Nonlinear Model Updating Applied to Reduced Order Models of Curved Beams

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Abstract

In the future it is expected that many advanced spacecraft and aircraft will have a digital twin, or a model that is used to predict life and that is updated as the vehicle ages. Methods already exist for creating and updating linear models, but advanced vehicles may contain nonlinear vibration absorbers, joints or thin panels that exhibit nonlinearity. New strategies are needed both to tune the digital twin to tests on the prototype structure and to update it as the structure ages. This work presents a case study on model updating of reduced order models (ROMs) using Nonlinear Normal Modes (NNMs) as a correlation metric between the numerical model and experimental data. The model updating procedure uses a gradient based optimization algorithm coupled with a harmonic balance continuation routine to compute NNM branches.

Keywords: Reduced Order Models, Nonlinear Dynamics, Harmonic Balance Method, Nonlinear Normal Modes, Model Updating

1 Introduction

This work presents a case study in which a recently proposed model updating strategy is used to update a Reduced Order Model (ROM) of a beam such that it captures the measured Nonlinear Normal Modes (NNMs) of the structure. The model updating procedure utilizes a gradient based optimization algorithm coupled with a Multi Harmonic Balance (MHB) continuation routine to compute NNM branches. Furthermore, within the MHB framework the algorithm provides analytic gradients describing how the NNM solutions change with respect to the nonlinear stiffness terms of the ROM, for details see [1]. This important aspect reduces the need for finite difference approximations of the gradients, which significantly reduces the computational cost and is more accurate. A brief review of reduced order model generation is presented and followed by a high level overview of the model updating procedure. Results from a case study of a curved beam are shown and discussed.

2 Theory

The geometrically nonlinear FE equations of motion for an n DOF system can be written as

$$\mathbf{M}\ddot{\mathbf{x}} + \mathbf{C}\dot{\mathbf{x}} + \mathbf{K}\mathbf{x} + \mathbf{f}_{NL}(\mathbf{x}) = \mathbf{f}(t) \tag{1}$$

where $\mathbf{M}$, $\mathbf{C}$, and $\mathbf{K}$ are the $(n \times n)$ mass, damping, and linear stiffness matrices respectively. The nonlinear restoring force $\mathbf{f}_{NL}(\mathbf{x})$ is a function of the displacement only. The full FE equations of motion can be projected onto a modal subspace using the coordinate transformation $\mathbf{x}(t) = \mathbf{\Phi}_m\mathbf{q}(t)$ in which the r^{th} nonlinear modal equation becomes

$$\ddot{q}_r + c_r\dot{q}_r + \omega_r^2 + \theta_r(q_1, q_2, ..., q_m) = \boldsymbol{\varphi}_r^T\mathbf{f}(t) \tag{2}$$

The nonlinear restoring force θ_r is a function of the modal displacements as $\theta_r(\mathbf{q}) = \boldsymbol{\varphi}_r^T\mathbf{f}_{NL}(\mathbf{\Phi_m}\mathbf{q})$. The nonlinear restoring force for a linear elastic system with only geometric nonlinearities can be accurately approximated as

$$\theta_r(\mathbf{q_1}, \mathbf{q_2}, ...\mathbf{q_m}) = \sum_{i=1}^{m}\sum_{j=i}^{m}\mathbf{A}_r(i,j)\mathbf{q_i}, \mathbf{q_j} + \sum_{i=1}^{m}\sum_{j=i}^{m}\sum_{k=j}^{m}\mathbf{B}_r(i,j,k)\mathbf{q_i}, \mathbf{q_j}, \mathbf{q_k} \tag{3}$$

where $\mathbf{A}_r$ and $\mathbf{B}_r$ are the quadratic and cubic nonlinear stiffness terms respectively.

Several other works review procedures for estimating a ROM from a finite element model [2,3,4]. In this work, we suppose that the finite element model is somewhat inaccurate, so that the coefficients of the ROM need to be adjusted to bring the ROM into agreement with measurements. Many prior works have shown that this may be necessary even in the best circumstances, since the FEM is not likely to perfectly capture imperfections in the flatness/curvature of thin sections, pre-stress, boundary conditions, etc. All of these features can cause the ROM coefficients to be inaccurate. Nonlinear normal modes are used in this work as the correlation metric between the numerical model and the hardware. Figure 1 shows a graphical representation of the method. For further information regarding the procedure see [1].

