Exploring Experimental Structural Dynamics in EMA/ME 540 at UW-Madison

Matthew S. Allen
Department of Engineering Physics
University of Wisconsin-Madison
1500 Engineering Drive, 535 ERB
Madison, WI 53706-1609
msallen@engr.wisc.edu

ABSTRACT

One of the highlights of the experimental structural dynamics course at the University of Wisconsin-Madison is a final project in which the students perform a modal test of a structure and explain how the identified modal parameters affect the performance of the structure. This is a 500 level course, typically taken by a half-and half mix of senior undergraduate students and graduate students. This paper discusses the topics that were covered in the course in the Fall of 2011 and shows highlights from the students' final projects. The different software packages that are used, both commercial and those created in Matlab for the course, are also discussed. The projects have included: a modal test on a wing in a wind tunnel to diagnose a vibration problem, a modal test of a wine glass to relate the modes of the glass to the sound that is heard when it is excited by friction between the rim of the glass and a finger rubbed around the circumference of the rim, and many others.

Keywords:

Laboratory; experimental; structural dynamics; experimental modal analysis; structural modification

1. Introduction

The goal of EMA/ME 540: Experimental Vibration and Dynamic System Analysis is to reinforce the students understanding of vibrations theory through hands on experiments while exposing them to state of the art structural dynamic testing methodologies and improving their laboratory skills. The course is intended for both undergraduates and graduate students, so it necessarily includes a review of undergraduate vibrations theory that takes the first four or five weeks of the course. Those concepts are then built upon over the next several weeks as the students are exposed to the discrete Fourier transform, experimental modal parameter identification, Ritz theory for continuous systems and structural modification theory. The students spend the last several weeks of the course working on a project of their choice the includes both testing and some analysis to show how the test results would be useful in the application of interest.

The instructor has found that it is critical to demand that the students think carefully about the meaning of the measurements that they are acquiring and strives to set this expectation with the very first lab report. This lab covers free vibration of mass-spring systems (carts with helical springs) that are conceptually extremely simple yet in fact can exhibit some interesting dynamics. The students receive a low grade on this report if they fail to thoroughly explore the meaning of their measurements. For example, Figure 1 shows a comparison between the measured and predicted time response for the SDOF system from a team who scored quite high. They took great care to carefully calibrate their analytical model, and hence they were able to observe how Coulomb damping begins to affect the response towards the end of the time record.
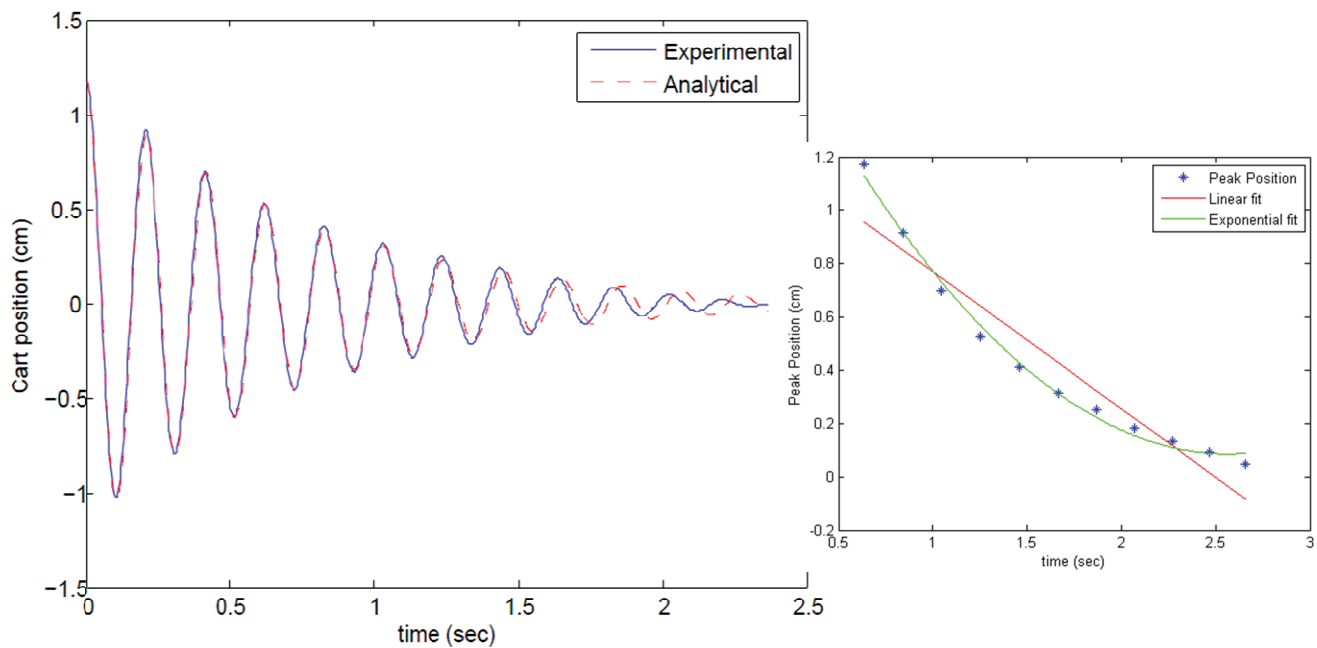
Figure 1: Time history from one team's report for Lab 1 that shows a thorough comparison between analysis and experiment. The period and effective damping in the system change at small amplitudes due to Coulomb friction in the system.

Further information regarding this course is presented in the syllabus and lab handouts which are provided in the Appendices of this document.

