Finite Element Model Calibration of a Nonlinear Perforated Plate

David A. Ehrhardt[1]
University of Bristol, Bristol, UK, BS8 1TR

Matthew S. Allen[2]
University of Wisconsin-Madison, Madison, WI, 53706

Timothy J. Beberniss[3]
Wright-Patterson AFB, Dayton, OH, 45433

Simon Neild[4]
University of Bristol, Bristol, UK, BS8 1TR

This paper presents experimental and computational results used during the model calibration of two curved perforated circular plates with geometric nonlinear stiffness characteristics. The initial geometry of each plate is experimentally identified using a static 3D Digital Image Correlation (3D-DIC) measurement system and linear natural frequencies and mode shapes are found from a roving hammer test. Due to the uncertainty in the stiffness characteristics from the manufactured perforations, the linear natural frequencies are used to update the effective modulus of the finite element model. Additionally, full-field experimentally measured nonlinear 'normal' modes (NNMs) obtained with a High Speed 3D-DIC system are used to identify modal interactions in the NNM and to update the boundary conditions so measurements of the structure's NNMs match computed NNMs. The updated models are then used to understand how the stress distribution changes at large response amplitudes providing a possible explanation of failures observed during testing.

1. Introduction

Model calibration is an important step in the development of computational models that are representative of physical structures. In this context, there is a large suite of test and analysis approaches which use a structure's linear modes of vibration to guide the calibration of computational models [1, 2]. It is beneficial to note here that these techniques can be centered on a structure's linear modes of vibration using the complex mode definition or a more specific subset of complex modes known as a structure's linear normal modes (LNMs) of vibration [3]. In many instances, the LNMs, which are dependent only on the mass and stiffness distribution of a structure, are adequate in the definition of the modes of vibration. The use of LNMs in model calibration is centered on a structure's natural frequencies and mode shapes. However, characteristics of these LNMs such as amplitude invariance and orthogonality break down when a structure behaves nonlinearly. This has motivated many studies focused on expanding the definition of a structure's LNMs to include nonlinear behavior resulting in several definitions of what are termed *Nonlinear Normal Modes*.

Two main definitions of nonlinear normal modes (NNMs) can be found in literature [4-7]. The first was developed by Rosenberg [4] for nonlinear conservative (i.e. un-damped) systems and limits a NNM to a *vibration in unison* of the nonlinear system. This definition was later extended by Kerschen et al [6] to include *non-necessarily synchronous periodic motions* of the nonlinear system. Although still centered on the conservative nonlinear equations of motion (computed using only mass and stiffness), this extended definition allows the inclusion of

[1] Post-Doc Research Assistant, Mechanical Engineering
[2] Associate Professor, Engineering Physics
[3] Aerospace Structures Engineer, Aerospace Systems Directorate, Air Force Research Lab
[4] Professor, Mechanical Engineering

internal resonances which can lead to non-synchronous motions of the nonlinear system. A generalization of Rosenberg's definition was proposed by Shaw and Pierre [5] which defines an NNM as a two-dimensional invariant manifold in phase space extending the NNM concept to damped systems. Jiang et al [8] presented a further expansion of the invariant manifold approach to include internal resonances by defining internally resonant NNMs as a $2m$-dimension invariant manifold where m is the number of modes retained for the definition of the invariant manifold. In this investigation the definition of a *non-necessarily synchronous periodic motion* of the conservative equations of motion is utilized for a NNM.

There have been several applications of NNMs in the field of structural dynamics. For instance, NNMs have been used to provide insight to guide the design of nonlinear vibration absorbers [9] as well as a structure with tunable bending-torsion coupling [10]. NNMs have also been used to characterize FE models of complicated, geometrically nonlinear structures aiding the creation of accurate nonlinear reduced order models [11, 12]. Kurt et al [13] numerically demonstrated the use of NNM backbone curves to guide the identification of nonlinear stiffness coefficients for a system with local nonlinearities. Of particular interest to this work, NNMs provide a tool to connect computational results with experimental measurements. For instance, NNMs have been used to correlate simulations [14] with experimental measurements [15, 16]. The summation of nonlinear behavior with the use of NNMs provide a compact means to compare experimental and computational results making NNMs a great tool for model calibration.

The implementation of NNMs for the purpose of model calibration requires advanced techniques in their analytical or numerical calculation as well as their experimental measurement. Analytical techniques include the method of multiple scales [6, 7, 17, 18], normal forms [19], and the harmonic balance approach [20], but are typically restricted to structures where the equations of motion are known in closed form limiting their application to simple geometries or low order systems. Numerical methods have also been developed to calculate a system's NNMs without the approximations required in the analytical approaches [21] and have been used to compute the NNMs of relatively complicated structures with local nonlinearities [16]. These techniques have been extended to the calculation NNMs of geometrically nonlinear finite element models (FEM) using an approach whereby the NNMs are calculated by coupling numerical continuation to transient dynamic simulation of full order FEM [22]. While numerical techniques are powerful, they are time consuming to implement for a large order FEM making application to iterative procedures (i.e. model calibration) difficult. Therefore in this investigation, the full order FEM created and updated in Abaqus® is used to determine nonlinear reduced order models (NLROMs) following procedures discussed in [23]. The low order NLROMs are then used to examine NNMs with the use of NNMCont as described by Peeters et al. [21].

Nonlinear normal modes can also be measured experimentally, but far less has been published in this area due to the difficulty of accounting for damping in the dynamic response of a structure which is needed to isolate a NNM. Recent work has sought to identify NNMs using phase separation techniques relieving the need to cancel damping in the measurement of a NNM [24]. Alternatively using a phase resonance approach, NNM backbones have been identified from the damped dynamics of a structure using the free decay of a response initiated near a NNM solution [15, 25] or the stepped forced response using a multi-frequency input force [26]. The free decay results presented in [15, 25] have shown good agreement between calculated and experimentally identified NNMs; however, modal [26] and shaker-structure interactions [27] demonstrate that there is no guarantee a lightly damped transient will follow a NNM. Alternatively, the force appropriation technique used to initiate a free decay in [15] has been extended to identify a NNM by incrementally increasing the input force amplitude and tracking the phase lag criterion along a NNM backbone [26]. This stepped-force technique allows the implementation of multi-frequency inputs to account for damping changes with response amplitude and is used in this investigation.

The goal of this work is to propose and implement a model updating framework that can be used to accurately capture the linear and nonlinear dynamic response of geometric nonlinear structures. The first step in updating a model is to decide which dynamic properties to measure and how to compare them between the model and experiment. In this work we focus on the physical parameters that have potential uncertainty and their effects on the global dynamics of the structure (i.e. LNMs and NNMs). Here, updates to the initial conditions, material properties, and boundary conditions are considered. The initial conditions are measured with static 3D digital image correlation and the difference between the calculated and measured LNMs and NNMs are used to update material properties and boundary conditions since LNMs and NNMs are closely tied to the physics of the real structure. The resulting models are shown to better represent the structure's linear and nonlinear dynamics. It is also shown that the changes made to the model based solely on its linear modal parameters may or may not improve the correlation of the model in nonlinear response regimes. This is an important consideration since finding a model that accurately represents an experiment requires the perturbation of uncertain parameters, which is not always possible in a linear regime. Hence, it is critical to simultaneously consider both the linear and the nonlinear behavior of the system in the model updating process.

