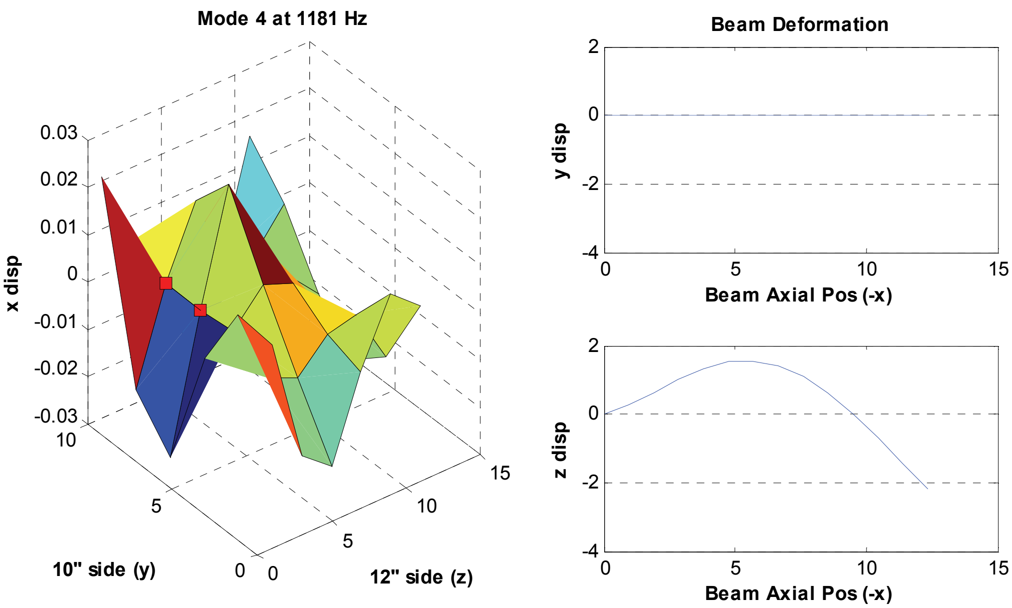

Figure 8: Deflection of Plate and Beam in 4th CMS estimated mode (after constraining plate motion to zero). In the left plot, the beam (not shown) is located midway between the red markers on the plate.

Another way of checking whether modal truncation is important is to check whether the plate's free modes span the space of the observed plate+beam modes. If this is not the case, then the approximation in eq. (3) will not be accurate. To check this, the FEA predicted modes for the plate+beam were projected onto the plate modes using ϕ. The largest difference between the projected shape and the measured shape was found and divided by the maximum absolute value of any coefficient in that shape. This gives the maximum percent error in each expanded shape, given in Table 6. Modes 1-16 of the plate+beam are accurately captured with 12 plate modes, but the higher modes are not. Modes 19 and above were not used in the CMS predictions shown here, so the errors in modes 17 and 18 are our focus. Those errors could be reduced to 10-15% by increasing the number of constraints from 12 to 16, but it was mentioned previously that the predicted natural frequencies did not change noticeably if that was done.

Max Error in Expansion		
12 Modal Constraints		
Mode	**Nat Freq**	**Max Err**
Zero for modes 1-6		
7	126.34	0.6%
8	207.2	1.6%
9	630.96	1.6%
10	688.82	1.3%
11	899.33	0.3%
12	1207.9	6.2%
13	1349.2	1.9%
14	1641.3	1.4%

Mode	**Nat Freq**	**Max Err**
15	1786.1	4.3%
16	1895.3	5.4%
17	2316.9	35.1%
18	2996.3	58.0%
19	3119.9	78.4%
20	3255.6	61.4%
21	3383.8	63.1%
22	3552.2	50.9%
23	3836.6	63.2%
24	4455.3	68.6%

Table 6: Error in expansion of the FEA mode shapes for the plate+beam onto the 12 plate modes.

4.1. Discussion

The fact that the CMS predictions have about the same level of error as the experimental predictions suggests that the errors in both methods are dominated by modal truncation. This is reassuring, especially considering the large errors that are sometimes encountered in substructuring predictions due to cross-axis sensitivity and such [16, 17]. The experimental modes presented here were obtained using a standard, inexpensive technique, yet they were adequate for use in CMS. The same cannot necessarily be said if one uses the conventional CMS or frequency based substructuring approach, which requires an estimate the rotations and moments at the connection points, since those are difficult to measure. On the other hand, it is quite disappointing that the natural frequencies are not more accurately estimated.