2. Course Outcomes: Final Projects

This section presents a sampling of slides from final project presentations that the students have presented over the past few years. Figures 2 through 5 describe a project in which a team sought to diagnose a vibration problem in the University's wind tunnel that had been observed when aluminum plates were added to the ends of a wing to force the flow to become more two-dimensional. In this configuration the wing exhibited excessive vibration making tests unsafe at high flow velocities. The team found that the lowest modes of the wing were most strongly excited (suggesting that this was not a flutter problem); apparently the addition of the end plates reduced these frequencies enough to significantly increase their response.
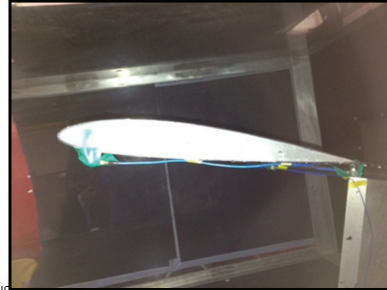
Figure 2: Slide 1 from wind tunnel vibration project presentation outlining the problem.
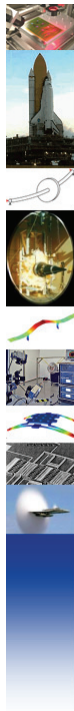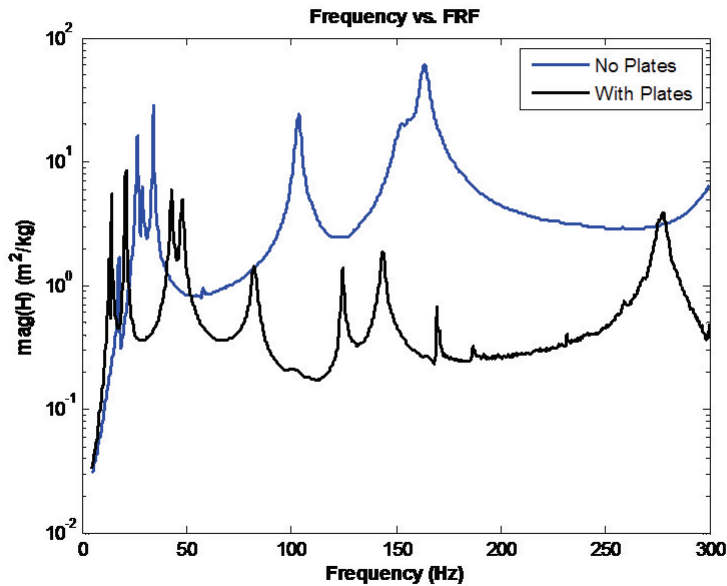
Figure 3: Slide 2 from wind tunnel vibration project presentation summarizing the differences between the frequency responses of the system with and without the end plates.
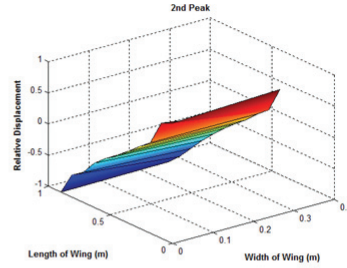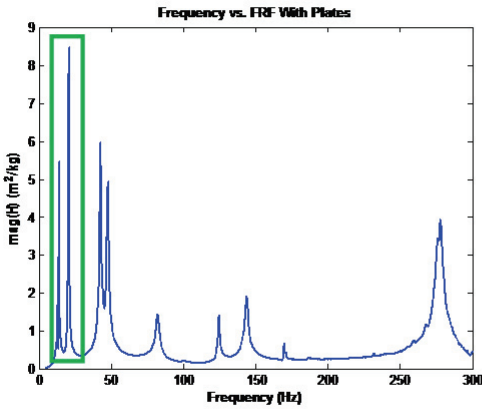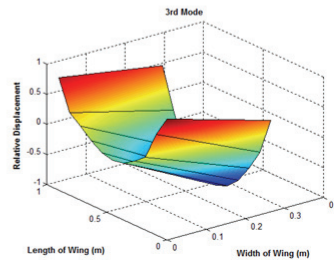
Figure 4: Slide 3 from wind tunnel vibration project presentation showing the shapes of the modes that are thought to be dominant.
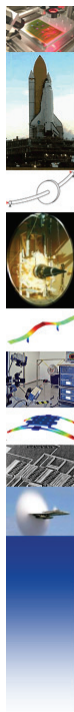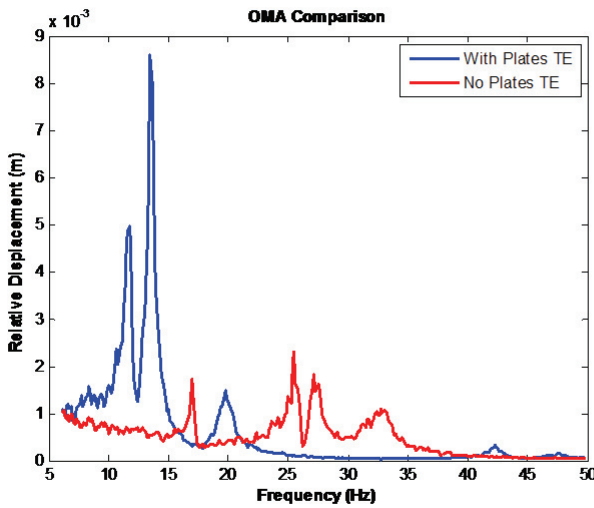
Figure 5: Slide 4 from wind tunnel vibration project presentation showing the difference in behavior of the system in operation with and without the end plates.

