Model Validation of a Bolted Beam Using Spatially Detailed Mode Shapes Measured by Continuous-Scan Laser Doppler Vibrometry

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The mode shapes of a structure are typically more sensitive than its natural frequencies to local features such as cracks, joints or other anomalies. However, when updating a finite element model for a complicated structure, the analyst is typically only provided with mode shape measurements at a very small number of sensors, so they must rely primarily on the frequencies. With the advent of the Laser Doppler Vibrometer (LDV), one can obtain an unprecedented level of spatial resolution in an automated manner. However, there still may be limitations on the information that can be obtained due to variation in the input to the structure from point to point, the large time required to acquire measurements at each point sequentially for some structures, and logistical issues. Continuous-scan laser vibrometry has the potential to overcome some of these limitations. The continuous-scan approach involves sweeping the laser spot continuously over the structure during a measurement, and then processing the measurement to estimate the mode shapes over a line or surface from that single measurement. This work explores the impact of a spatially detailed measurement set on model validation of a simple structure comprised of a straight beam with a bolted lap joint. The natural frequencies of the system are found to change by 1-3% due to the joint, but when the experimental natural frequencies are compared with those of the model the expected trends are not observed, so it is difficult to ascertain whether the joint model is correct based on the natural frequencies alone. On the other hand, mode shapes measured by CSLDV clearly reveal the presence of the joint, showing a 16-26% reduction in a few of the shapes in a 10-20 centimeter region surrounding the joint. The observed changes in the mode shapes correlate well with those estimated by the model.

I. Introduction

Experimental modal analysis is routinely used in the aerospace industry to validate finite element models of aircraft components and space systems. NASA [1] and the US Air Force [2] mandate a validation procedure, in part to assure that the satellite or component of interest survives the launch environment [2]. Typically, a modal test is performed on the actual hardware to extract its modes and they are then compared with those of the finite element model to ascertain whether the model is capable of reproducing the measured dynamics. If the discrepancies are large enough, then the model must be refined to better describe the system. If adequate agreement is obtained between the model and test, then the model can be tuned or updated to reproduce the measured dynamics even more closely and the updated model is then used to predict the loads on the structure and to determine whether it is likely to survive. In the initial creation of the model and in the updating process, there is never enough information nor resources to perfectly model the structure. For example, there are typically uncertainties associated with material properties, complex geometric features must be simplified, and there are uncertainties regarding the stiffnesses and perhaps even linearity of joints in the structure. The modeling difficulty is compounded by the fact that the analyst typically has only very limited information from the test. The modes of the structure can typically only be extracted

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over a limited frequency range, and at a few hundred points or less, even though the model may have millions of degrees of freedom and dozens of uncertain parameters. So the analyst is faced with an updating problem that may be significantly underdetermined; there may be many combinations of the uncertain parameters that reproduce the measured modes to within the expected precision, but some of those combinations may result in quite different estimates of the loads and stresses in the component.

With the advent of Laser Doppler Vibrometry, one can measure the response of a structure at hundreds or even thousands of points in an automated fashion, providing much more spatial information than was previously available. There are some limitations, of course, such as the fact that the points at which measurements are obtained must be in the line of sight of the laser, and the quality of the measurements depends on the reflective properties of the surface and other factors. Traditional scanning laser vibrometry (SLDV) also requires an automated excitation device, and may require excessive time to acquire the measurements [3], since the measurement at each point must encompass a number of time constants and a number of averages. Allen and Sracic recently developed a method that can greatly accelerate the acquisition of spatially dense measurements using Continuous-Scan Laser Doppler Vibrometry (CSLDV) [4-6]. A number of other CSLDV approaches are also available [7-11], the earliest of which dates back to the early nineties. These are compared and discussed to some extent in [6]. The procedure in [4-6] or [9, 12] involves sweeping the laser spot continuously over the structure while measuring the response to an impact force, and extracts the modes of the system at hundreds of points along the scan path from a few response measurements.

Structural modification theory [13] suggests that the eigenvectors of a structure are more sensitive to local modifications than the eigenvalues (which contradicts the assumption used when deriving the structural modification equations). Specifically, Ewins showed that if the system of interest has symmetric mass and stiffness matrices, and a modification is made to either the stiffness, [*K*], or mass, [*M*], matrices <u>that does not alter the eigenvectors of the system</u>, then the sensitivity in the *r*th natural frequency with respect to a change in parameter *p* is

$$\frac{\partial \omega^{mod}{}_r^{\,2}}{\partial p} = \{\phi\}_r^{\,T}\left[\frac{\partial[K]}{\partial p} - \omega_r^{\,2}\frac{\partial[M]}{\partial p}\right]\{\phi\}_r \,. \tag{1}$$

Specifically, adding a rigid mass Δm_n to the *n*th nodal degree of freedom changes each natural frequency squared by the following amount.

$$\Delta\omega^{mod}{}_r^{\,2} = -\omega_r^{\,2}\phi_{nr}^{\,2}\Delta m_n \tag{2}$$

These sensitivities show that the natural frequencies change as the square root of the modification. A similar analysis for the eigenvectors gives [13],

$$\frac{\partial\{\phi\}_r}{\partial p} = \sum_{s=1}^{N} - \frac{\{\phi\}_s^{\,T}\left(\frac{\partial[K]}{\partial p} - \omega_r^{\,2}\frac{\partial[M]}{\partial p}\right)\{\phi\}_r}{\left(\omega_s^{\,2} - \omega_r^{\,2}\right)}\{\phi\}_s \,. \tag{3}$$

revealing that the eigenvectors or mode shapes change with the first power of the modification. Hence, by comparing the mode shapes of a structure with those of a finite element model, one may discover local errors in the model that are not revealed by the natural frequencies. On the other hand, one should bear in mind that the natural frequencies of a structure can be measured quite precisely. If the experimental fixturing is properly designed, then although the frequencies are less sensitive, they may still be more reliable than the mode shapes. However, current laser vibrometry methods promise to provide both accurate and detailed spatial information so one may finally be able to measure mode shapes with competing accuracy.