2. Background

2.1. Nonlinear Normal Modes

The concept of nonlinear 'normal' modes (NNMs) has seen much interest due to their usefulness in interpreting a wide class of nonlinear dynamics. While definitions are limited to conservative or weakly damped systems, NNMs provide an excellent summary of a mode of vibration's dependence on response amplitude. The reader is referred to [4, 6, 7, 21] for an in-depth discussion of NNMs in regards to their fundamental properties and methods of calculation. A summary of the methods used in this work is provided next.

Here, NNMs are numerically calculated using shooting techniques and pseudo arc-length continuation with step size control as implemented in NNMCont [21]. This method of calculation solves for periodic solutions of the nonlinear equations of motions, presented in Eqn. (1). Calculation begins in the linear response range and follows the progression of the system's dependence on the energy of the response. In Eqn. (1), [M] is the mass matrix, [K] is the stiffness matrix, and f_{nl} is the nonlinear restoring force that is a function of *x*. This method is capable of following the frequency-energy evolution around sharp changes in the NNM with the ability to track bifurcations, modal interactions, and large energy dependence of the frequency of vibration.

$$[M]\{\ddot{x}(t)\}+[K]\{x(t)\}+f_{nl}(\{x(t)\})=0 \quad (1)$$

The NNMs of a structure can be measured with the use of force appropriation and an extension of phase lag quadrature as discussed by Peeters et al. in [14]. It was shown that an appropriated multi-point multi-harmonic force can be used to isolate the dynamic response of a structure on an NNM. The nonlinear forced response of a structure with viscous damping can be represented in matrix form by Eq. (2), where [C] is the damping matrix and $p(t)$ is the external excitation. As discussed before, an un-damped NNM is defined as a periodic solution to Eq. (1). So, if the forced response of a structure is on an NNM, then the defined conservative equation of motion is equal to zero. Therefore, the forced response of a nonlinear system is on an NNM if the input force is equal to the structural damping for all response harmonics. As with linear force appropriation, the appropriated force can be simplified to single-point mono-harmonic components to produce a response in the neighborhood of a NNM, providing a practical application to experimental measurement. This simplification breaks down when the input force is not able to properly excite all modes in the response requiring careful consideration of input force location and harmonics needed. This technique has been experimentally demonstrated in [26].

$$[M]\{\ddot{x}(t)\}+[C]\{\dot{x}(t)\}+[K]\{x(t)\}+f_{nl}(\{x(t)\})=p(t) \quad (2)$$

2.2. Perforated Plate Description

The structure under investigation is a circular perforated plate with rolled ends which is shown in Fig. 1a. A mechanical punch was used to create the circular perforations in a flat 16 gauge (1.52 mm thick) 409 stainless steel plate. The center each perforation was located at the vertex of an array of equilateral triangles with 10.16 mm long edges. Once this process was completed, the plate was formed around a 317.5 mm diameter mold with the excess trimmed so a lip of 24 mm remained. The plate was then welded to an 89 mm high cylinder made from a 14 gauge (1.9 mm thick) 409 stainless steel plate that was cold rolled to the 317.5 mm diameter as shown in Fig. 1b. The final experimental setup shown in Fig. 1b is meant to simulate in situ conditions. The entire assembly was then bolted to a fixture with twelve 6.4 mm evenly spaced bolts attached to a rigid base. The linear natural frequencies and mode shapes were determined using a roving hammer test with 37 evenly spaced impacts and two light weight accelerometers mounted in the center of the plate.

The initial motivation of this work was to characterize nonlinear behavior of the fundamental mode of vibration at large response amplitudes as described in [26]; however, during testing an unexpected failure occurred in the center of the first plate tested (PP01) as shown in Fig. 1d in blue. A detailed image of the failure is shown in Fig. 1e, where the failed ribs are outlined in blue. This failure was repeated in the second plate tested (PP02). The region of this failure has been emphasized on the image of PP01 in Fig. 1d in green for comparison. A detailed image of the failure seen in PP02 is shown in Fig. 1f where the failed ribs are outlined in green. Although there is a slight difference in the location of the failure between the two plates (potentially from geometric differences discussed in Section 3.1), the failures occurred repeatedly and in an unpredicted location for the expected stress distribution. Using the measured LNMs and NNMs, a dynamically representative model is created which points to the potential mechanism of this observed failure.

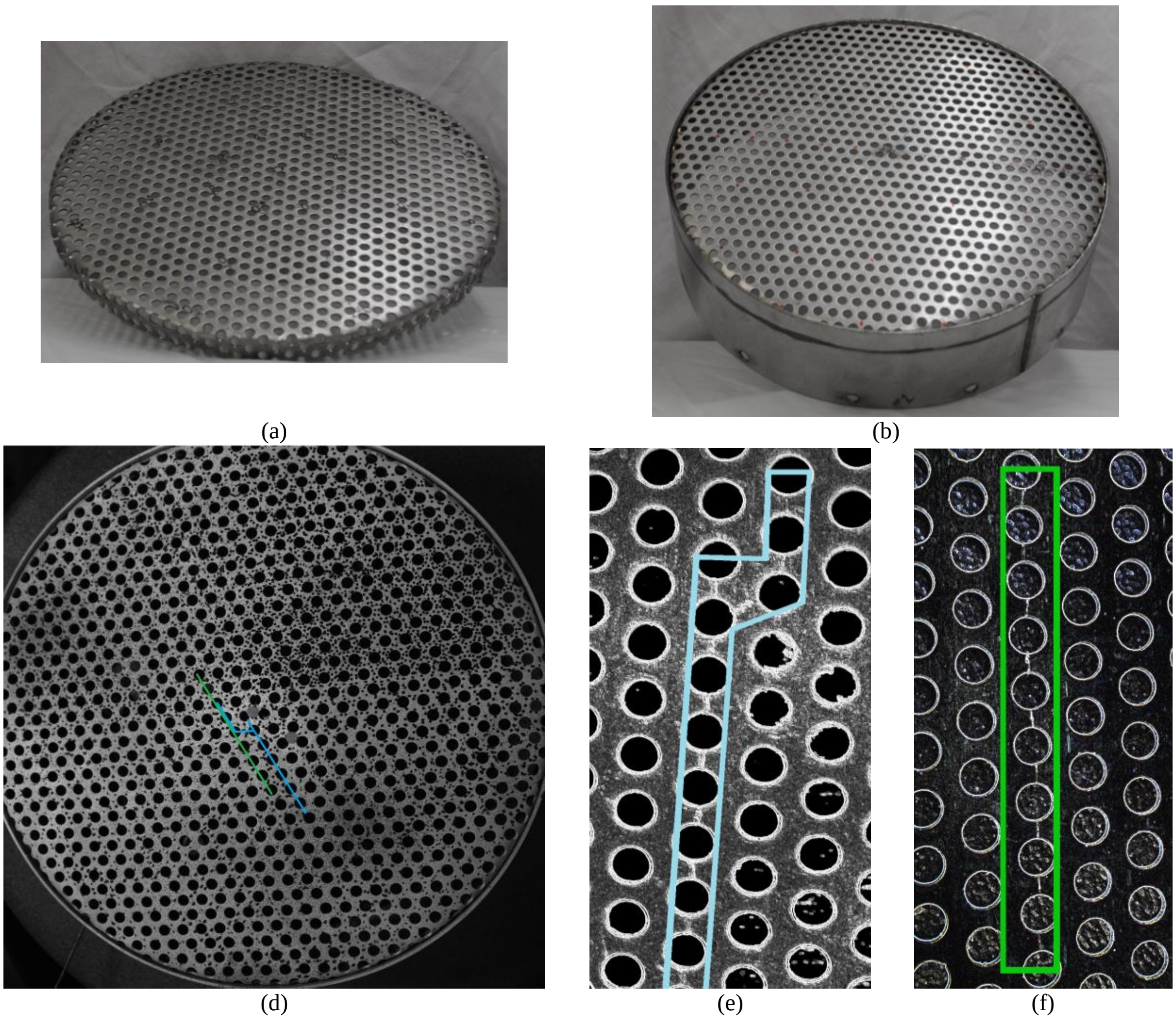


Figure 1: Experimental setup of the perforated plates. a) Perforated plate before welding into test configuration, b) Perforated plate welded into the supporting cylinder, d) Region of failure in both plates, and e & f) Detail of failures in both plates tested. The image has been modified using a threshold adjustment and edge detection algorithm in ImageJ [28] to emphasize the cracked ribs.