Another team studied the vibration and noise emitted by a crystal glass when a finger is rubbed along the rim of the glass. They performed a modal survey of the glass, taking care to mass load the glass as little as possible, and then acquired measurements of the sound emitted as the glass was excited by friction between a finger and the rim of the glass. They found that the frequency that dominated the audible spectrum corresponded to a pair of 4-lobe ovaling modes of the glass.
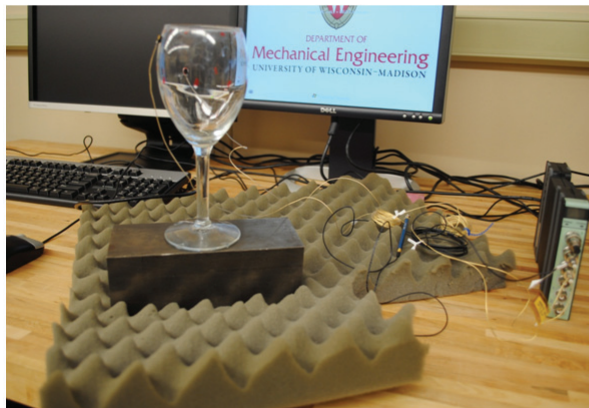
Figure 6: Slide 1 from crystal glass project presentation showing experimental setup.
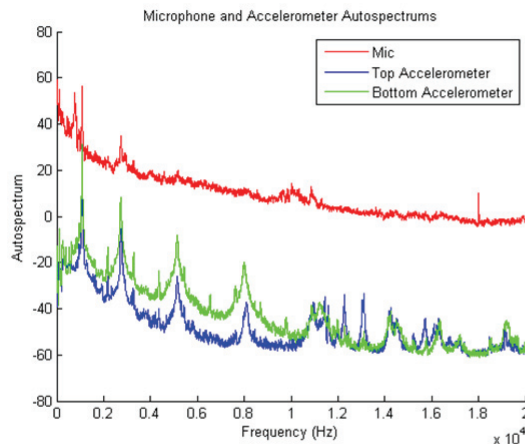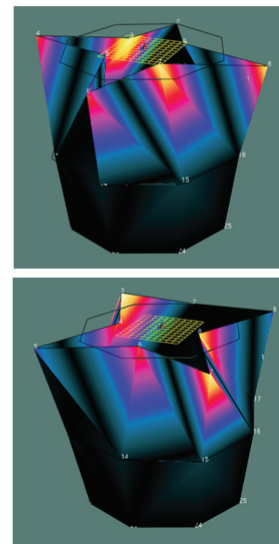
Figure 7: Slide 2 from crystal glass presentation highlighting experimental results. The modes that contribute most to the audible sound were found to be a pair of four-lobe ovaling modes.

3. Conclusions

The goal of this course is to develop the students' understanding of vibration theory and their experimental abilities. Their performance on the final project is usually a good indicator of their understanding of structural dynamics and how it impacts the performance of their chosen system of interest, as well as their persistence and work ethic. Results to date have been good suggesting that many of the students obtain a good level of proficiency as a result of the class. The instructor also has several graduate research assistants who were former students in this class and has found that they tend to be quite capable with measurement and analysis of measurements following the class. Of course there is certainly room for improvement. The primary challenge at the moment is how to construct a review of vibrations theory that allows the class to draw from a large pool of students (with varied pre-requisites) while encouraging a solid mastery of the important material. The instructor is also constantly seeking new ways to encourage the students to think more deeply and seek further insights into the issues that they confront in their projects. On the other hand, the instructor himself has learned a tremendous amount over the past few years teaching this course and so he suspects that this is a never-ending quest.

4. Acknowledgements

The author would like to thank Dr. Peter Avitabile for the many helpful comments and materials that he supplied when Dr. Allen first created this course in 2009. His comments and ideas were extremely influential in shaping this course. The author would also like to thank Tom Carne for the many questions he answered and the insights and advice that he offered regarding this course.

5. Appendix: Course Syllabus

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Syllabus: ME/EMA 540 – Fall 2011
Experimental Vibration and Dynamic System Analysis

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Instructor: Prof. Matt Allen
      Department of Engineering Physics
Office Hours: M/Tues 2-3PM, Th 12:30-1:30PM, or by appointment

Prerequisites: ME 340, ME 440, EMA/ME 545 or Instructor Consent

Course Home Page: https://ecow2.engr.wisc.edu/courses/course/view.php?id=1531#section-2 Lecture notes, homework, lab handouts and solutions will be posted on course web site.

Textbook: Mechanical and Structural Vibrations: Theory and Applications, Jerry H. Ginsberg [1]. An excellent reference and used for homework assignments.

Online course description:

Application of digital data acquisition to the investigation of mechanical components, structures and systems using time histories, transforms and response functions to characterize free, forced and transient inputs. Introduction to sensors, instrumentation and methods appropriate for dynamic system response.

Evaluation:

- **Homework sets** – A few homework sets will be assigned throughout the semester to help you review some important concepts from vibrations. Homework sets are due as noted on the course website when they are assigned (syllabus lists expected due dates, which are subject to change). Late homework will not be accepted unless prior arrangements have been made with the instructor. Consulting with your peers is allowed so long as it is done responsibly.
  - If you are ill or otherwise unable to turn in an assignment, contact Prof. Allen immediately by phone or email to make arrangements to turn the assignment. Late homework will not be accepted unless prior arrangements have been made.

- **Exams**
    - None planned, although the instructor reserves the right to give a few quizzes for homework credit if he deems that it would be beneficial.
- **Lab Reports**
    - Most labs will be completed within the scheduled class time, although they require analysis that will be performed outside of class. Lab reports are required for most of the labs and these will be done in teams.
    - Lab reports will be evaluated based on their professionalism, clarity and the insight that they demonstrate into the subject matter. A professional report will contain a short introduction, results, and conclusions, will be neat and will contain enough explanation to guide the reader through your results. ***To receive an "A" you must show that you have thought deeply about the results and what they mean; a cursory coverage of the required tasks will earn only a passing grade.***
        Brevity and efficiency in the text of your report will be rewarded, although you are encouraged to include as many figures/discussion as needed to present your results. Avoid providing a "data dump;" present your results so that the conclusions and key findings clearly stand out. You do not need to include a detailed description of the procedure that you used unless your procedure deviated from that given to you in the lab assignment (as it may in the later labs).