This work explores these issues, using CSLDV to acquire highly detailed mode shape measurements from a simple beam with a bolted joint. The effect of the bolted section on the natural frequencies and mode shapes of the beam is quantified and used to validate a finite element model of the bolted structure. The results show that the spatially detailed mode shapes provided by CSLDV reveal information about the joint that is not readily manifest by the natural frequencies of the structure. The changes in the mode shapes are quite small, so measurements of the mode shapes using conventional techniques with a coarse measurement grid would also be unlikely to reveal the differences between this bolted joint and a straight beam.

II. Experimental/Analytical Model Validation Exercise

Figure 1 shows a schematic of the system under consideration, which consists of two steel beams joined in a lap joint with two steel bolts. The longer beam was 66.25 cm long, 2.54 cm wide, and 0.47 cm thick, while the shorter was 35.56 cm long and had the same cross section. The beams overlap over the last 5.08 cm, giving the structure a total length of 96.7 cm. Measuring from the left end of the beam, the bolted section comprises the region of the beam for which $61.2 < x < 66.3$ cm. Figure 2 shows a picture of the actual beam and two views of the bolted section. The mating surfaces were milled, showing some residual scarring from the milling process. For the tests described here the bolts were very tight to minimize nonlinear effects of the joint.

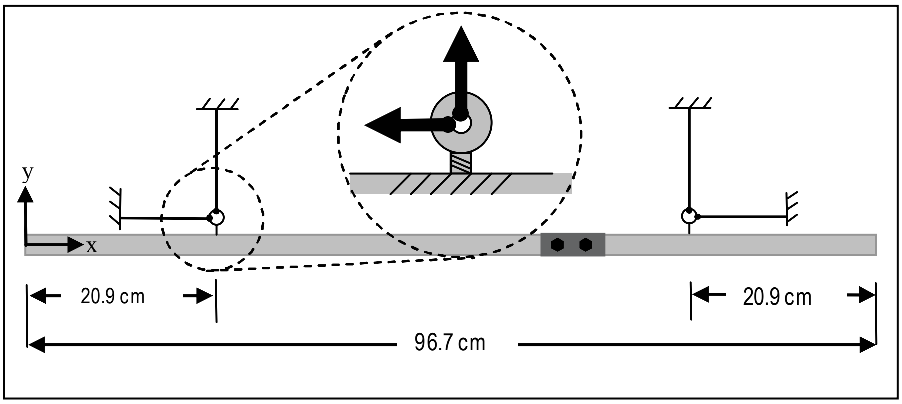

Figure 1: Schematic of experimental test structure.

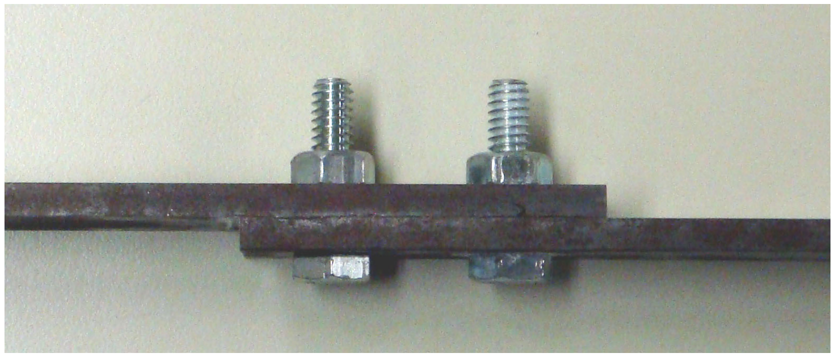

Figure 2: Photographs of bolted joint in the steel beam.

The bolted structure was modeled in a very approximate sense using one-dimensional beam elements. The joint was modeled simply by increasing the thickness of the beam material in the region of the joint. This model was found to capture the modes of the beam very well (see the following section), so more detailed modeling was not pursued. The models presented here include a straight beam (no joint section), a beam where the jointed section was made twice as thick as the rest of the beam, and a third model where the thickness of the jointed section was three times the beam thickness. These models are denoted 1x, 2x and 3x respectively. One would expect the 2x model to capture the stiffness of the actual structure if the constraint between the two beams were perfect. The mass loading effect of the joint is expected to be about 20% larger than that of the 2x model, due to the mass of the bolts.

A. Natural Frequencies

The beam was tested using CSLDV using the methods described in [6] to obtain its first six elastic modes. A free-free condition was simulated by suspending the beam in a wooden frame using thin bungee cords, such that the natural frequencies of the beam's rigid body modes were low (1 to 2.5 Hz) relative to the first elastic mode. To simplify the setup, two eyelets were installed 20.9 cm from each end of the beam, as shown in Figures 1 and 2, to which the bungee cords were attached. Two tests were performed in which the scan speed, or frequency with which the laser swept across the length of the beam, was set at 48Hz and 90Hz respectively. The data acquisition sample rate was 25kHz, but the 48Hz measurements were resampled at 12.5kHz, so the number of points extracted along one sweep of the beam were respectively, 267 and 285. The 48Hz measurements were resampled because the bandwidth of the measurement was found to be less than 12.5kHz, and retaining superfluous frequency lines has been found to increase the noise in CSLDV measurements [14]. Five input locations were chosen, and the response of the beam to about three different impulses was measured for each location, resulting in a total of 14 and 15 time responses in the two cases respectively, each of which constitutes a single-input-multiple-output measurement set. No reliable averaging technique exists for CSLDV measurements, so the entire set of measurements was processed globally as if it were a MIMO data with 14 or 15 inputs. The AMI modal parameter identification routine was used to identify the modes of the structure from this measurement set [15-17]. CSLDV results in significant time savings when compared with conventional measurement techniques. For the 90 Hz measurements, only 10.22 seconds of response data were needed to acquire each of the 15 time records, so a total of 2.5 minutes of measurement data was required to obtain the 285 by 14(15) FRF matrices. Using conventional scanning laser vibrometry, 12.1 hours of measurements would have been required to obtain the same resolution. Furthermore, hammer testing would not have been practical for such a large number of points so one would be forced to use a shaker or some other automated excitation device, which would complicate the setup and might modify the structure.

