Simultaneous Regression and Selection in Nonlinear Modal Model Identification

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Abstract

Nonlinear reduced order models (ROMs) have demonstrated to be an effective approach to the modeling of structures undergoing geometrically nonlinear response. These models allow for the nonlinear response of large finite element (FE) models to be approximated at a significantly lower computational cost. A popular approach to creating a ROM is the Implicit Condensation and Expansion (ICE) method which identifies nonlinear modal model parameters using regression of static force-displacement data from FE model simulations. A drawback of these models is that the number of coefficients to identify increases cubically with the number of modes in the ROM, posing a key challenge for identification of critical ROM parameters which is required for model updating. This work utilizes the method of least absolute shrinkage and selection (LASSO) to identify sparse solutions of ROM coefficients during the ICE regression step. It will be shown that the number of nonlinear coefficients of a ROM can be drastically reduced while maintaining accurate response predictions.

Keywords: Reduced Order Models, Nonlinear Dynamics, Finite Element Analysis, Nonlinear Normal Modes, Parameter Identification

1 Introduction

High fidelity finite element (FE) models are often used to simulate the complex response of nonlinear dynamical systems. Numerical integration comes at a high computational cost making it infeasible for large systems, especially when the response is of interest over a long time duration as in the case of random loads. An alternative is to create a reduced order model (ROM), which can be dramatically cheaper to integrate. ROMs express the nonlinear dynamical system in a reduced (modal) subspace. The ROM can be integrated at a significantly lower computational cost because the dimension of the system is typically much less than the full order system and it may admit a larger timestep. ROMs can be generated from FE element models using a variety of approaches, one popular approach is Implicit Condensation (IC) [1,2], in which the nonlinearities of the FE model are represented as quadratic and cubic polynomials.

The Implicit Condensation method uses static force-displacement data from the full order FE model to identify the parameters in a nonlinear modal model using polynomial regression. The conventional approach is to use least squares (LS) regression to find these parameters. A drawback of IC and LS regression is that the number of parameters increases cubically with the number of modes in the ROM. However, it is often the case that only a small subset of the parameters contribute significantly to the nonlinear response of the system. If one could identify and eliminate the parameters that are are unimportant, the computational efficiency and simplicity of the ROM could be improved significantly.

This work makes use of the Least Absolute Selection and Shrinkage Operator (LASSO) to simultaneously identify the ROM coefficients and select those that are most important. Originally developed by Tibshirani [3], LASSO is a popular tool in machine learning and data analytics for generating sparse predictor models. LASSO produces sparse solutions by penalizing the one-norm, i.e. sum of absolute values, of the coefficient vector. As the penalization term is increased the regression solution becomes more sparse such that the nonzero terms correspond to the most important parameters. A brief overview of the theory of nonlinear modal parameter identification and lasso will be presented followed by a short numerical evaluation of the method.

2 Theory

The geometrically nonlinear elastic FE equations of motion for an n DOF system can be written as

$$\mathbf{M}\ddot{\mathbf{x}} + \mathbf{C}\dot{\mathbf{x}} + \mathbf{K}\mathbf{x} + \mathbf{f}_{NL}(\mathbf{x}) = \mathbf{f}(t) \tag{1}$$

where $\mathbf{M}$, $\mathbf{C}$, and $\mathbf{K}$ are the $(n \times n)$ mass, damping, and linear stiffness matrices respectively. The nonlinear restoring force $\mathbf{f}_{NL}(\mathbf{x})$ is a function of the displacement only. The full FE equations of motion can be projected onto a modal subspace using the modal coordinate transformation $\mathbf{x}(t) = \mathbf{\Phi}_m \mathbf{q}(t)$ in which the r^{th} nonlinear modal equation becomes

$$\ddot{q}_r + c_r \dot{q}_r + \omega_r^2 q_r + \theta_r(q_1, q_2, ..., q_m) = \boldsymbol{\varphi}_r^T \mathbf{f}(t) \tag{2}$$