- **Final Exam Period: 12/17/2011, Saturday, 2:45PM - 4:45PM**
    - In lieu of a final exam, each lab team will give a presentation to the class on their final project.
- Grades in the course will be decided with the following weighting:
    - Homework Sets   20%
    - Lab Reports   60%
    - Final Project   20%

General Topic Areas Covered:

- Review of Free and Forced Harmonic vibration of SDOF and MDOF systems.
- Discrete Fourier Transform, Nyquist sampling theorem and analog to digital conversion.
- Transient modal testing for continuous systems (including impact, step relaxation, etc…).
- Ritz Series models for continua and relationship to experimentally derived models and structural modification.
- Common vibration sensors including: piezoelectric accelerometers and load cells, Laser Doppler Vibrometers (LDV)/ Interferometers, strain gauges, etc…
- Identification of modes with close natural frequencies using Mode Indicator Functions (MIFs) and MIMO identification.
- Introduction to vibration of nonlinear systems.

Final Project: Each team will chose a system to study for their final project. The final project focuses on performing an accurate dynamic test and explaining how the modal parameters that you identify affect the performance of the system. The list below gives a sampling of ideas for a final project.

- Test a crystal glass to identify the shape of the mode(s) that contribute to sound generation when a finger is rubbed along the top of the glass. What other modes are present in the frequency range that do not generate sound? Why?
- Hard Drive: Perform modal survey of the case. How do modes compare with the nominal drive frequency and its harmonics? At what spindle speed would you expect to encounter vibration problems with the hard drive?
- Sporting equipment such as a tennis racquet (or similarly for baseball, hockey, etc…): Compare the dynamic properties of two racquets (or one racquet in multiple configurations, e.g. with a vibration absorber.) How would the modes that you observed affect the way a ball is hit or the way it feels to the user? (e.g. simulate the impact of a ball with the racquet using the modal model that you've developed.)

- Perform a modal test on a guitar and use the result to explain the amplification of certain notes in the sound produced.
- Use operational modal analysis to find the modes of the ERB or the 2nd floor of ME.
- For RC enthusiasts: Perform a Modal test of an RC plane to identify the first few bending and torsional modes. Compare the natural frequencies and predict which bending-torsion mode combination is most likely to lead to flutter at high airspeeds.

Links:

- Articles giving easy to understand explanations of various concepts from vibrations and experimental modal analysis:
    - http://macl.caeds.eng.uml.edu/umlspace/mspace.html
- U Michigan Matlab tutorial:
    - http://www.engin.umich.edu/group/ctm/basic/basic.html
- Some very basic Matlab tutorials:
    - http://www.cyclismo.org/tutorial/matlab/
- If you need additional help, Matlab tutors are generally available in Wendt library during the walk-in tutoring on Sunday, Monday, Tuesday, and Wednesday nights. See http://studentservices.engr.wisc.edu/classes/tutoring/ for more details.

Semester Schedule: ME/EMA 540 – Fall 2011

* NOTE: All of the dates below are approximate. The instructor will adjust the schedule as necessary.

Date*	Week	Topic	Due Dates*
9/6	1	Introduction, Start SDOF Free (1.1-1.4)	HW#1 Assigned
9/8		Finish SDOF Free, MATLAB example (2.1-2.2)	
9/13	2	MDOF Systems: Free Response / Generalized Eigenvalue Problem (4.1, 4.2.1-3)	*HW#1 due* HW #2 Assigned
9/15		MDOF Systems: Orthogonality (4.3.1,2,4,6)	HW #3 Assigned
9/20	3	Lab 1: SDOF/MDOF Transient Response (2145ME)	*HW#2 due*
9/22		SDOF Forced Response (2.3.1,3.1-2)	
9/27	4	MDOF Systems: Frequency Domain TF, Intro. to Experimental Modal Analysis (5.1-2, Notes)	*HW#3 due* HW #4 Assigned
9/29		MDOF Systems: Vibration Absorber, Sensors: Accelerometer, Strain Gauge & LDV (5.3, Notes)	*L1 Report Due*
10/4	5	Fourier Series, DFT, Sampling, FRF Estimation (3.7 all)	*HW #4 Due* HW# 5 Assigned
10/6		Lab 2: Frequency Response of SDOF/MDOF Sys.	
10/11	6	Lab 2: Continued (2163 ME)	
10/13		Intro to Continuous Systems / Ritz (6.1.1)	
10/18	7	Ritz Method for Beams (6.1.3-4, 6.2)	HW #6 Assigned *L2 Report*
10/20		Lab 3: Sensor Comparison (B331 EH)	*HW #5 Due*
10/25	8	Ritz Method Continued (6.3)	
10/27		MAC, MSF & Support Conditions (Notes)	
11/1	9	Modal Parameter Estimation (1/2 lecture) (Notes) Lab 4a: Impact Modal Test of F-F Beam (2163 ME)	*HW #6 Due*
11/3		Lab 4a: Continued	*L3 Report Due* HW #7 Assigned

11/8	10	Lab 4a: Continued	
11/10		Structural Modification / Substructuring (~9.1~)	Assign/Discuss Projects
11/15	11	Lab 4b: Structural Modification	**L4a Report Due**
11/17		Operational Modal Analysis (Notes)	
11/22	12	Lab 5: Output-only Modal Test of Downhill Skis	**L4b Report Due** *Choose Projects/Teams*
11/24-		**Thanksgiving Recess 11/24-11/27**	
11/29	13	Intro to Nonlinear Vibrations (Notes)	
12/1		Testing/Analysis for final projects	**HW #7 Due**
12/6	14	Testing/Analysis for final projects	**L5 Report Due**
12/8		Testing/Analysis for final projects	
12/13	15	Testing/Analysis for final projects	
12/15		Testing/Analysis for final projects	
12/17		**Project Presentations During Final Exam Period, 12/17/2011, Saturday, 2:45PM - 4:45PM**	**Each group present a summary of their final projects**

6.   Appendix: Sample Laboratory Assignments:

Lab 1: Free Vibration of SDOF / MDOF Systems

Lab Report

You will be required to turn in a lab report summarizing your findings. Pay special attention to the questions in bold-italics throughout the lab handout. You do not need to discuss the hardware setup in detail or other items that were given in this handout.