The CSLDV measurements were processed and the mode shapes, natural frequencies and damping ratios of the first six modes were extracted using the procedure described in [4-6]. In each case the mode shapes and natural frequencies were acquired and compared with the FEM described previously. Table 1 shows the natural frequencies of each of the FEMs compared with those obtained experimentally. The experimental frequencies are all very consistent (identical to the number of significant figures shown in the table), the largest difference being only 0.03%, although one should bear in mind that the support conditions were not modified between these two tests, so any bias in the frequencies due to the support conditions would not be visible here. The ratio of E/ρ in the FEM was chosen such that the first natural frequency obtained by the 2x FEM was equal to the average of those estimated experimentally. All of the frequencies are quite similar; there is no more than a 3% difference between any natural frequency of the 1x and 3x models, and the experimental frequencies all fall in that range. This is more easily seen in Table 2, which shows the percent differences between each experimentally obtained frequency and those of the

2x FEM. The percent differences between the 1x and 3x FEM models are also shown. One would expect the experimental frequencies to differ from the 2x model by only a very small amount if that model is indeed accurate. The differences in the frequencies are small, although they are large relative to the differences between the 1x and 3x FEM models. The 3rd mode's frequency differs the most from the 2x model, which is especially troubling because the model predicts that this mode should be insensitive to bolted section; it was discovered that the beam's first in-plane bending mode was at about 145 Hz (1.5% higher than the FEM 2x frequency), so perhaps the closeness of these frequencies is causing the system to be sensitive in this frequency range. The other modes vary from the FEM in a seemingly random pattern, so it is impossible to conclude which of the FEM models is best based on the frequencies alone.

Mode	**FEM**			**Experiment**			
	1x	**2x**	**3x**	**48 Hz**	**90 Hz**	**Mean**	**Difference**
1	26.6	26.8*	26.7	26.8	26.8	26.8*	0.014%
2	73.2	72.6	71.4	72.5	72.5	72.5	0.026%
3	143.1	142.9	142.6	141.1	141.0	141.0	0.029%
4	235.9	234.2	232.1	234.8	234.8	234.8	0.016%
5	351.2	349.4	347.0	352.1	352.0	352.1	0.021%
6	488.6	486.5	483.6	492.8	492.8	492.8	0.004%

*Table 1: Natural Frequencies of FEM model of Beam compared to frequencies measured experimentally with CSLDV. *The FEM was tuned so that the first natural frequency of the 2x FEM matched the mean experimental frequency.*

Mode	**1x**	**2x**	**3x**	**48 Hz**	**90 Hz**
1	-1.0%	ref	-0.4%	-	-
2	0.8%	ref	-1.7%	-0.18%	-0.20%
3	0.1%	ref	-0.2%	-1.30%	-1.33%
4	0.7%	ref	-0.9%	0.25%	0.23%
5	0.5%	ref	-0.7%	0.78%	0.76%
6	0.4%	ref	-0.6%	1.30%	1.29%

Table 2: Percent difference between 2x FEM natural frequencies and those of the 1x and 3x FEM models. The differences between the experimental frequencies and the 2x FEM are also shown.

Figures 3 through 8 show the shapes of the first six elastic modes of the beam obtained by the Algorithm of Mode Isolation [15] from the CSLDV responses with the 48 Hz scan frequency. While one can find mass-normalized mode vectors using CSLDV [5], the uncertainty in the scale factor for a mode is typically much higher than the uncertainty in the shape. In these figures, the modes have been scaled to have the same norm as the 2x FEM mode vector, so only the shape is of interest.

The FEA model shows that the 1st, 3rd and 6th modes of the FEM were not very sensitive to the bolted section (Figures 3, 5 and 8), which seems reasonable since the bolted section was located between 61.2 and 66.3 cm from the left end, and those two modes do not deform the beam significantly in that region, nor do they displace it much, suggesting that the stiffening and mass-loading effects of the joint are negligible for those modes. The experimentally measured 3rd and 6th mode shapes agree very well with the FEM, the latter being contained within the scatter band of the experimental measurements over most of the beam.

On the other hand, the 2nd, 4th and 5th mode shapes are all sensitive to the bolted section; the FEM shows a 16% reduction in the amplitude of the shape near the bolted section for the 2nd mode, and that reduction becomes slightly larger for the 4th and 5th mode shapes. The experimental measurements agree very well with the model for the 2nd and 4th mode shapes, and fairly well for the 5th, although that mode shape is fairly noisy, which is probably explained by the fact that it is aliased to a noisy part of the spectrum by the CSLDV process [6].