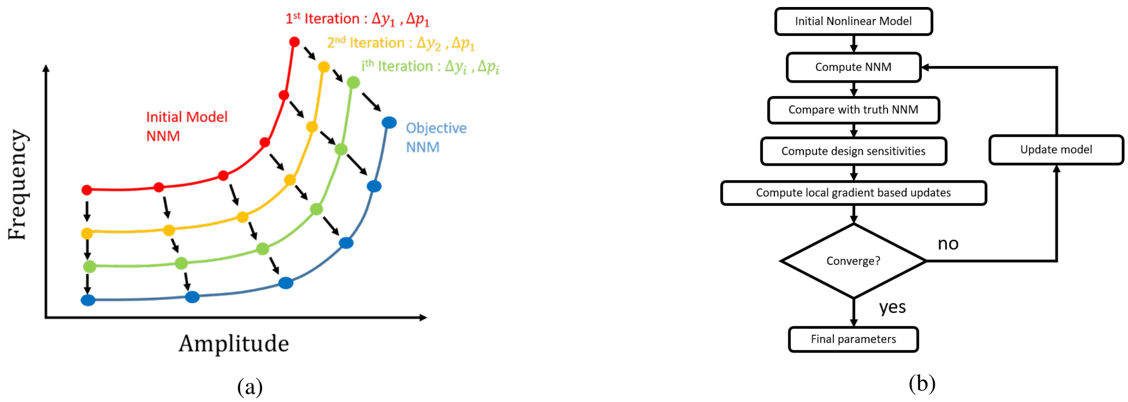

Figure 1: Representation of the model correlation procedure: (a) graphical representation. (b) procedural flow-diagram.

2.1 Numerical Results - Curved Beam

The test structure, depicted in subplot (a) of Fig. 2, is a curved beam with length of 139.7 mm, thickness of 1.397 mm, width of 12.7 mm, and radius of curvature of 3175 mm. The beam is made of Polylactide (PLA) with a nominal modulus of elasticity of 3516 MPa, and density of 1248 kg/m^3. The first NNM was measured using force appropriation and a laser vibrometer to capture the displacement along the length of the beam. The NNM is expressed in terms of frequency versus the initial displacement of the center of the beam in Fig. 2(b) and Fig. 2(c) shows the initial deformation of the beam at various frequencies. A reduced order model (ROM) of the system is desired, but experience has shown that it is challenging to estimate an accurate ROM of curved structures like this from a FEM, so instead we will seek to update a nominal ROM to capture the experimentally measured behavior.