2.3. Model Description

In this investigation, an initial finite element model (FEM) was built using the nominal measured geometries of the perforated plate described in the previous section and is termed the Flat model. It is initially assumed that the welded boundary between the plate and steel cylinder provide fixed boundary conditions. If a detailed representation of the stress distribution is needed, a very fine mesh would be required in order to model each of the perforations. However, Jhung and Jo [29] found that a perforated plate behaves dynamically identical to a non-perforated plate of the same dimensions, as long as the elastic properties are adjusted appropriately. A reduced elastic modulus and density were calculated based on the perforation geometry as detailed by Jhung and Jo. For the triangular perforation pattern of this plate a new elastic modulus of 168 GPa and density of 5120 kg/m^3 was found. The resulting meshed Abaqus® model is shown in Fig. 2, and has 1440 S4R shell elements.

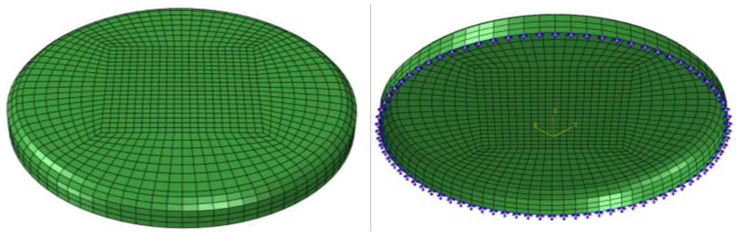


Figure 2: Meshed dynamically equivalent plate model

Although the resulting 8886 degrees of freedom (DOF) model is substantially reduced when compared to the number of DOF needed to model the perforations (~400000 DOF), it is still prohibitively large to run dynamic simulations. Therefore, a nonlinear reduced order model (NLROM) is created using the Implicit Condensation and Expansion (ICE) method [30-32]. In this method, the expected geometric nonlinearity due to large amplitudes of deformation is implicitly accounted for using nonlinear static solutions in Abaqus®. The nonlinear coefficients are determined using specific levels of applied modal forces and decomposing the resulting displacement onto the preselected modal basis for the NLROM and implicitly accounting for membrane effects. A system identification procedure is then utilized on the resulting restoring force/modal displacement relationship. The ICE method produces an N DOF system of equations in the modal domain as shown in Eqn. 3, where n is the number of r modes included in the modal basis.

$$\ddot{q}_r + \omega_r^2 q_r + \sum_{i=1}^{n}\sum_{j=1}^{n} B_r(i,j) q_i q_j + \sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{k=1}^{n} A_r(i,j,k) q_i q_j q_k = 0 \quad (3)$$

This type of NLROM should be adequate for this investigation since: 1) the focus of this investigation is near a single mode and 2) the nonlinear effects due to stiffness should dominate the geometric nonlinearity of this thin structure. The creation of NLROMs for the nominal dimensions of the plate has been discussed in [23] where it was shown that an accurate NLROM should include the first and sixth mode resulting in a two degree of freedom (2-DOF) representation of the structure. For this work, Eqn. 3 can therefore be summarized with a 2-DOF spring-mass system as shown in Fig. 3. In this context, it is beneficial to view the nonlinear behavior as a coupling between the first and sixth mode of vibration. Accordingly, in linear response regimes the system response remains uncoupled using the linear modes of vibration; however, in nonlinear response regimes, the system response now includes both the first and sixth mode. The 2-DOF NLROMs created using the ICE method are used to find NNMs of the structure by implementing continuation techniques discussed in [21] for a further comparison with experimentally measured NNMs.

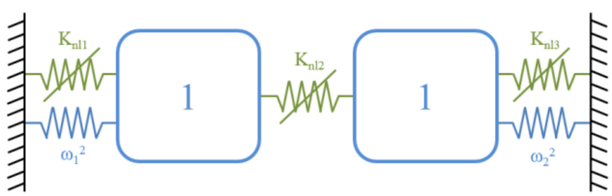


Figure 3: Schematic of modal domain NLROM.

2.4. Modeling Considerations

Although the plate is a relatively simple structure compared to the vehicle it is attached to, validating the FEM with experimental measurements involves some engineering judgment and physical insight to the inherent uncertainty of the physical assembly. Uncertainties in initial geometry, material properties, and boundary conditions are expected to dominate errors between the model and experimental structure. Therefore, a general model, shown schematically in Figure 4, is considered. In Figure 4, the areas of potential uncertainty are shown in orange: K_R represents the stiffness of the boundary in the radial direction, and the orange line represents the initial geometry. The starting point for the model updating procedure uses the nominal geometry (shown in black), nominal material properties (as predicted by [29]), and fixed boundary conditions (K_R = infinity). Variations in the initial geometry (shown in orange) are taken into account with the use of full-field static 3D digital image correlation coordinate measurements of the plate surface. The remaining error between the model and measurement is accounted for by tuning the modulus of elasticity and boundary conditions. All six degrees of freedom at the boundary are initially considered; however, it was found that only K_R is important. The reduced density was not updated since it can be computed from the geometric properties of the perforations (i.e. size of hole and count) and hence should be quite accurate. On the other hand, the effective modulus is dependent on any residual stresses from the addition of perforations, imperfections of the perforation location geometry, and curvature from the forming processes. Similarly, the stiffness, K_R, has potential variation due to the flexibility of the cylinder to which the perforated plate is welded.

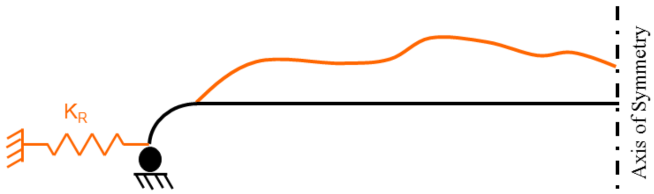


Figure 4: Boundary Condition Schematic

3. Updating Results

3.1. Initial Conditions and Linear Updating

Although subjected to similar mechanical loads during manufacturing, each plate will have variations in the final geometry which are important to identify during model calibration. Using static 3D digital image correlation, the initial geometry of the surface of each plate is measured resulting in a dense point cloud describing the shape. It is assumed that the lip of the plate (i.e. the edge welded to the cylinder shown in Fig. 1) remains near circular and would have little effect on the final structural response since the primary deformation is along the Z-axis (as defined in Fig. 5). The surface measurement is directly applied to the FEM using bi-harmonic interpolation resulting in two FEMs describing the initial shape of PP01 and PP02 as seen in Fig. 5. The resulting initial curvatures show only include the updated surface of the model neglecting the 25mm lip which is unchanged. It is interesting to note that each plate shows a slightly different asymmetry in the peak deformation. This is most noticeable in the XY plot of the curvature (Figs. 5a and 5c) where the largest deformation (dark red) is skewed in the positive Y direction and the negative X direction. For comparison, the peak curvature observed for PP01 (3.73mm) is lower than the peak curvature observed for PP02 (4.84mm) resulting in slightly different natural frequencies between the two plates seen in Tab. 1.