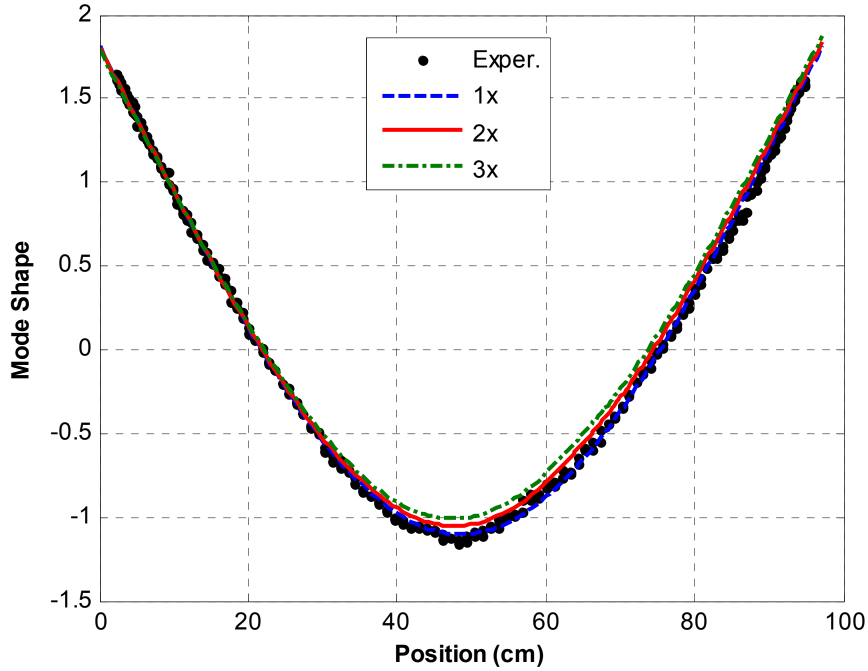

Figure 3: Experimentally measured 1st bending mode measured using 48Hz scan frequency, compared to modes of 1x, 2x and 3x FEM models. The bolted section is located between 61.2 and 66.3 cm.

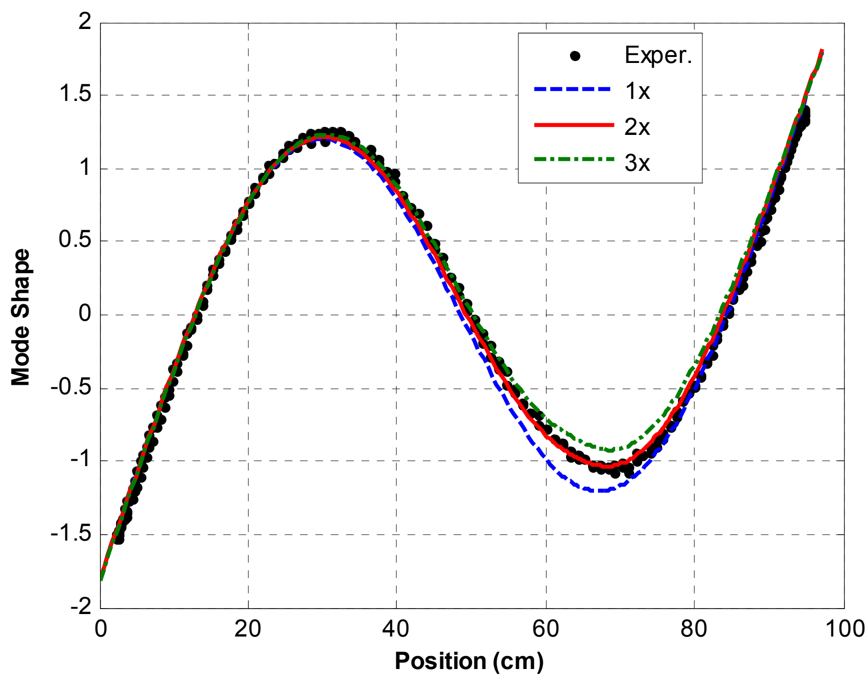

Figure 4: Experimentally measured 2nd bending mode measured using 48Hz scan frequency, compared to modes of 1x, 2x and 3x FEM models. The bolted section is located between 61.2 and 66.3 cm.

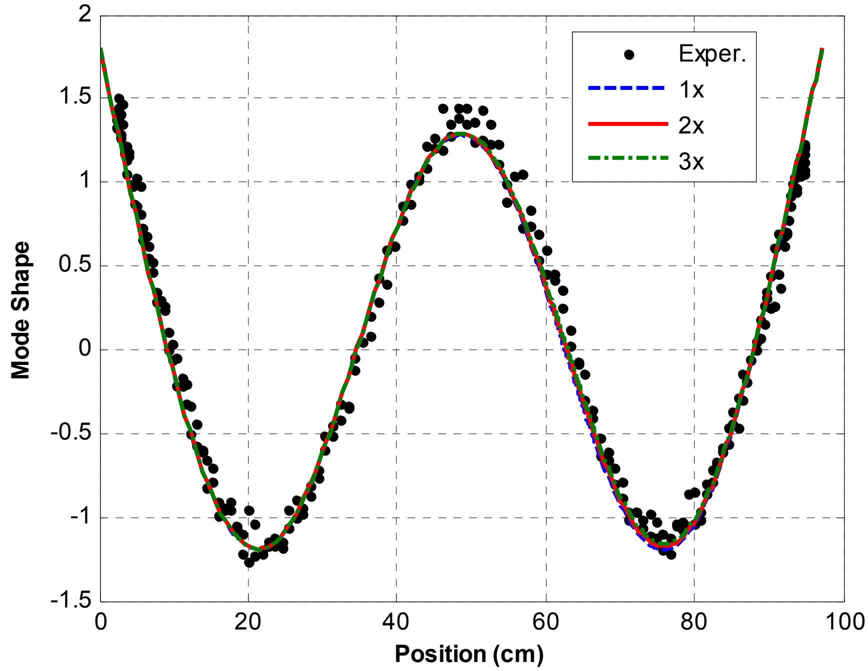

Figure 5: Experimentally measured 3rd bending mode measured using 48Hz scan frequency, compared to modes of 1x, 2x and 3x FEM models. The bolted section is located between 61.2 and 66.3 cm.