Objective

The purpose of this lab is to understand the basic features of the free response of underdamped single-degree-of-freedom (SDOF) and multi-degree-of-freedom (MDOF) systems. You will acquire measurements from a real system and estimate the system's parameters (e.g. ω_n, ζ, etc…) for both SDOF and MDOF systems. For the SDOF case, you will carefully compare the measured response to the analytical response that is expected. <u>Your key focus with that comparison is to seek to ascertain to what degree the analytical model is capable of describing the motion of the real system</u>.

In this lab, translational single DOF mass-spring systems will be examined. It is important to note that all physical systems contain some damping, even if no external damper is attached (you will not use the external damper in this lab).

Figure 8 shows the rectilinear system that will be used for this experiment. Figure 9 shows a schematic version of the actual systems used in this experiment.
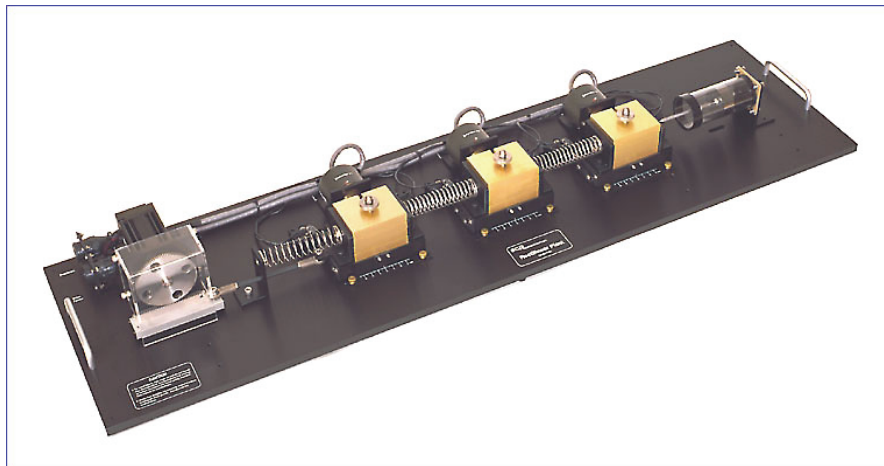
Figure 8: Rectilinear lumped-element dynamic system
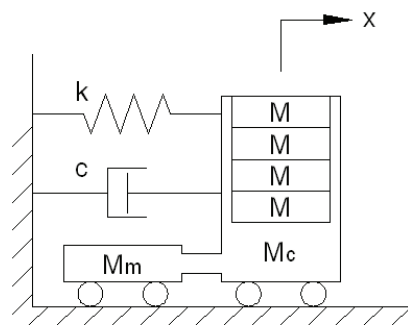
Figure 9: Translational system schematic

None of these parameters will be given to you except for the mass of the brass blocks (M = 0.59 kg, even if we gave you the others they wouldn't be accurate). All of the other parameters must be determined from experimental data. Of course, one could determine the stiffness of the springs using statics and weigh the parts to estimate Mm and Mc the system, but there are some potential disadvantages to that approach. Instead, we shall use dynamic measurements to determine the unknown stiffness k and the mass of the entire cart Mm+Mc=m_1. Note that you will not connect the external dashpot, but the system still has some damping. You will also investigate a 2-DOF system whose schematic is shown below. Note that you should use identical springs so $k_1 = k_2$ and you should not connect the dashpot.

Figure 10: 2-DOF translational system schematic

INSTRUCTIONS:

1) **SDOF SYSTEM:** Use the data acquisition system to capture and save the response of the system to various initial displacements from equilibrium. See the appendix for detailed instructions regarding how to setup the data acquisition system.
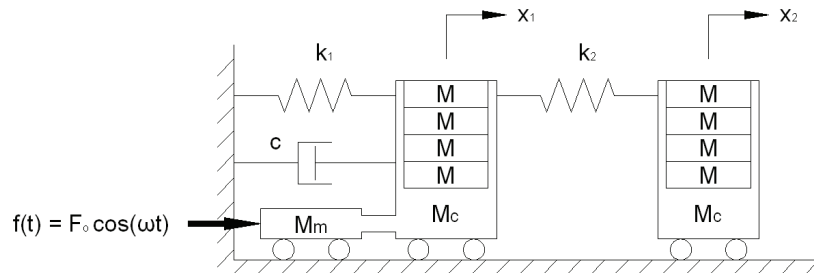   a) Record the response of the system to initial displacements of about 1, 2 and 3 cm.
   b) Add a known mass to the system (m_2, so the total mass of the system is now m_1+m_2) and repeat the experiment once again for initial displacements of about 1, 2 and 3 cm.
      i) **ANALYSIS:** Find the natural frequency and damping ratio for the system with and without mass and use them to compute the mass, stiffness and damping (*m, c* and *k*) constants of the system. ***How uncertain are the natural frequency and damping ratio? What about (m, c and k)?*** Make sure your units are correct. ***Are your answers reasonable?***
      ii) **ANALYSIS:** Create a plot of the measured response for at least one of the cases in (a) (no mass added) and compare it with the analytically reconstructed response (see detailed instructions in the Appendix for hints).
         (1) It is likely that, even after very carefully tuning your model, you may still not have perfect agreement between the test and model. Think carefully about the following questions to explore this.
         (2) ***How does the measured response compare with theory? Pay special attention to the damping envelope of the response and the frequency of the oscillations.***
         (3) ***If there are discrepancies, are these due to errors in your estimates for ζ and ω_n, or are you observing physics (e.g. Coulomb damping) that cannot be described by a linear, viscous model? If we assume viscous damping, how uncertain is the damping ratio?***
2) **MDOF SYSTEM:** Remove any masses from the carts and connect a second spring between the first and second carts. Make a note of what type of spring you used (thick or thin).
   <u>a)</u> Displace one of the carts about 2 cm and record the response. You should now see a multi-harmonic response since the system is now a 2-DOF system. Include a plot in your report.
   <u>b)</u> Displace both carts to the right by about the same amount and observe the response. If you displace them exactly in the pattern of the first mode of vibration, then they should oscillate in unison at the same frequency. Can you realize this experimentally? Record the best (closest to harmonic) response that you can obtain on the computer and save it so you can plot it for your lab report. ***Use your plot to estimate the mode shape and natural frequency of the first mode.***
   <u>c)</u> Displace the carts in opposite directions by about the same amount and record the response. ***Can you displace them such that they oscillate in a pure second mode? Record the best response that you can obtain. Is this more or less difficult than the previous case? Why or Why not? Use your plot to estimate the mode shape and natural frequency of the second mode.***