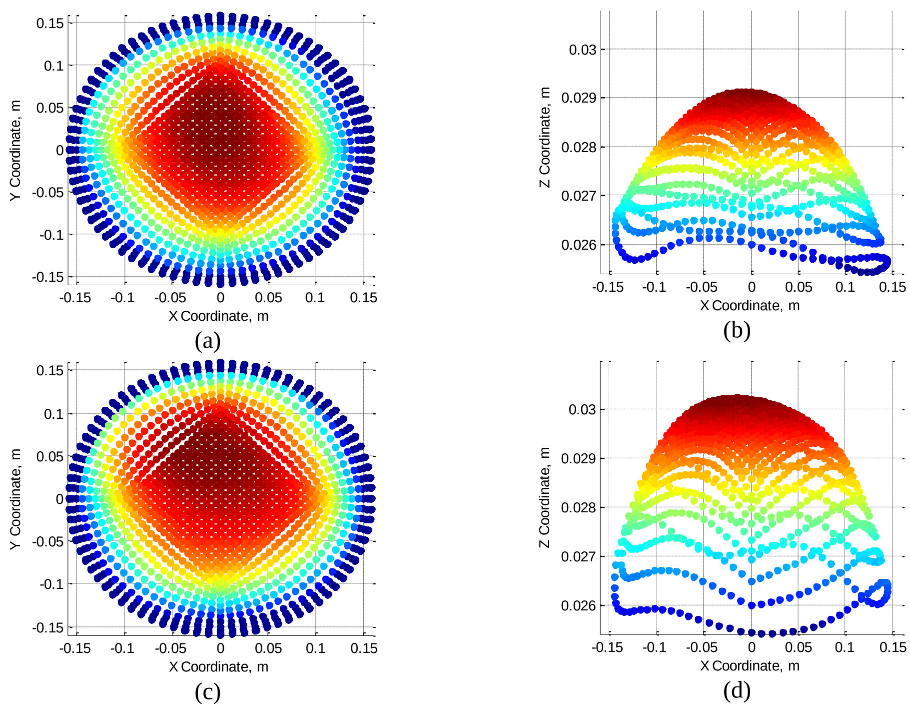


Figure 5: Initial geometry of each plate measured with 3D digital image correlation. a-b) PP01, c-d) PP02

Since there are several areas of uncertainty between the model and the experiment, there is potential for the model calibration process to deliver non-physical results or lead to a non-optimal result. Therefore, in this investigation, we will use experimental and numerical data from PP01 to guide each model calibration step, and blindly apply the resulting values to PP02 to gain insight into the robustness of the model calibration for the curved plates. In addition to the two models created with the measured geometry of PP01 and PP02, a nominal (Flat) model is created to demonstrate the difference in nonlinear behavior. During the model calibration of the Flat model, an average between the PP01 and PP02 experimentally identified natural frequencies is used to guide the model calibration steps in an attempt to produce the 'best' flat model that could be used for both plates.

Table 1 provides a summary of the first 10 natural frequencies for the results obtained during the first stages of model calibration. Using the material properties previously defined as a starting point, the Flat (NUM 1) model was created to reduce the percentage error between the measured natural frequencies. Several combinations of physical parameters were used to update the Flat model based on the expected experimental uncertainties (i.e. boundary conditions and modulus of elasticity). The best updated Flat (NUM 1) model is presented in Tab. 1, where K_R was not changed (i.e. boundary was infinitely stiff) and the modulus of elasticity decreased to 132 GPa (decrease of 21%). While some linear modal frequencies are captured accurately, the resulting model does not appear to capture the linear natural frequencies in Modes 1, 3, and 9, as shown in Tab. 1. However, the MAC value between the resulting Flat model and experimentally measured PP01 mode shapes for Modes 1, 3, and 9 show a good agreement. This information provides an indication of the distributed nature of the error (i.e. geometry of the structure) which would preserve the distribution of mass and stiffness thereby minimally changing the mode shapes.

The same procedure was applied to the PP01 (NUM 1) model to reduce the percent error between the measured natural frequencies for PP01 (EXP). The best updated model produced a reduction in the modulus of elasticity to 96 GPa (decrease of 43%) with no change in K_R. The resulting update brought the errors between natural frequencies within 6%, and had varying effects on the MAC values. The reduction in modulus for PP01 (43%) compared with the Flat model (21%) emphasizes the importance of the initial curvature in the updating process when examining the effective modulus. The resulting modulus update for PP01 was then blindly applied to the PP02 (NUM 1). Both PP01 and PP02 models show better agreement in the first ten natural frequencies when compared with the Flat model as seen in Tab. 1 leading to the conclusion that the model is a good representation of the experimental setup after simply updating the elastic modulus and curvature. The larger frequency error but good agreement in MAC value observed in PP02 further emphasizes the influence of curvature on the effective modulus. Since only minor differences are observed between the mode shapes of PP01 and PP02, only the modes shapes of the experimentally measured PP01 and updated PP01 model are presented in Fig. 6. Here a good agreement is seen between both sets of modes.

In the absence of a measurement of the initial curvature, the updated Flat model may be all that is available to understand the dynamics of the structure. Since the mode shapes of the measurement and flat model match well, one may be tempted to factor in the frequency difference in any further analysis and stop updating at this point; however, the flat model misses important nonlinear characteristics of the dynamic response as discussed in Section 3.3. For the sake of discussion, all three models (Flat, PP01, and PP02) will be used throughout all stages of updating.

Table 1: Perforated Plate Correlation Results

Model	Flat				PP01				PP02			
Mode #	f_n, Hz, EXP AVG	f_n, Hz, NUM 1	Mean % Err.	MAC	f_n, Hz, EXP	f_n, Hz, NUM 1	% Err.	MAC	f_n, Hz, EXP	f_n, Hz, NUM 1	% Err.	MAC
1	213.75	154.01	27.95	0.980	205.36	202.26	1.51	0.990	222.13	210.77	5.11	0.983
2	339.18	319.52	5.80	0.870	327.84	328.86	-0.31	0.983	350.51	350.80	-0.08	0.976
3	356.80	319.52	10.45	0.835	348.65	352.10	-0.99	0.955	364.95	363.78	0.32	0.990
4	506.18	527.64	-4.24	0.903	489.17	512.93	-4.86	0.887	523.19	548.75	-4.88	0.912
5	526.68	528.62	-0.37	0.945	510.23	528.16	-3.51	0.972	543.12	564.80	-3.99	0.857
6	571.87	604.38	-5.68	0.945	572.63	559.94	2.22	0.900	571.12	573.49	-0.41	0.869
7	711.78	777.36	-9.21	0.930	697.90	736.59	-5.54	0.844	725.66	783.02	-7.90	0.903
8	715.14	777.36	-8.70	0.906	699.95	737.37	-5.35	0.779	730.33	787.95	-7.89	0.897
9	796.45	922.05	-15.77	0.933	814.12	821.08	-0.86	0.851	778.78	840.18	-7.88	0.820
10	852.27	922.05	-8.19	0.941	827.68	832.57	-0.59	0.854	876.85	849.89	3.07	0.867