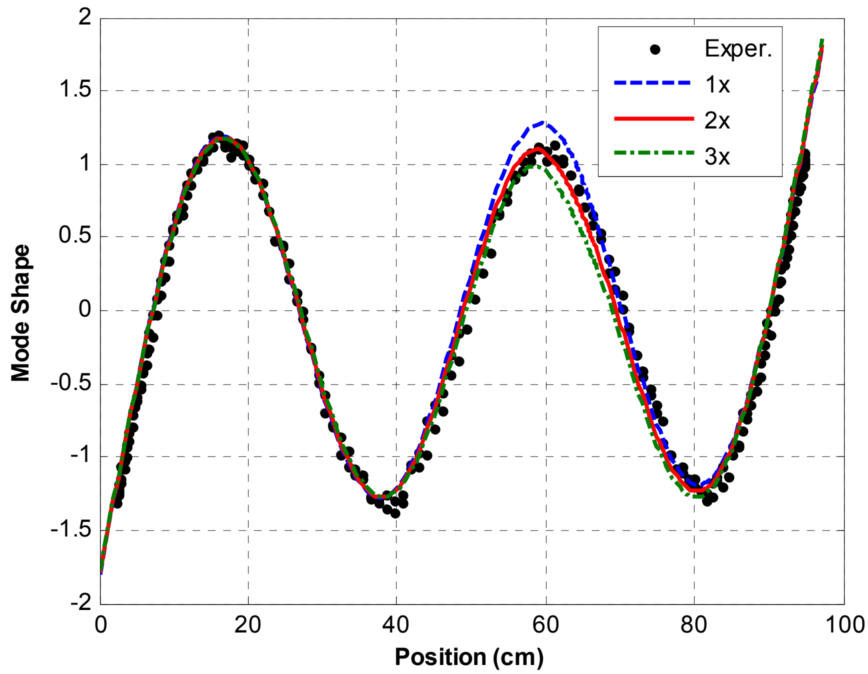

Figure 6: Experimentally measured 4th bending mode measured using 48Hz scan frequency, compared to modes of 1x, 2x and 3x FEM models. The bolted section is located between 61.2 and 66.3 cm.

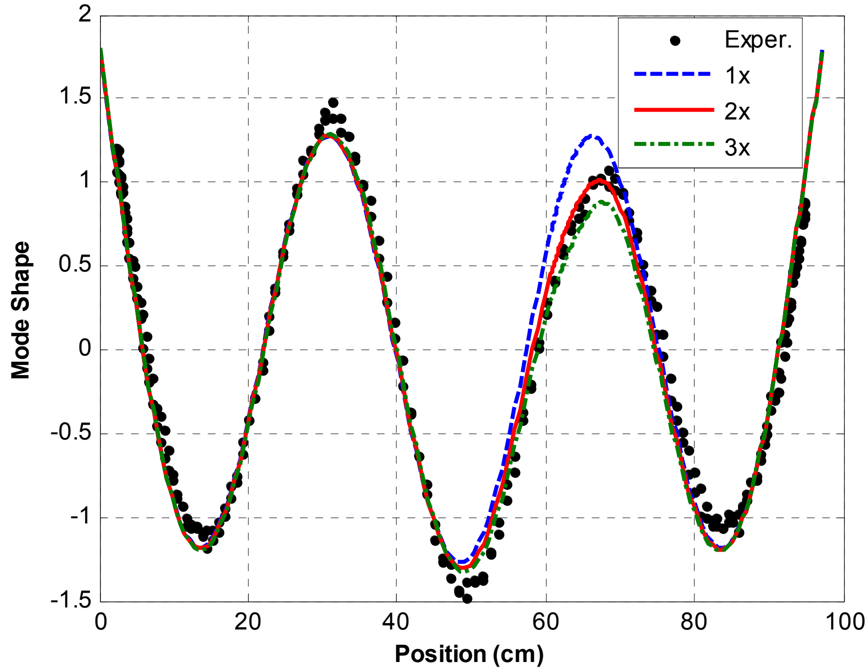

Figure 7: Experimentally measured 5th bending mode measured using 48Hz scan frequency, compared to modes of 1x, 2x and 3x FEM models. The bolted section is located between 61.2 and 66.3 cm.

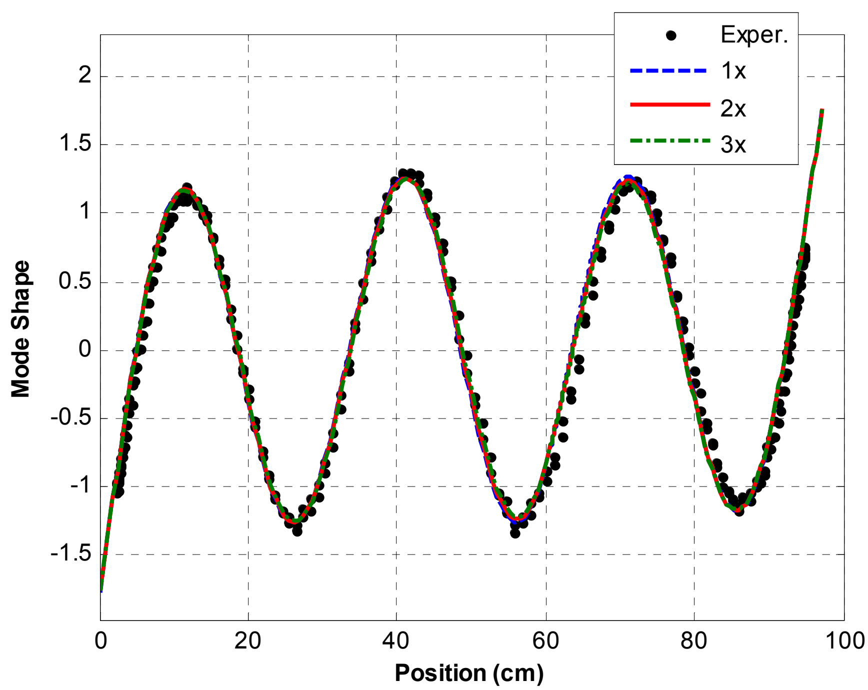

Figure 8: Experimentally measured 6th bending mode measured using 48Hz scan frequency, compared to modes of 1x, 2x and 3x FEM models. The bolted section is located between 61.2 and 66.3 cm.