The plate motion shown in Figure 8 suggests that the modal basis used to describe the plate was not adequate to completely constrain its motion to ground. This problem arises because the plate deforms near the point where the beam is connected. In order to understand the effect of this localized deformation on the CMS predictions, another model was created where this region was stiffened. In this model, a 4 inch by 4 inch square of the plate surrounding the point where the beam connects (outlined with a dashed red line in Figure 2) was made to be three times as thick (1.875 inches) as the rest of the plate (0.625 inches). This significantly increases the stiffness of the plate near the connection point and should reduce the localized kinking shown in Figure 8. This model was used in the CMS procedure as described above and the fixed-base natural frequencies of the beam were estimated. The errors in the CMS predicted frequencies for y-direction bending, the 1st, 3rd and 5th natural frequencies, were only -0.3%, -0.5% and -0.8% respectively using this modified plate model, as compared to -3.8%, -3.8% and -3.9% for the regular plate as was previously shown in Table 5. Similarly, the errors in the z-direction bending frequencies reduced from -14.4 and -12.3% to -1.3 and -2.1% respectively. This suggests that localized bending of the connection point was responsible for the errors observed in the natural frequencies in Tables 3 and 5. Fortunately, one can address this difficulty by designing the fixturing to minimize localized bending whenever using the proposed CMS approach.

5. Conclusions

This work presented a new method of estimating the fixed-base modes of a structure from measurements on the structure and a flexible fixture, based on modal substructuring with modal constraints. The proposed approach was evaluated using experimentally measured modes of a simple plate-beam system. A finite element model was also created to determine how the method would perform if the experiments were perfect. The comparison between the finite element and the experimental results suggests that the proposed method is not very sensitive to the errors that are inherent to experimental modal analysis. The method approximates the motion of a fixture as a sum of contributions from its free modes, and then constrains each modal motion to ground. For the system considered here, the results were always qualitatively reasonable even if far too few constraints were used, and the fidelity of the prediction increased as the number of constraints increased. Even though the plate+beam system had pairs of modes where the beam and plate motion was highly coupled, the beam acting as a vibration absorber, those modes merged smoothly to a single mode near the true natural frequency as the number of constraints increased. However, one drawback of this approach is that each constraint eliminates a mode of the fixture+subsystem, so the experimental modal database limits how many constraints can be used. Also, for the system studied here, there were still moderate errors in the predictions of the fixed-base natural frequencies near this limit. The analysis revealed that those errors were caused by modal truncation, which was exacerbated by the fact that the plate deforms locally near the point where the beam is connected. This can and should be addressed by designing the experimental fixturing to spread out the load near the connection point. The metrics presented here did help to reveal these issues, and so they should be used to check the validity of the CMS predictions when applying this technique to real systems. Also, in many applications of interest there is not so much coupling between the modes of the system and fixture, but in either case the results presented here suggest that this is a simple and effective method for estimating the fixed-base modes of real structures from experimental measurements.

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7. Appendix

Mode	Experiment	FEA Model	MAC	MSF
1	130.66	126.34	0.9966	1.03
2	224.0	207.2	0.9960	1.01
3	628.3	631.0	0.9928	0.98
4	693.8	688.8	0.9942	1.04
5	902.4	899.3	0.9655	1.06
6	1254.4	1207.9	0.9676	0.91
7	1350.2	1349.2	0.9266	1.05
8	1657.8	1641.3	0.9675	1.06
9	1770.7	Torsion?		
10	1797.8	1786.1	0.9586	0.74
11	1906.1	1895.3	0.9893	0.98
12	2311.9	2316.9	0.9765	1.08
13	2995.7	2996.3	0.9713	1.10
14	3107.7	3119.9	0.9800	1.03
15	3233.0	3255.6	0.9617	0.99
16	3424.5	3383.8	0.9681	1.48
17	3522.8	3552.2	0.9752	0.91
18	3845.5	3836.6	0.9704	0.73
19		4455.3		
20		4866.9		
21		5102.3		
22		5395.4		
23		5717.8		
24		5827.2		

Table 7: Comparison of experimentally measured natural frequencies with those from the finite element model.

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