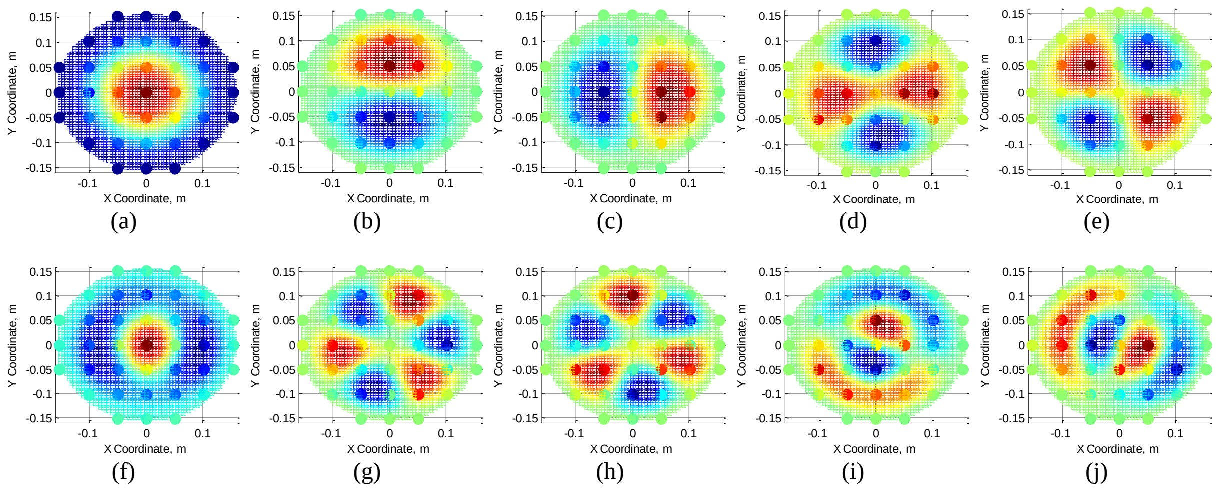


Figure 6: Experimental and numerical mode shapes of the first 10 modes

3.2. Examination of Experimental NNM Backbone Curves

The NNMs for the perforated plate can be obtained experimentally using phase quadrature relationships as described in [26]. In this investigation, a mono-frequency base excitation was applied to the structure as the phase was tracked between the input voltage to the shaker and the response velocity at the center of the plate to follow the NNM at higher amplitudes. The resulting experimental NNM for PP01 is shown in Fig. 7a. For the experimental setup, we were able to capture 12Hz of spring softening before 4.5Hz of spring hardening. Above this level of response, PP01 failed as previously described. At a higher level of response amplitude along the NNM (i.e. at the point labeled 'Point 1' in Fig. 7a), a multi-frequency response is observed in Fig. 7b, although only mono-frequency voltage was used to drive the shaker, the measured base motion also shows higher frequency content bringing into question the validity of the mono-frequency excitation. To provide context, the amplification factor from the base motion to the response at the center of the plate at 1^*f_{in} is 360 (i.e. 360 times more response is observed than base motion) and is 328 at 3^*f_{in} describing the dominant behavior of the structure. The large amplification factor almost 10Hz from the 1^{st} linear mode of vibration and 7Hz from the 6^{th} linear mode of vibration (closest to 3^*f_{in}) provides an indication to the level of nonlinearity in the response of the structure. An examination of the phase between the response of the plate at 1^*f_{in} and 3^*f_{in} (4.43deg compared to 0 degrees for perfect NNM measurement) provides an indication of how well the NNM of the structure is captured at the largest amplitude of deformation.

In this experimental setup we had the benefit of measuring full-field dynamic displacements using high speed 3D digital image correlation from a previous setup [33] capturing images at 4000 Hz. The full-field measurement of the response of the plate provides an indication of which linear modes of vibration are participating in the multi-frequency dynamic response shown in Fig. 7b. A qualitative examination of the physical deformation at 1^*f_{in} (Fig. 7c) reveals a distinctive Mode 1 shape. Similarly, at 2^*f_{in} (Fig. 7d) a combination of Mode 1 and Mode 6 is observed, whereas at 3^*f_{in} (Fig. 7e) only a Mode 6 shape is observed. Finally, at 4^*f_{in} (Fig. 7f) a distinctive Mode 10 shape is observed. This measurement shows a strong participation of Mode 1 and Mode 6 in the dynamic response at 1^*f_{in} and 3^*f_{in} (i.e. the dominant frequencies) giving a good indication of the important modes participating at this response level. The fact that Mode 6 is shown to be important in the measured nonlinear response of the plate reinforces previous numerical studies [23]. However, a deeper understanding of how Mode 6 participates in the dynamic response is sought and will be discussed in the next section.

