Application of the Bouc-Wen Model to Bolted Joint Dynamics

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Abstract

Various numerical models have been developed to capture the dynamic, hysteretic behavior of different mechanical systems. One such semi-physical model is the Bouc-Wen model, which relates the input displacement to the output restoring force in a hysteretic way. This formulation is intended for any form of hysteresis and was originally applied to force – deflection and flux – current diagrams of mechanical and ferromagnetic hysteresis. Built-up structures are also known to show hysteretic behavior due to the slipping that occurs between interfaces bolted together. This paper tests how effective the Bouc-Wen model is in capturing the power-law damping behavior observed in bolted joints by comparing it with another commonly used numerical model – the Iwan model. While the Iwan element has been proven to be robust and well-suited at capturing the power-law increase in energy dissipation and slow decrease in stiffness with vibration amplitude exhibited by bolted interfaces, numerical integration of the same is currently computationally expensive. Time integration of the Bouc-Wen model, on the other hand, is much more efficient, thus warranting the proposed study.

Keywords: Non-linear dynamics, Bouc-Wen model, Bolted joints, hysteresis, parameter identification

1 Introduction

Mechanical fasteners have long been known to be an important source of stiffness and energy dissipation in built-up structures. The slipping between two surfaces that are bolted together results in frictional dissipation of energy. As a result, the restoring force versus displacement graph of systems consisting of bolted joints shows non-linear, hysteretic behavior. At low force amplitudes, the edges of the contact patch slip, which is known as micro-slip. As the force amplitude increases, the area of contact reduces, ultimately resulting in relative motion between the surfaces, also known as macro-slip. The change in stiffness and damping that occurs due to bolted interfaces has also been found to be a function of the amplitude of the response [1]. In order to capture this observed dynamics, lumped hysteretic models are used. One such model prevalent in bolted joint dynamic analysis is the Iwan model [2], which consists of a parallel arrangement of spring-slider units known as Jenkins elements. Solving the equation of motion of a system consisting of an Iwan model requires implicit integration schemes that can be time consuming. Thus, alternative hysteretic models that would reduce the computational effort while still providing the accuracy obtained with Iwan models must be considered.

A particular semi-physical, hysteretic model that has found applications in multiple areas, especially in civil and mechanical engineering, is the Bouc-Wen model [3]. It consists of a non-linear, first-order ordinary differential equation (ODE) that relates the input displacement to the output restoring force in a hysteretic way. This differential equation can be easily integrated using explicit ODE solvers to obtain the non-linear restoring force. The existing literature predominantly focuses on applying the Bouc-Wen model to capture the steady-state hysteretic behavior of various non-linear systems. In bolted joints, however, the non-linearity is also characterized by power-law dissipation behavior - a behavior in which the log of the energy dissipated (or effective modal damping ratio) increases linearly with the log of the vibration amplitude, and a decrease in natural frequency of the system with increase in response amplitude [1]. This work analyses the effectiveness of the Bouc-Wen model in capturing this power-law behavior by comparing it against Segalman's four-parameter Iwan model [4], which was derived specifically to simulate bolted joint dynamics.

The next section describes the mathematical representation of the Bouc-Wen model and the system identification method adapted from [5] to estimate the Bouc-Wen parameters. The conference presentation will elaborate on the case study presented here, in which the response of the Bouc-Wen model is compared against that of the Iwan model when an impulsive force is applied on a mode of a built-up structure that exhibits non-linearity.

2 Theory

The equation of motion for a single degree-of-freedom (SDOF) system consisting of a hysteretic non-linear element can be written as

$$m\ddot{x} + c\dot{x} + k_f x + f_{NL}(x, z) = f(t) \tag{1}$$

where m, c, and k_f are the mass, linear damping, and linear stiffness of the system respectively. The non-linear restoring force $f_{NL}(x, z)$ is a function of the displacement x and a hysteretic state variable z. The Bouc-Wen formulation for the non-linear restoring force is given as