The mode shapes for the 90 Hz scan frequency are shown in the Appendix. When scanning at 90 Hz the mirror system was found to behave somewhat nonlinearly, which changes the position vector that is used when plotting the mode shapes. To correct for this, the sinusoidal signal describing the position of the laser spot [6] was replaced with a sinusoid plus a 3rd harmonic (270 Hz), whose amplitude was 7% of the first. While this correction was found empirically, the mirror system was tested by using a second LDV to measure its motion when driving it sinusoidally, and those tests revealed about this same level of harmonic distortion. The resulting mode shapes are shown in Figures 9 through 14. The mode shapes obtained at the 90 Hz scan frequency agree very well with the 2x FEM, as was the case for the 48 Hz modes. The scatter in the 5th mode shape found with the 90 Hz scan frequency is considerably smaller, but most of the rest of the modes are quite similar to those found at 48 Hz.

Table 3 shows the Modal Assurance Criterion [13] or MAC between each of the experimental mode shapes and the FEM shapes for each of the models. The MACs were computed by interpolating the FEM modes onto the points measured by CSLDV. The largest MAC value is highlighted in gray. The MAC values are quite insensitive to the shape change, as evidenced by the fact that the lowest MAC between the experimental shapes and any of the FEM models is greater than 0.96. However, the results do show that the MACs are generally largest between the experimental shapes and those of the 2x model, especially if one disregards the 3rd and 6th modes, which did not change appreciably between the three FEM models.

Mode	**MAC between 48 Hz and**			**MAC between 90 Hz and**		
	1x	**2x**	**3x**	**1x**	**2x**	**3x**
1	0.9991	0.9973	0.9931	0.9956	0.9982	0.9969
2	0.9914	0.9962	0.9930	0.9870	0.9944	0.9914
3	0.9827	0.9841	0.9852	0.9850	0.9841	0.9836
4	0.9837	0.9857	0.9790	0.9673	0.9810	0.9806
5	0.9638	0.9773	0.9765	0.9631	0.9759	0.9728
6	0.9796	0.9807	0.9810	0.9706	0.9720	0.9735

Table 3: MAC values between mode shapes obtained experimentally by CSLDV at 48 and 90 Hz scan frequencies and those of the 1x, 2x and 3x Finite Element Models.

III. Conclusions

This work has explored the use of mode shapes and natural frequencies when validating a model of a beam with a simple lap joint. Continuous-scan laser vibrometry was used to acquire measurements of the mode shapes of the beam with very high spatial resolution and relatively low scatter, and these were used to aid in the model validation process. The objective of the validation exercise was to determine how thick the bolted section of the beam should be to accurately reproduce the mode shapes and natural frequencies of the system. The analytical model showed that the natural frequencies of the beam were somewhat sensitive to the joint, varying by a little more than 1% over the range of thicknesses studied, so one would expect to be able to use them to calibrate the model. However, the experimental frequencies did not follow the expected trends, so one could gain very little information regarding what thickness was correct using the frequencies. The reason for this discrepancy is not known. Some possible explanations include the support conditions for the test, model inaccuracy due to simplifications in the model, or bias in the signal processing or modal parameter extraction routine.

On the other hand, the mode shapes obtained by CSLDV gave a wealth of information regarding the joint. It was shown to change the mode shapes of only those modes that involved significant bending of the overlapping section, reducing the amplitude of the shape by 10-20% in the region of the joint for those modes. The analytical model predicted that modes that do not involve bending of the joint would not change, and the experimental shapes confirmed this. The resolution of the mode shapes was sufficient that one could see fairly clearly that the 2x model agreed most closely with the measurements based on plots of the mode shapes. The MAC values of the FEM and experimental mode vectors confirmed this.

The results here confirm the usefulness of spatially detailed mode shapes when correlating a model with experiment. One interesting question is whether mode shapes of adequate quality could have been obtained using other methods. First consider impact testing with a roving hammer. It would be very difficult to obtain mode shapes that are consistent enough to detect the 10-20% changes that were observed here, as illustrated by Ribichini et al. [9]. Another alternative is a test involving fixed accelerometers. Accelerometer sensitivities typically have uncertainties of about 5% or less, and consistency across a set that are calibrated simultaneously could be even better, so the changes observed here probably could have been captured with fixed accelerometers, so long as their locations were carefully controlled and care was taken in calibration. However, one would need at least two sensors in the region of interest to have strong confidence that the observed changes weren't simply a measurement anomaly. Finally, conventional SLDV could have been used to obtain this same level of spatial resolution, but one would need to use an automated excitation device such as a shaker or automated hammer, and one might need many hours of measurements to obtain the desired spatial resolution. Anomalies are possible at single points using SLDV due to variations in the surface reflectivity or in the input signal, so one would need a surplus of measurements to be sure that the variations in the mode shape in any given region were meaningful.

Future works will explore whether CSLDV can be used to detect structural damage using the mode shape information and will apply the technique to more complicated joints and structures.

Appendix

Mode shapes of beam found using CSLDV with a 90 Hz scan frequency.