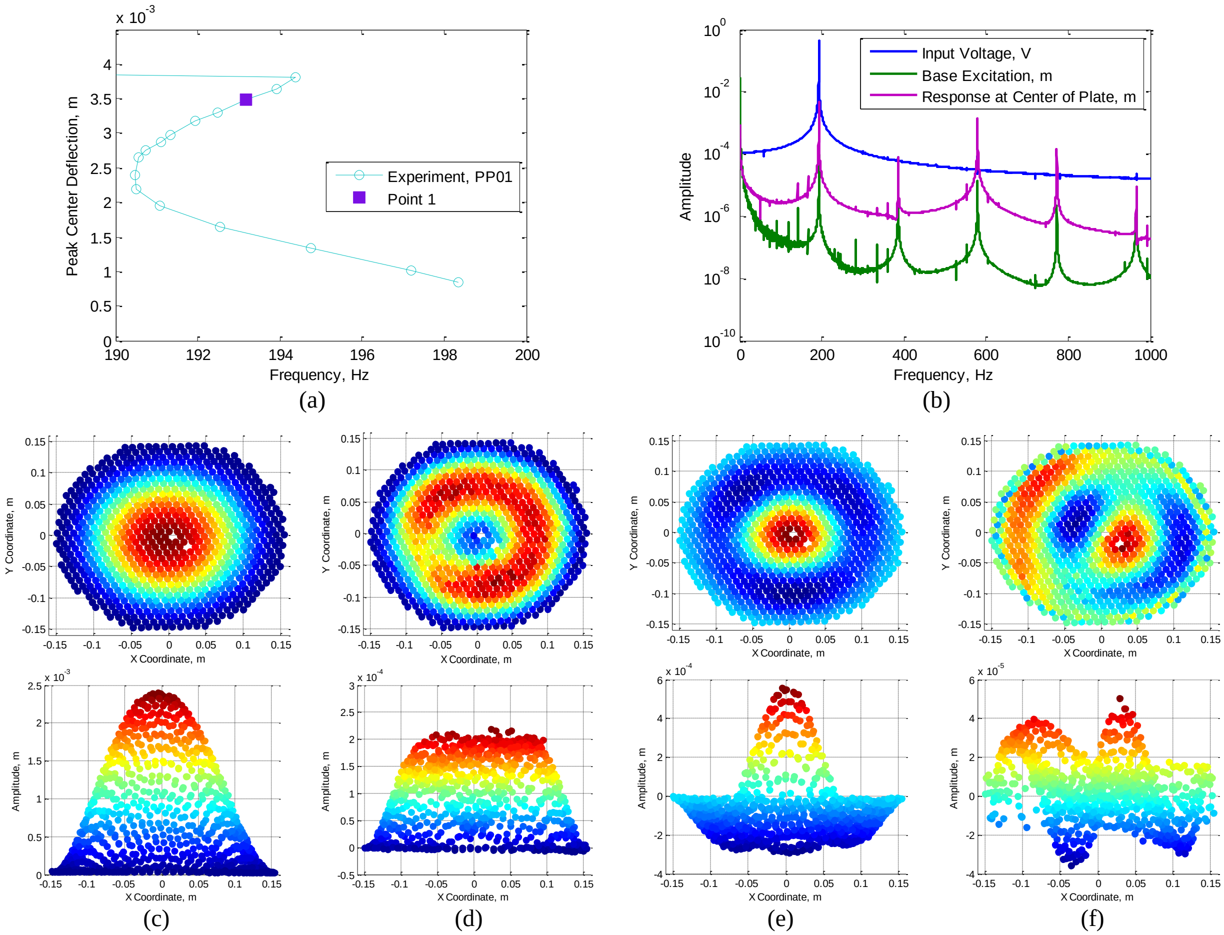


Figure 7: Experimental NNM backbone curve. a) Peak displacement vs. frequency, b) FFT of input (acceleration and voltage) and response velocity, c) Deformation at 1**f*$_{in}$, d) 2**f*$_{in}$, e) 3**f*$_{in}$, and f) 4**f*$_{in}$

3.3. 2DOF NLROMs and Comparison with Experimental Results

From a previous numerical study it has been shown that Mode 6 must be included in the formation of the NLROM to predict the nonlinear behavior of the plate [23], which is also confirmed in the measured NNMs shown in the previous section. Therefore, 2-DOF NLROMs are created using Mode 1 and Mode 6 as a basis for the three models created in this work. The inclusion of Mode 6 to the NLROM requires consideration of the level of the maximum static deflection each mode undergoes to implicitly account for membrane effects and build the NLROM. It was previously found that a deformation of Mode 1 and Mode 6 for a converged NLROM was 1*thickness and 0.25*thickness of the plate, respectively. The first NNM of all models are compared with the measured NNM of PP01 and PP02 in Fig. 8. It is observed that the general characteristics of the NNM are captured (i.e. spring softening to hardening), but there is a discrepancy to the amplitude of deflection where hardening begins and the amount of softening observed in the NNM for the PP01 and PP02 models. It is of interest to note that the Flat model completely misses the nonlinear behavior predicted and observed in the perforated plate and only shows spring hardening behavior.

More insight to the dynamic response of the plate can be gained by examining the time series of a period of the predicted and measured response at different amplitudes. In Fig. 8b-f the full field displacement is projected onto the mode shapes which have been normalized to the peak z-deflection to preserve the relative scale of the deformation of each mode in the physical response. The time series of Mode 1 (dash) and Mode 6 (dot) are shown for the two experimental measurements and the three models near a peak deflection of 0.1mm (purple), 0.2mm (green), and 0.3mm (orange) in Figs. 8b-f. In Fig. 8b, the experimental time series of the measured NNM at these points for PP01 is shown. As expected, Mode 6 becomes more pronounced at higher levels of response amplitude and primarily oscillates at three times the fundamental frequency. It is interesting to note that at all levels an asymmetry is observed in the amplitude of the peak deformation of the time series of both modes. A similar behavior is observed in the PP02 experimental results shown in Fig. 8c; however, a lower amplitude of Mode 6 is at a similar level of peak center deflection. A comparison of the experimental results from PP01 (Fig. 8b) and the numerical results of PP01 (Fig. 8d) reveals an over prediction of the participation of Mode 6 at higher levels of response amplitude. This provides an indication of missing or inappropriately accounted for coupling between Mode 1 and Mode 6. Also, the phase relationship between Mode 1 and Mode 6 is 180 degrees different for the model while the experiment predicts an in-phase behavior. The phase difference can be traced back to the phase of the calculated linear modes when compared to the measured modes of vibration. The NNM of the Flat model (Fig. 8f) shows minimal participation of Mode 6 in the period of the response emphasizing again the important difference observed in the Flat model. At this point it is difficult to determine the next step in the model updating procedure, but a consideration of the experimental setup points to a potential need to relax the boundary conditions due to the assembly of the perforated plate.