SUMMARY OF WHAT TO INCLUDE IN YOUR LAB REPORT:
   1.) Plots
      a. comparing the response of the SDOF system with $m = m_1$ and $m = m_1+m_2$.
      b. comparing analytical response of SDOF (tuned to agree as closely as possible) to measured response with $m = m_1$.
      c. of MDOF system response to ICs $x_1 \approx 2$, $x_2 \approx 0$.
      d. of MDOF system response with ICs that you determined to isolate each of the two modes.
   2.) Results:
      a. Identified natural frequencies and damping ratios for SDOF system with $m = m_1$ and $m = m_1+m_2$. Also discuss the uncertainty and how you estimated it.
      b. m_1, k_1, c_1 and c_2 computed from the modal parameters above
      c. identified natural frequencies and mode shapes of the MDOF system, found from the plots in (1d).
   3.) Answers to the questions found in bold-italics throughout this lab report.
   4.) You will also include any other plots, tables and analysis that is important to understanding the uncertainty in your results, why your results deviate / or do not deviate from the analytical model, etc…

LAB 4A: IMPACT MODAL TEST OF FREE-FREE BEAM

In this lab you will perform a modal test to determine the first six modes of a beam in free-free conditions. The beam is made of aluminum and is two inches wide and between 3/16 and ¼ inches thick. Each team will have a beam of a slightly different length.

BEFORE CLASS:

Please review the video posted on the course website that describes how to perform a hammer modal test using the B&K software. Download a copy of the **ModalHammer_ME540.plt** template to use for this lab.

IN THE LAB:

Perform a roving input modal test to find the FRFs of the beam. Create a measurement grid on your beam consisting of N_o points, where N_o is at least 20, as shown in Figure 1. Fix a low sensitivity (5 mV/g) accelerometer to the back side of the beam **behind measurement point 1**. This will be your reference node.
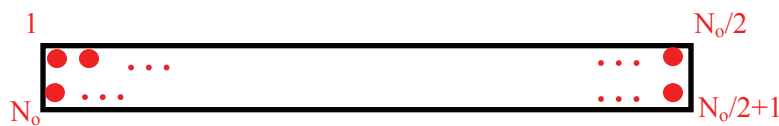

Figure 1: Measurement grid for modal test of free-free beam.

Choose a sample rate and number of samples to avoid leakage (the anti-aliasing filters on the data acquisition hardware should take care of aliasing, although it is still wise to avoid exciting modes that are outside of the measureable bandwidth.). Before you begin saving a complete set of measurements, estimate the natural frequencies of the first six modes and the damping ratio of the first two modes of the beam and determine how long you will need to sample to obtain adequate frequency resolution in the FRF for those modes. Also view the autospectrum of the input force from the hammer, to be sure that it adequately excites the frequency band of interest. Set up an exponential window and record the parameters of the exponential window that you have used. Once you are comfortable that you know how to acquire good FRFs, save anything that you need to answer the questions below and then you may begin acquiring your N_o point measurement set.

Initially, do not use a response or hammer window. Focus on getting a good set of measurements to make sure you know the measurement process. Later there will be time to improve the measurements using a window if that is necessary.

QUESTIONS

1.) *How did you assure that your sample rate and number of samples were adequate to avoid leakage? Make a note of the settings that you used and list them in your lab report.*
2.) *What was the bandwidth of the input force supplied by the hammer? Save a plot of the autospectrum of the input from one of your measurements and include it in your lab report.*

Use the B&K software to acquire and save a complete set of N_o FRFs of the beam. Use **three averages per point**. Make sure that each of the three measurements that you are averaging is reasonably clean. Severe double hits will visibly contaminate your measurements. Use the "Undo Hit" feature when needed and go back and re-test any points where the measurement is noisy.

Save your project file (*.pls) file so you can go back and view/export your measurements as needed. Then export a set of FRFs to a *.uff file that you can load into Matlab (see the instructions at the end of this handout).

ANALYSIS

1.) **Peak Picking for first few modes:**
   a. Load your FRFs into Matlab using the "loaduff" function on the course website. If you have correctly numbered your input and response points, this should create an N_f x N_o FRF matrix from your measurements, where N_f denotes the number of frequency lines in each measurement. If you run into difficulty, you may need to assemble your FRF matrix manually (see instructions at the end of this handout).
   b. Create and plot a composite or average of your FRFs using the "comp_FRF" function, or by simply computing the average of the magnitude of your N_o FRFs.
   c. Use peak picking and the half-power bandwidth method to estimate the natural frequency and damping ratio of the first **two** modes of the beam. Estimate each mode shape by assuming that it is proportional to the FRF at the peak. (Note that this is essentially the same procedure that the B&K software uses to produce the mode shape animations that you saw in the lab.)

2.) **AMI Identification:**
   a. Use the AMI modal parameter identification algorithm to extract the natural frequencies, damping ratios and mode shapes of the first six modes from the measurements. ***Include a plot of AMI's GUI (figure 1000) in your lab report.***
   b. ***How well does AMI's modal model reconstruct your measurements? If there are discrepancies, what might they be attributed to?***

3.) **Compare Experimental Results with Analytical Model:**
   a. Estimate the analytical bending natural frequencies of your beam using the measured dimensions and nominal material properties for aluminum. Compare the analytical bending natural frequencies with those that you measured. There is some uncertainty regarding E and ρ, so you can adjust those to bring the results into as good of agreement as possible. ***After adjusting those parameters, how do the natural frequencies compare? What do you attribute the differences to?***
   b. **Bending Modes:** Generate the analytical mode shapes of the beam using the equations in the textbook for bending, and plot them with your experimentally obtained mode shapes overlaid. ***Include a plot of your mode shapes in your lab report. How do they compare? What do you attribute the differences to?***

Note that the comparison will be easier if you create two sets of the analytical mode shapes as follows

$\{\phi_{exper}\}_r$: ($N_o \times 1$), experimentally measured mode shape vector for rth mode

$\{\phi_{an}\}_r$: ($N_o \times 1$), analytically derived mode shape vector for rth mode, mass normalized

$\{\phi_{an,fine}\}_r$: ($N_{plot} \times 1$), analytically derived mass-normalized mode shape vector for rth mode at 100 points evenly distributed between 0 and L.