$$f_{NL} = (1 - \alpha)k_i z \tag{2}$$

$$\dot{z} = A\dot{x} - \beta|\dot{x}||z|^{n-1}z - \gamma\dot{x}|z|^n \tag{3}$$

where $\alpha = \frac{k_f}{k_i}$ is defined as the ratio of post-yield to pre-yield stiffness, which, for a bolted joint, corresponds to stiffness at macro-slip (i.e. the linear stiffness of the system without the joint) to the stiffness when the joint is stuck. A, β and γ are the Bouc-Wen parameters to be identified. An adaptation of the parametric identification method presented in [5] has been used to identify the Bouc-Wen parameters for a given load-displacement curve. The system to be identified could be an SDOF system or a non-linear mode of a structure. Assuming the modes to be uncoupled and the non-linearity to be weak, the non-linear mode can be written as an SDOF system consisting of a non-linear element that is independent of other modes. The system under consideration is excited by a harmonic force having fixed amplitude and frequency, and the steady-state force-displacement curve is obtained. Because the Iwan joint is not velocity dependent, a very low forcing frequency can be chosen so that the excitation is nearly quasi-static and the steady-state hysteresis curve is obtained within the first few periods of integration. The same approach is used with the Bouc-Wen model. The amplitude of the force is chosen to be low enough for the system to be in micro-slip.

Firstly, α and k_i can be obtained from the low amplitude linear frequency, ω_0, and the high amplitude slip frequency, ω_∞ using the following relations:

$$\alpha = \left(\frac{\omega_\infty}{\omega_0}\right)^2 ; k_i = m\omega_0^2 \tag{4}$$

The parameters A, β and γ are then calculated using the method of least squares, given by Eqn. 5,

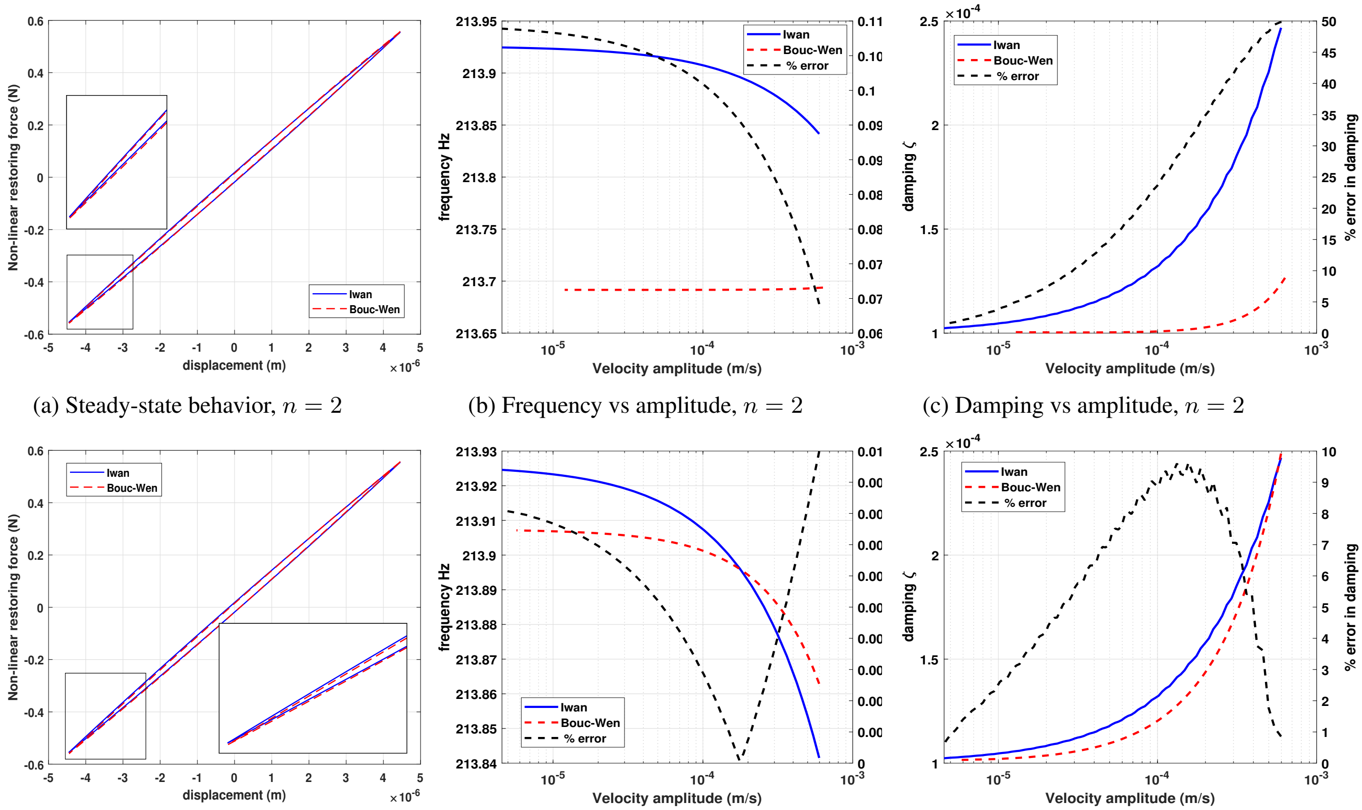
$$\frac{\triangle \mathbf{F_{nl}}}{\mathbf{(1-\alpha)k_i}} = \begin{bmatrix} \triangle \mathbf{x} & -\triangle \mathbf{x}|\mathbf{z_i}|^{\mathbf{n-1}}\mathbf{z_i} & -\triangle \mathbf{x}|\mathbf{z_i}|^{\mathbf{n}} \end{bmatrix} \begin{bmatrix} A \\ \beta \\ \gamma \end{bmatrix} \tag{5}$$