The nonlinear restoring force θ_r is a function of the modal displacements as $\theta_r(\mathbf{q}) = \boldsymbol{\varphi}_r^T \mathbf{f}_{NL}(\mathbf{\Phi_m q})$, which for a linear elastic system with only geometric nonlinearities can be accurately approximated as

$$\theta_r(\mathbf{q_1}, \mathbf{q_2}, ...\mathbf{q_m}) = \sum_{i=1}^{m}\sum_{j=i}^{m} \mathbf{A}_r(i,j)\mathbf{q_i}, \mathbf{q_j} + \sum_{i=1}^{m}\sum_{j=i}^{m}\sum_{k=j}^{m} \mathbf{B}_r(i,j,k)\mathbf{q_i}, \mathbf{q_j}, \mathbf{q_k}, \tag{3}$$

where $\mathbf{A}_r$ and $\mathbf{B}_r$ are the quadratic and cubic nonlinear stiffness terms respectively. These are estimated by applying a series of static forces in the shapes of the modes to the full FE model. Each static load case is the response to a loading in the shape of up to three linear modes, although the nonlinearity causes other modes to also be excited. The displacement is then projected onto the reduced linear modal basis as $q_r = \boldsymbol{\varphi}_r^T \mathbf{M}\mathbf{x}$ where the training data for the regression problem can be collected and written in the following matrix form

$$\mathbf{G} = \begin{bmatrix} q_1^2[1] & q_1q_2[1] & \cdots & q_m^2[1] & q_1^3[1] & q_1^2q_2[1] & \cdots & q_m^3[1] \\ q_1^2[2] & q_1q_2[2] & \cdots & q_m^2[2] & q_1^3[2] & q_1^2q_2[2] & \cdots & q_m^3[2] \\ & & \cdots & & & & \cdots & \\ q_1^2[N] & q_1q_2[N] & \cdots & q_m^2[N] & q_1^3[N] & q_1^2q_2[N] & \cdots & q_m^3[N] \end{bmatrix} \tag{4}$$

where $[\cdot]$ represents the load case number. The regression problem to identify the parameters for the r^{th} mode can be formulated in the following form, $\mathbf{G}\Theta_r = \mathbf{b_r}$, where $\mathbf{b_r}$ is the nonlinear modal force and Θ_r are the vector of unknown nonlinear stiffness coefficients $\mathbf{A}_r$ and $\mathbf{B}_r$. For a given system the number of nonlinear coefficients, N_Θ, is a function of the number of modes m in the reduced linear basis set given by $N_\Theta = \frac{2}{3}(2m^3 + 3m^2 + 4m)$. Clearly, the number of nonlinear terms that must be found scales as $\mathcal{O}(m^3)$. For large basis sets the number of terms becomes extremely large with many possibly unimportant terms.

2.1 Estimating the Nonlinear Stiffness Terms

The conventional approach to the nonlinear stiffness parameter identification is to use least squares (LS) regression to solve for the parameters by minimizing the mean squared error or L_2 norm.

$$\min_{\Theta_r} ||\mathbf{G}\Theta_r - \mathbf{b_r}||_2^2 \tag{5}$$

The nonlinear stiffness terms in Eq (3) are found from LS regression using the pseudo inverse of the data matrix $\mathbf{G}^+ = (\mathbf{G}^T\mathbf{G})^{-1}\mathbf{G}$, and so $\Theta_r = \mathbf{G}^+\mathbf{b_r}$. Similar to LS regression, LASSO determines the solution that minimizes the mean squared error on the L_2 norm of Eq (5) *as well* as the L_1 norm of the coefficient vector.

$$\min_{\Theta_r} ||\mathbf{G}\Theta_r - \mathbf{b_r}||_2^2 + \lambda||\Theta_r||_1 \tag{6}$$

where λ is the LASSO penalty term that regularizes the regression. This is no longer a single step solution and requires an optimization routine to be solved, however it remains a convex optimization problem so a unique and global solution is obtained.

2.2 Numerical Case Study - Flat Beam

The first numerical study is a flat clamped-clamped beam that exhibits geometric nonlinearity. This beam has been used in previous studies for evaluating ROM building procedures. A FEM was created for a 228.6 mm (9 in.) long beam using the material properties for steel with a cross section of 12.7 mm (0.5 in.) wide by 0.787 mm (0.031 in.) thick. Then a two-mode ROM including modes 1 and 3 of the flat beam was used to demonstrate the method. First, the traditional least squares method was used to create a ROM. Then LASSO was used for various values of λ, each value resulting in a different size model (different number of ROM coefficients retained). The nonlinear stiffness coefficients obtained for the first modal equation are shown in Fig. 1(a) as a function of the penalization term, λ. Each coefficient is normalized with respect to the coefficients estimated using LS. The ROM becomes more sparse as the regularization term is increased. In subplot (b) the mean squared error (MSE) of each model on a set of cross-validation data is plotted, this MSE was also normalized with respect to the MSE of the least squares ROM on the cross-validation data set. The λ value that provides a minimum MSE is marked, corresponding to a ROM with 4 terms, additionally the λ value that provides a MSE of one standard deviation larger than the minimum is also identified. Interestingly, the ROM with the minimum MSE is not the least squares solution (λ approaching zero), but a ROM with only 4 coefficients, all cubic, in the first modal equation. The MSE values between the LS estimate and optimal ROM estimate remains nearly constant with a difference of less than 0.03% between them. As the penalization term is increased past the one-standard deviation MSE, there is a significant increase in error. This occurs as the number of coefficients in the first modal equation drops below 3.