To scale these modes for plotting, define the following:

$$\left(S_{exper}\right)_r = \left\|\{\phi_{exper}\}_r\right\|$$
$$\left(S_{an}\right)_r = \left\|\{\phi_{an}\}_r\right\|$$

Where $\|x\|$ is the norm of x (use Matlab's "norm" function). Then you can plot the following to obtain a comparison of the mode shapes that is not affected by the scale of the vectors.

$$\frac{\{\phi_{exper}\}_r}{\left(S_{exper}\right)_r} \qquad \frac{\{\phi_{an,fine}\}_r}{\left(S_{an}\right)_r}$$

For modes 1 and 2 your plots should show the shape you got using peak picking, AMI's shape, and the analytical shape. For modes 3-6 compare only the AMI shapes and the analytical shapes.

      c. **Torsion Modes:** You should identify at least one torsion mode of the beam. Plot it(them) as well but you do not need to compare it(them) with an analytical shape.

      d. Compute the Modal Assurance Criterion (MAC) matrix between your experimental mode shapes and your analytically predicted bending mode shapes (see MAC_plot.m). ***Are all of your mode shapes distinguishable on the measurement set that you used?***

      e. Compute the Modal Scale Factors (MSFs) between your experimental and analytical bending mode shapes (you don't need to do this for the torsion modes). ***How well do the scales of the shapes compare?*** To mass-normalize the analytical shapes from Appendix C in Ginsberg, divide them by $(\rho A L * \mu_{nd,k})^{\wedge}(1/2)$, where

$$\mu_{nd,k} = \int_0^1 \psi_k(x')^2 \, dx'$$

The values of $\mu_{nd,k}$ are 1.0359, 0.99845 and 1.0001 for k = 1, 2 and 3 respectively, and $\mu_{nd,k}$ =1 for k>3.

APPENDIX:

Definition of MAC and MSF between mode vectors *cr* and *dr*, where $()^H$ is the transpose for real vectors (or the Hermetian or complex conjugate transpose for complex vectors).
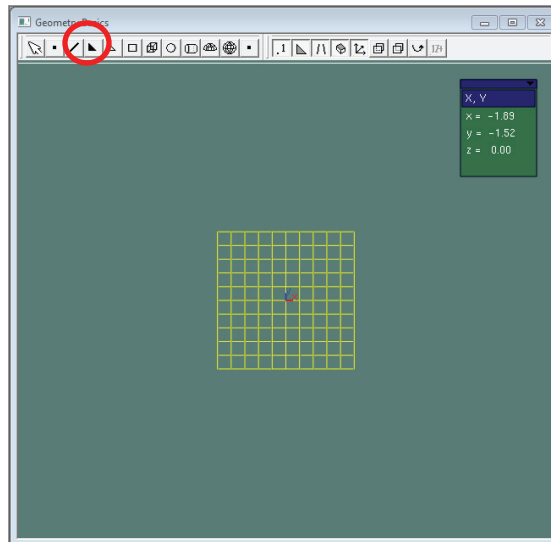
$$MAC_{cdr} = \frac{\left| \{\psi_{cr}\}^H \{\psi_{dr}\} \right|^2}{\{\psi_{cr}\}^H \{\psi_{cr}\} \{\psi_{dr}\}^H \{\psi_{dr}\}} = \frac{\left(\{\psi_{cr}\}^H \{\psi_{dr}\} \right) \left(\{\psi_{dr}\}^H \{\psi_{cr}\} \right)}{\{\psi_{cr}\}^H \{\psi_{cr}\} \{\psi_{dr}\}^H \{\psi_{dr}\}}$$

$$MSF_{cdr} = \frac{\{\psi_{cr}\}^H \{\psi_{dr}\}}{\{\psi_{dr}\}^H \{\psi_{dr}\}}$$

B&K Labshop Intructions:

How to Create Geometry in PULSE Labshop

Navigate to Setup>Geometry in the task bar on the left side of the screen. The following screen should appear. Since the test specimen is a beam, a simple rectangle will represent this geometry well enough.

In the geometry window above, click on the rectangle button above (red circle), then drag and click to create the arbitrary rectangle. Note that you should first view the XY plane before creating the geometry. To do this, right click in the window and select view>x,y.