where $\triangle \mathbf{x}$ is a vector of the difference between two consecutive displacements, $\triangle \mathbf{f_{NL}}$ is the difference between consecutive force value and $\omega_\mathbf{i}$ is the vector of hysteretic displacement calculated as $w_i = f_{NL}/K_w$ for each time instant. Equation 5 is of the form $\mathbf{Y} = \mathbf{\Phi\Theta}$ which can be solved for $\mathbf{\Theta}$ by taking the inverse of $\mathbf{\Phi}$. To ensure that the matrix $\mathbf{\Phi}$ is well-conditioned, each column of $\mathbf{\Phi}$ is normalized to its corresponding maximum value. The least-squares problem is solved iteratively for different values of n. In most of the existing literature, an arbitrary value of $n = 2$ has been found to be good enough. However, while the steady-state behavior of a system consisting of a Bouc-Wen element may not be sensitive to the parameter n, the ring-down response is, as will be shown in the following numerical example.

3 Numerical Case Study - Sumali Beam

The identification method described in section 2 was applied to an assembly of two beams bolted together, commonly referred to as the Sumali beam. This structure consists of two thin, identical, stainless steel beams of length 508 mm, width 50.8 mm, and thickness 6.35 mm, that overlap and are joined with four bolts. The first three elastic bending modes of this structure are non-linear. The parameters of a modal Iwan model, which approximates each of these non-linear modes, were obtained in [6], and have been used in this work as a benchmark against which the fitted Bouc-Wen model can be compared. The second bending mode of the Sumali beam was considered. The response of a modal Iwan model for this non-linear mode was numerically simulated for a harmonic force input using the Newmark-β integration method. The steady-state hysteresis loop obtained was then used to calculate the Bouc-Wen parameters A, β and γ by iteratively solving the least-squares problem, given by Eqn. 5. The system was then excited by an impulsive fore and the time response obtained by integration was post-processed using the Hilbert filter in order to obtain the amplitude-dependent damping and natural frequency.

Figure 1 shows the effect n has on the accuracy of the Bouc-Wen model. The hysteresis loops obtained using the Bouc-Wen model show some deviation from the Iwan model, with the percentage error in the estimation of energy dissipated over a cycle (obtained by calculating the area under the curve) being 3.46% when $n = 2$ and 7.86% when $n = 1.1$. However, significant difference in accuracy can be observed in the prediction of the ring-down response. When $n = 2$, the Bouc-Wen model gives a maximum error of 0.11% in the natural frequency estimate and a maximum error of 50% in the damping estimate. When n is



(a) Steady-state behavior, $n = 2$
(b) Frequency vs amplitude, $n = 2$
(c) Damping vs amplitude, $n = 2$
(d) Steady-state behavior, $n = 1.1$
(e) Frequency vs amplitude, $n = 1.1$
(f) Damping vs amplitude, $n = 1.1$

Figure 1: Comparing the quasi-static behavior and dynamic behavior obtained using two different values of the Bouc-Wen parameter n, demonstrating the sensitivity of the ring-down response to n

changed to 1.1 and the other Bouc-Wen parameters are re-calculated using the least-squares method, the errors in both frequency and damping drop to a maximum of 0.01% and 9.3% respectively. There is a marginal improvement in the agreement in natural frequency but a significant one in the prediction of damping. Thus, in order for the Bouc-Wen model to be effectively used to capture the power-law dissipation observed in bolted joints, it appears the parameter n must also be optimized.

Acknowledgments

This material is based in part upon work supported by the National Science Foundation. Any opinions, findings, and conclusions or recommendations expressed in this material are those of the author(s) and do not necessarily reflect the views of the National Science Foundation.

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