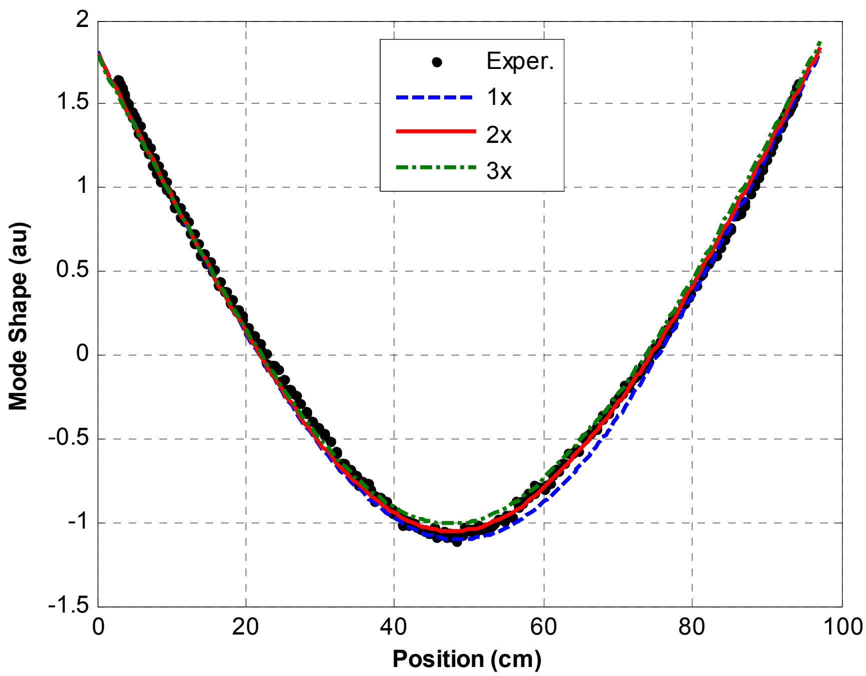

Figure 9: Experimentally measured 1st bending mode measured using 90Hz scan frequency, compared to modes of 1x, 2x and 3x FEM models. The bolted section is located between 61.2 and 66.3 cm.

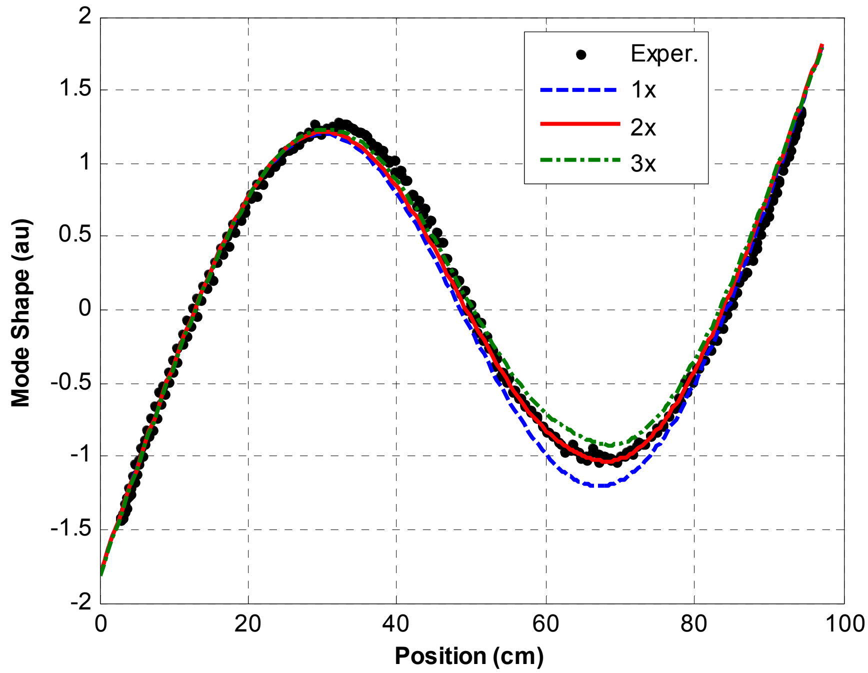

Figure 10: Experimentally measured 2nd bending mode measured using 90Hz scan frequency, compared to modes of 1x, 2x and 3x FEM models. The bolted section is located between 61.2 and 66.3 cm.

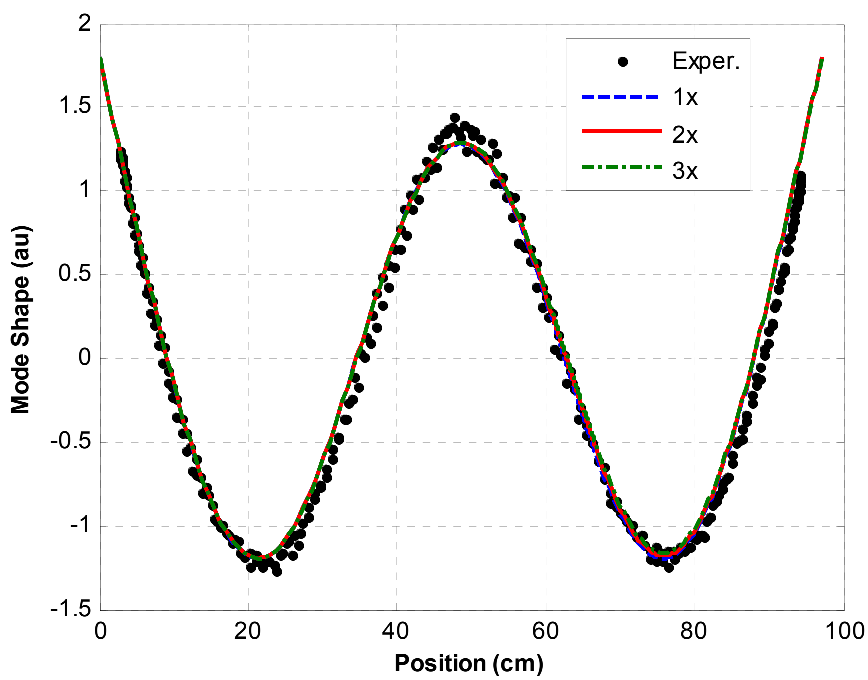

Figure 11: Experimentally measured 3rd bending mode measured using 90Hz scan frequency, compared to modes of 1x, 2x and 3x FEM models. The bolted section is located between 61.2 and 66.3 cm.