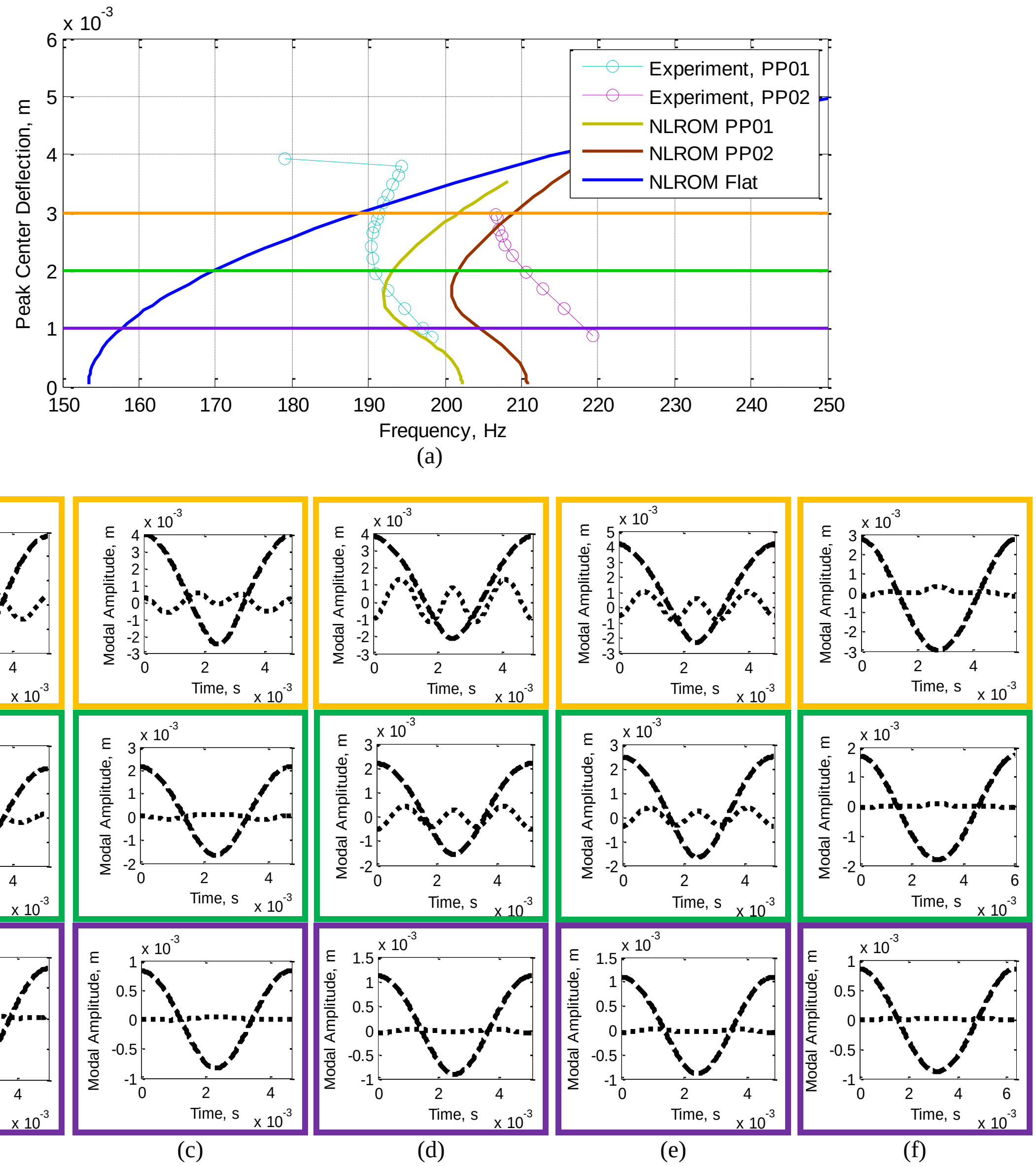
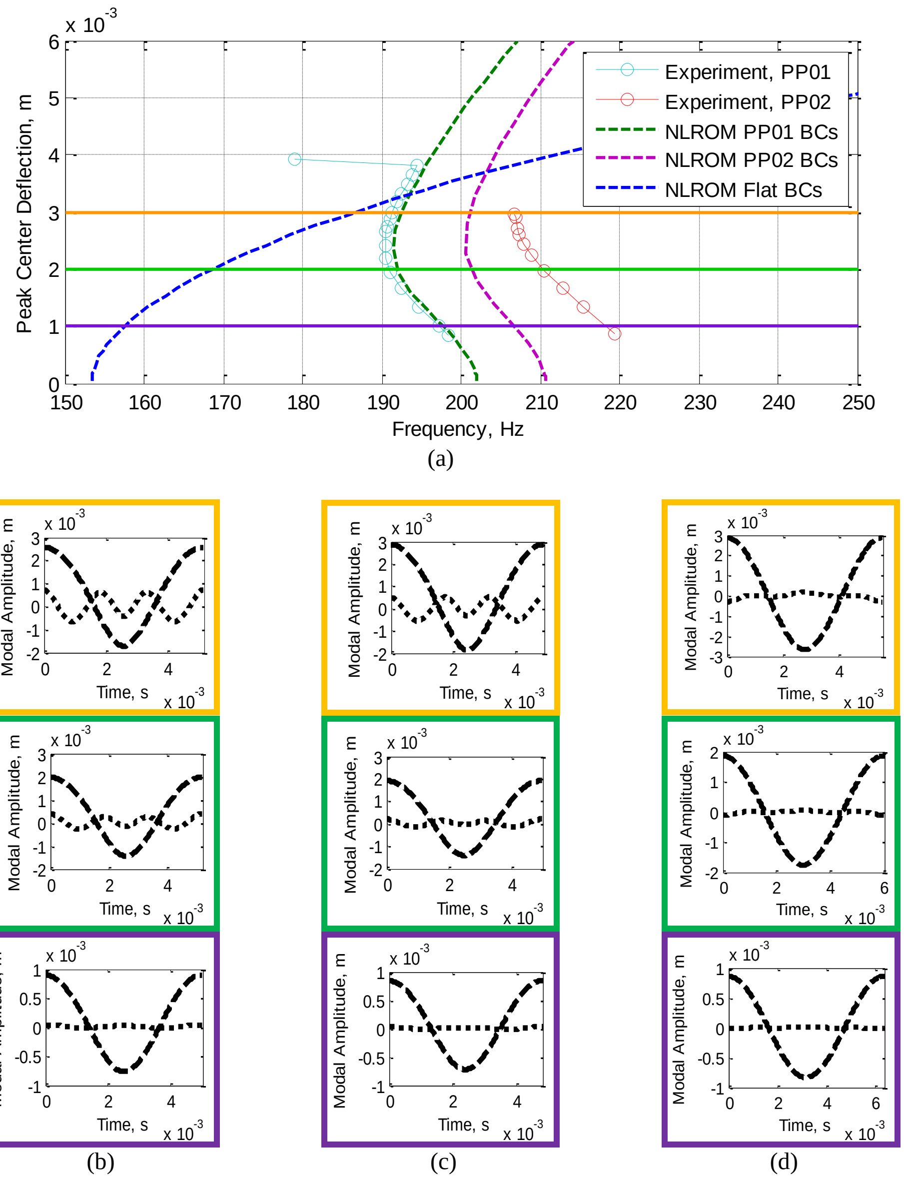


Figure 8: a) Initial and measured NNMs and time series results for Mode 1 (dash) and Mode 6 (dot) at 0.1mm (purple), 0.2mm (green), and 0.3mm (orange) deflections for b) PP01 Experiment, c) PP02 Experiment, d) PP01 Model, e) PP02 Model, and f) Flat model.

3.4. Updating Based on Nonlinear Normal Mode Backbone Curves

Using the NNM of PP01, the difference between the measured and predicted peak deflection at the center of the plate at the 'turning point' from spring softening to spring hardening is reduced. Using a forward difference gradient based optimization leads to a reduction in boundary stiffness to K_R = 650000 N/m. This boundary condition is blindly applied to PP02 and the Flat model. Both curved models show a better agreement with the experimentally measured NNMs as seen in Fig. 9a. The time series of PP01 (Fig. 9b) also shows better agreement with the experimentally measured time series of PP01 (Fig. 8b). The relaxation of the boundary conditions improves the amplitude of Mode 6, and the phase between Mode 1 and Mode 6. The updated boundary conditions applied to the PP02 model (Fig. 9c) which shows a better representation of the contribution of Mode 6 to the dynamic response, but does a worse job predicting the asymmetry observed in Mode 1 when compared with the experimental results previously presented in Fig. 8c. It is interesting to note that the reduction in boundary conditions shifted the asymmetry of the modes in the time domain for the Flat model (Fig. 9d) toward the measured asymmetry. While the updated models do not provide an exact comparison with experimental results, the resulting NLROMs describe the behavior of the perforated plates well.



(a)

(b) (c) (d)

Figure 9: a) Updated NNMs and time series results at 0.1mm (purple), 0.2mm (green), and 0.3mm (orange) deflections for b) PP01 Model, c) PP02 Model, and d) Flat model

So as not to invalidate all previous updating results, the relaxation of the boundary conditions should have limited effect on the linear natural frequencies. At this point it is beneficial to revisit a comparison between the linear natural frequencies of the NLROMs and the experimental measurements. With the relaxation of the boundary conditions, the natural frequencies of all models show minimal change. This emphasizes that the boundary conditions were not fully perturbed during linear testing and was missed in the first updating step. A better representation of the perforated plates could be found by using both PP01 and PP02 results during the updating providing a more optimum result; however, the model updating steps have shown a level of robustness since the primary focus has been on creating the best model to match PP01.