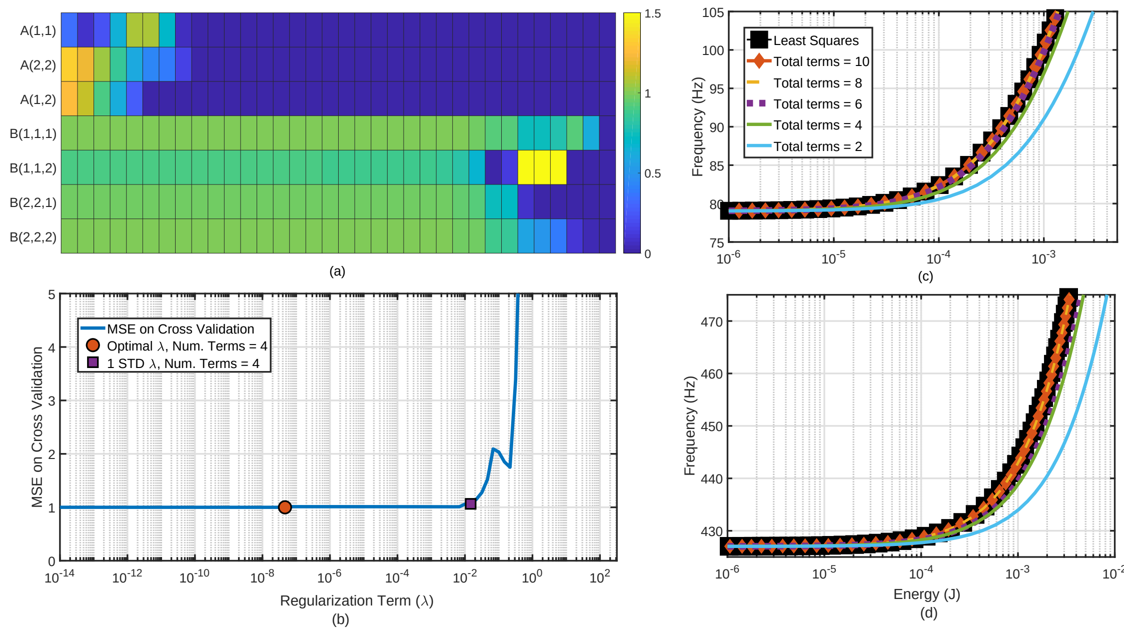

Figure 1: Results of the LASSO procedure; (a) The nonlinear stiffness coefficients of the 1st modal equation estimated, normalized to the least squares estimate, versus penalization term. (b) The MSE of the 1st modal equation versus penalization term normalized to the LS MSE. (c) 1st NNM of select ROMs estimated using LASSO. (d) 2nd NNM of select ROMs estimated using LASSO.

To quantify the accuracy of the ROMs found by LASSO, the nonlinear normal modes were computed for a ROM generated using conventional LS regression and each of the ROMs generated using LASSO. The LS results, which were shown to be nearly identical to the true NNMs of the full FEM, are used for reference. Subplots (c) and (d) of Figure 1 contain the NNMs that originate at the first two modes of the system. The legend indicates the total number of terms retained in the ROM. For the first NNM, the LASSO solutions provide near identical predictions for ROMs with as few as 6 parameters. The 2nd NNMs show that the results start to deviate when fewer than 8 parameters are retained. Hence, using LASSO it was possible to obtain a ROM that retains high accuracy for both NNMs but is 8/14=57% as large. Note that as the MSE seen in the regression analysis increases, the deviation from the nominal NNMs also occurs.
In the IMAC presentation a more detailed comparison of the MSE during cross-validation and ROM accuracy will be presented. Additionally, the procedure will be demonstrated on two additional structures in which much larger ROMs are needed: a curved beam and a multi-bay panel.

Acknowledgments

This work was supported by the Air Force Office of Scientific Research, Award # FA9550-17-1-0009, under the Multi-Scale Structural Mechanics and Prognosis program managed by Dr. Jaimie Tiley.

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