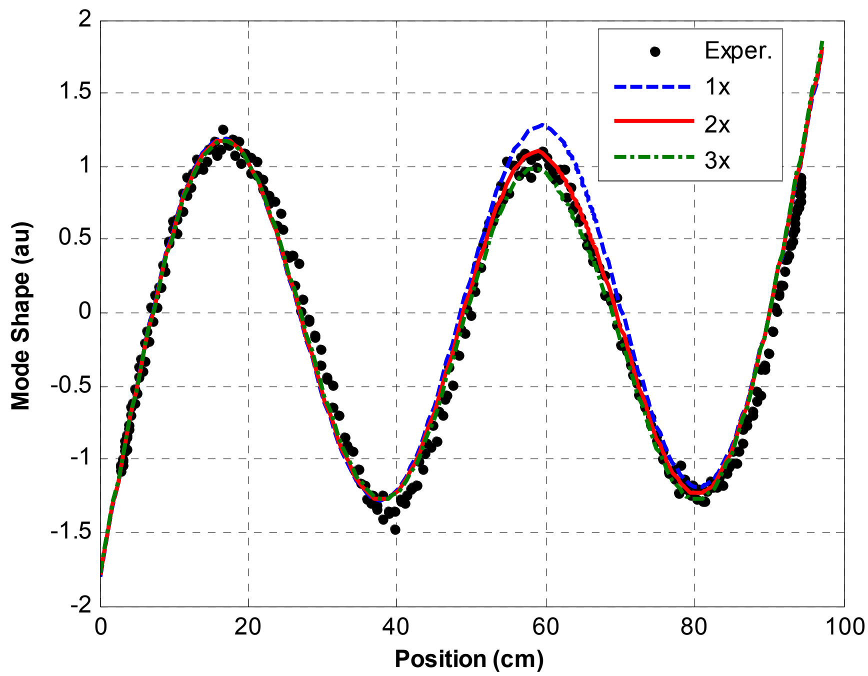

Figure 12: Experimentally measured 4th bending mode measured using 90Hz scan frequency, compared to modes of 1x, 2x and 3x FEM models. The bolted section is located between 61.2 and 66.3 cm.

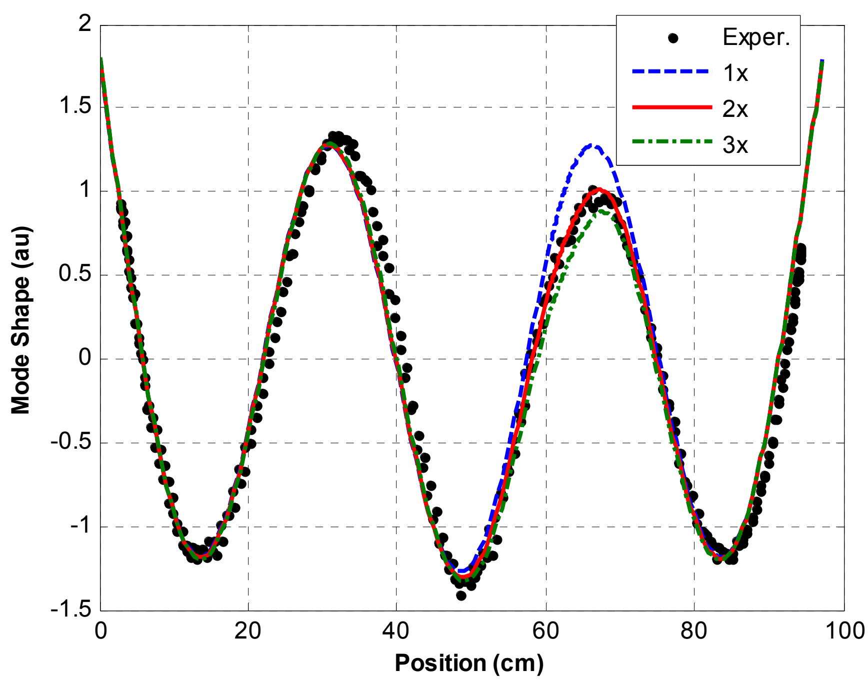

Figure 13: Experimentally measured 5th bending mode measured using 90Hz scan frequency, compared to modes of 1x, 2x and 3x FEM models. The bolted section is located between 61.2 and 66.3 cm.

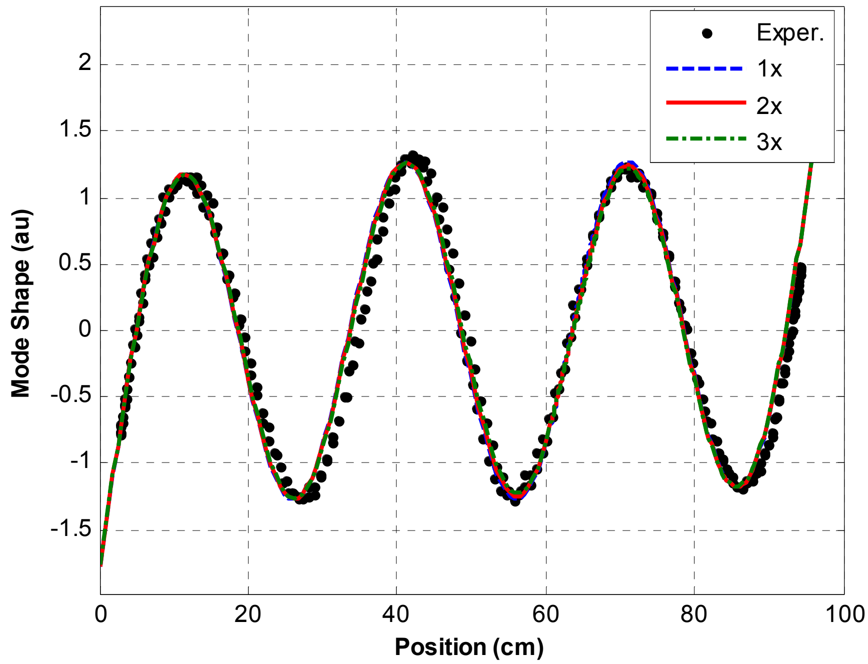

Figure 14: Experimentally measured 6th bending mode measured using 90Hz scan frequency, compared to modes of 1x, 2x and 3x FEM models. The bolted section is located between 61.2 and 66.3 cm.

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