Table 2: Frequency results from boundary condition update

Model	Flat			PP01			PP02		
Mode #	f_n, Hz, EXP AVG	f_n, Hz, NUM 3	% Err.	f_n, Hz, EXP	f_n, Hz, NUM 3	% Err.	f_n, Hz, EXP	f_n, Hz, NUM 3	% Err.
1	213.75	153.45	28.21	205.36	202.03	1.62	222.13	210.71	5.14
2	339.18	317.43	6.41	327.84	327.89	-0.01	350.51	350.48	0.01
3	356.80	317.43	11.03	348.65	351.37	-0.78	364.95	363.47	0.41
4	506.18	524.51	-3.62	489.17	511.94	-4.66	523.19	548.16	-4.77
5	526.68	525.87	0.15	510.23	526.34	-3.16	543.12	564.36	-3.91
6	571.87	601.63	-5.20	572.63	558.33	2.50	571.12	572.31	-0.21
7	711.78	774.59	-8.82	697.90	735.73	-5.42	725.66	782.54	-7.84
8	715.14	774.59	-8.31	699.95	737.8	-5.41	730.33	787.65	-7.85
9	796.45	924.26	-16.05	814.12	824.29	-1.25	778.78	837.55	-7.55
10	852.27	924.26	-8.45	827.68	835.38	-0.93	876.85	847.43	3.36

3.5. Stress Distribution of the Final Updated Models

Using the updated NLROMs, we can now explore potential causes of the failures observed in the experimental testing of PP01 and PP02. Before examining the stress distribution throughout increasing nonlinear responses, it is beneficial to note that a circular flat plate with fixed boundary conditions undergoing distributed loading is expected to have the highest stresses near the clamp when deformations are on the order of half of the plate thickness. However, small differences in the boundary conditions will reduce edge stresses while increasing the deflection and stresses at the center of the plate [34]. The plates presented here show an interesting combination of these effects due to the changes in the boundary conditions we have implemented as well as internal forces predicted in the plates. It was demonstrated in Fig. 7, that along the NNM backbone a dominant interaction with Mode 6 is observed at $3*f_{in}$ with secondary effects observed at $2*f_{in}$ and $4*f_{in}$. Since the failures observed were in the center of the plate, the modal interaction is thought to change the stress distribution at higher response amplitudes so a 'hot spot' forms near the center of the plate.

This is demonstrated using the updated NLROMs by expanding the membrane effects condensed during the ICE procedure and projecting the predicted deformation of the modal NLROM at several levels along the NNM to find the deformation of the structure in the physical space. This deformation is then applied to the structure using a nonlinear static analysis in Abaqus®. The internal stresses due to the predicted in- and out-of-plane deformations are resolved within the analysis. Plots of the peak stress observed over a period of the response at four levels of deformation along the NNM are shown in Fig. 10 for the three updated models. For the flat plate (Fig. 10d) the low amplitude deformation (purple) shows a high von Mises stress at the center and edge of the plate indicating the importance of the relaxed boundary condition at low amplitudes. At higher levels of deformation (blue to red) the stiffening from in-plane stretching becomes more apparent as the stress concentrates near the edge of the plate. For PP01 (Fig. 10b) and PP02 (Fig. 10c), the von Mises stress is highest at the edges for low amplitude of deformation (purple) indicating a stiffer boundary condition due to the high curvature near the edge of the plate. At higher levels of deformation (blue to red), the Mode 6 interaction concentrates the highest level of stress just offset of the center of the plate forming a 'hot spot'. The location of this larger stress corroborates the failures observed in the experiment and provides a final emphasis to the difference between a flat and curved geometrically nonlinear structure.

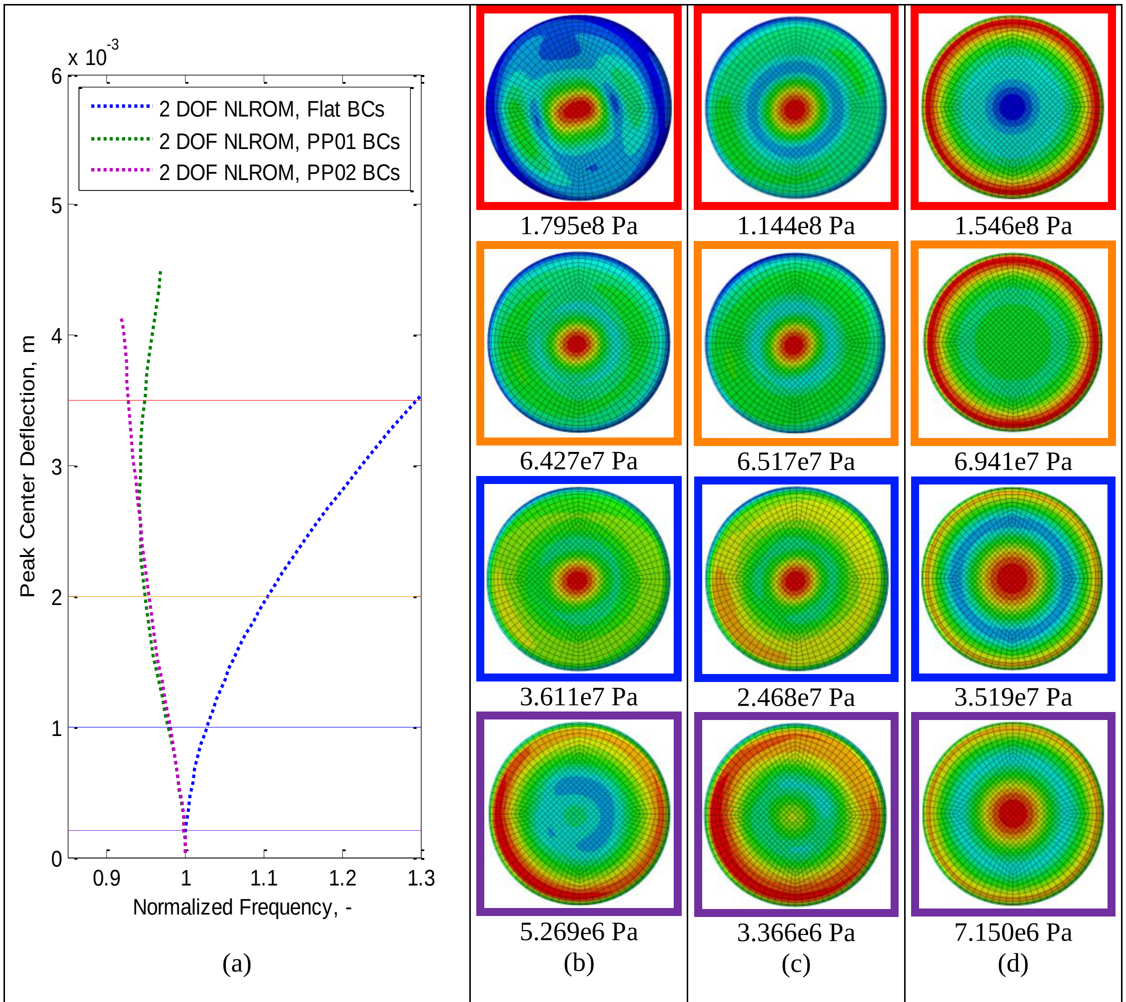


Figure 10: Von Mises Stress Distribution and corresponding peak stress at 0.02mm (purple), 0.10mm (blue) 0.20mm (orange), and 0.35mm (red) deflections along the NNM for the b) Flat model, c) PP01, and d) PP02.

4. Conclusions

This work has explored the use of NNM backbones as a metric for finite element model calibration in nonlinear response regimes for two axi-symmetric perforated plates. Full field measurements and previous numerical work has shown the importance of the coupling between mode 1 and mode 6 at larger response levels. Through the examination of single point responses at the center of the perforated plate, the frequency-amplitude relationship is presented and used for model updating. For the nominally flat plate, the fundamental natural frequency of the final updated model is still outside acceptable frequency error ranges; however, there is gained insight to the final stress distribution at increasing amplitudes of response pointing to a more optimal design. The curved plates agree well throughout the updating process, and final stress distributions point to a potential mechanism of the observed experimental failures. This work has shown the benefit of using NNMs to characterize nonlinear behavior and guide model updating for geometrically nonlinear structures. Additional emphasis is placed on the importance of accounting for potential uncertainties in the experiment before changing nonlinear characteristics to obtain a more complete picture of how the structure behaves nonlinearly.

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