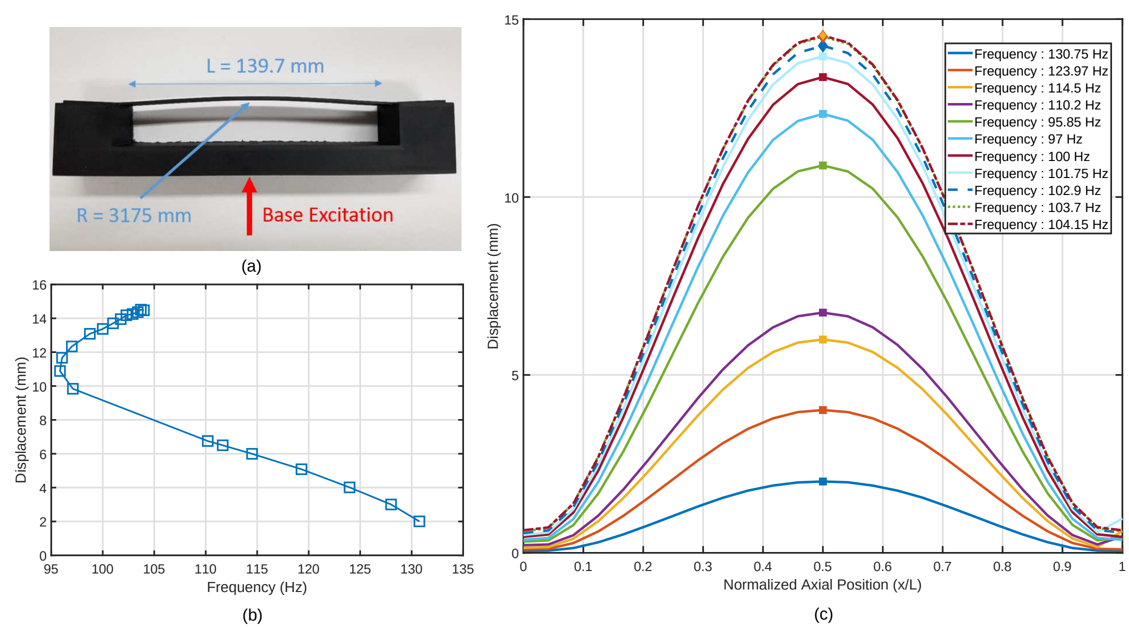

Figure 2: Experimental setup and data; (a) beam with dimension and excitation. (b) center node vertical displacement versus frequency. (c) full field displacement solutions acquired during test.

A reduced order model was created from a finite element model using the nominal geometry (including the radius of curvature) and material properties. The beam was meshed with 2-node beam elements and the base was approximated as a fixed boundary condition, which is known to be a poor approximation since the stiffness of the block has a significant effect on the axial stiffness and hence the nonlinearity. This also neglects the fillets between the beam and block. After estimating a 1-mode ROM from this nominal FEM, the nonlinear stiffness coefficients were updated to correlate the NNMs of the ROM with the experimental data.

The results of the model correlation procedure for the ROM are presented in Fig. 3. The two plots on the left display the quadractic and cubic nonlinear stiffness terms ($\mathbf{A_{1,1}}$ & $\mathbf{B_{1,1,1}}$) during the model updating. Results are presented when updating using only a single harmonic in the harmonic balance solution and when including three harmonics. The coefficients estimated from the correlation procedure for the 1-harmonic and 3-harmonic model are consistent up to the 17th iteration, at which point the coefficients continue to change by an additional 4% for the 3-harmonic solution. The plot on the right of Fig. 3 presents the NNM for the 3-harmonic solution. The nominal ROM is quite distant from the true NNM, almost completely missing the initial softening of the NNM. After model updating, the ROM's NNM is comparable to the target data with frequency errors below 2% at each displacement level.

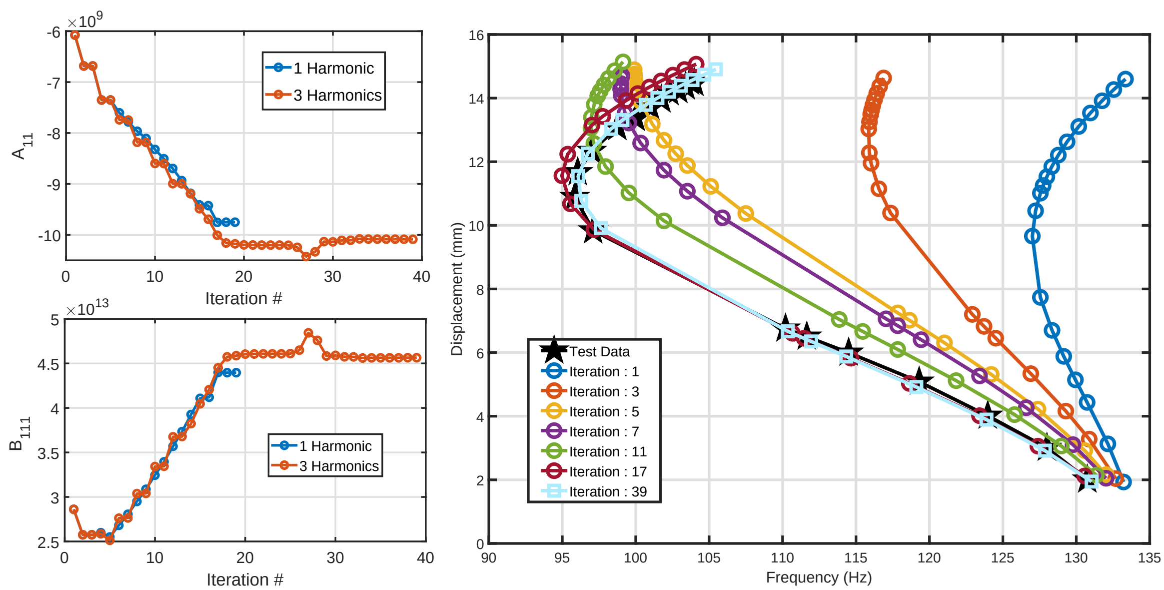

Figure 3: Results of the model correlation procedure.

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