The window (bottom left) with the rectangle properties will allow you to change the dimensions of the beam (adjust accordingly).
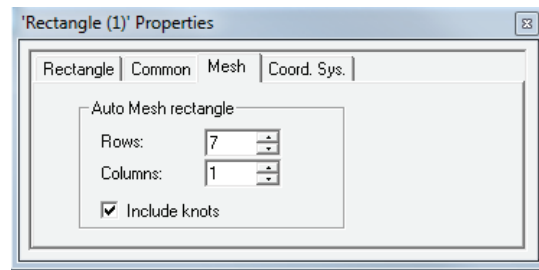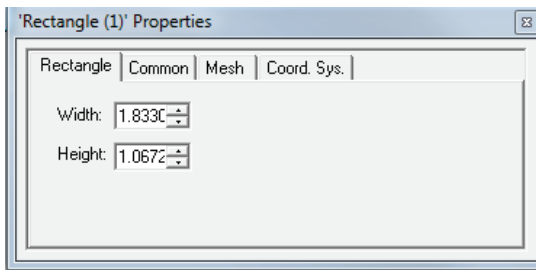
Now, select the Mesh tab (top right). Selecting the number of rows will create various points on the beam. These points will represent the inputs for the experiment. Note that the relative distance between the points is irrelevant for this lab. Keep track of the point number and its location on the beam. The measurements will be saved referencing these nodes.
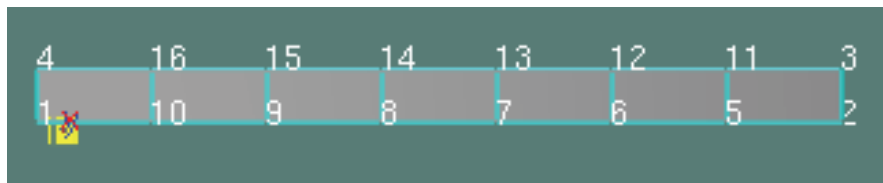
After collecting measurements at each grid point, you can export all the results to a .uff file. Export the measurements by navigating to **Measurements>Export** in the side task bar. The following screen should appear.
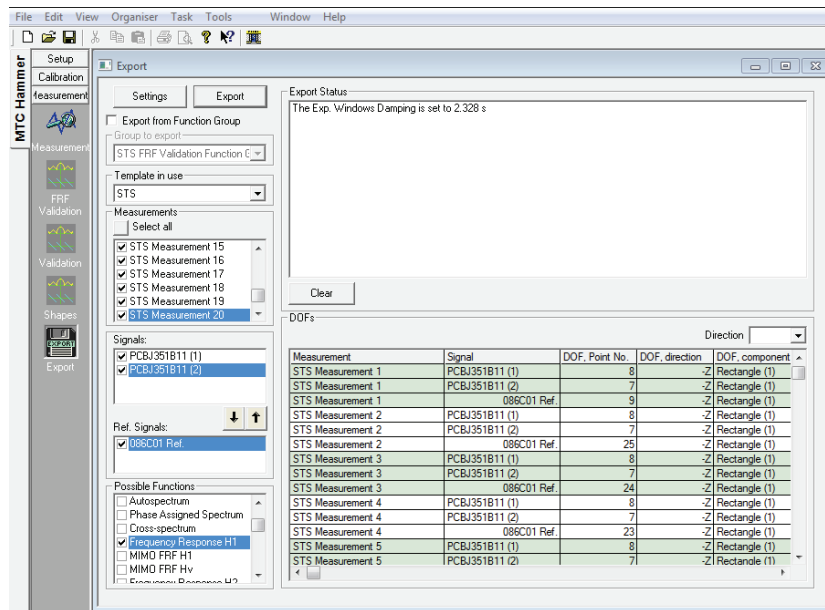
Be sure to select all measuments to export, and that you are exporting only the Frequency Response Functions H1. Next, click on the **Settings button**, which brings up a window shown below.
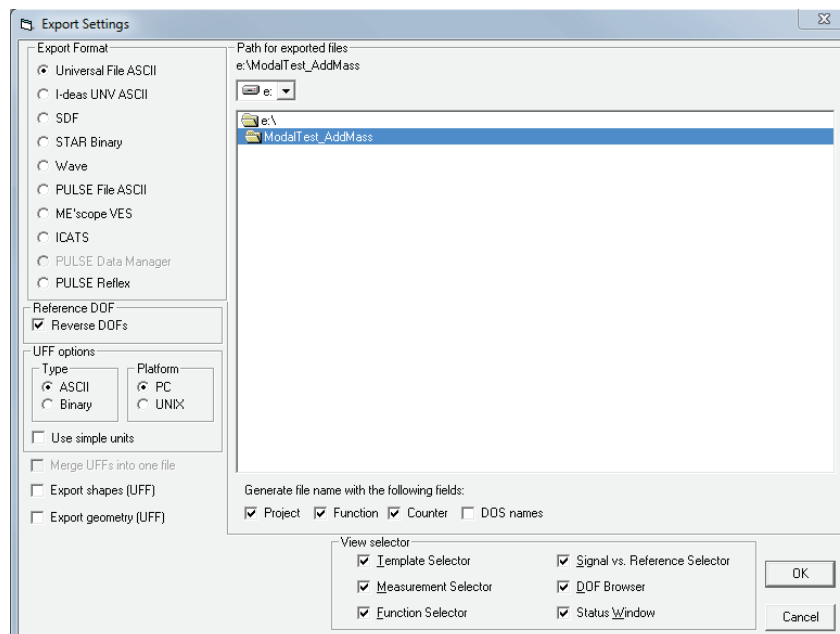
Export in ASCII Universal File format, type ASCII, and PC platform. Select which folder you'd like to export to. Once this is done, return to the prior screen and select **Export**.

Load the UFF file into MATLAB using 'loaduff.m' (call in Matlab is >>data = loaduff). Note that the data will be structured in cells, and each measurement is located in a cell. For example:

data{*n*} displays all of the fields in the *n*th measurement

data{n}.measData is the complex amplitude of the FRF (H_{np}) where p is the drive point and n is the response point.

data{n}.x is the frequency vector.

The measurement number, units and other information regarding the measurement is stored in the cell structure. Use the variable editor in Matlab to look at these.

Note that if you have done everything correctly you can use the shortcut:

```
[data,dinfo,H,ws,rsps,refs] = loaduff;
```

Then, `H` will be the FRF matrix with your measurement points listed in ascending order in the columns and `ws` will be the corresponding frequency vector in radians per second. The names of your reference and response measurement points will be in `rsps` and `refs`. For example, if `rsps.name{5}='+3x'` then `H(:,5)` corresponds to the FRF at measurement point 3 in the +x direction.

Pay close attention to the order of your measurement points when plotting. You may need to re-order the columns of H (or from the mode shapes that you get from H) in order to plot the mode shapes correctly.

Additional Help

To use 'pickpeaks.m' you first need to plot the magnitude of the average of all your FRFs. For example:

Haverage = sum(H,2)/size(H,2);

or use

Haverage = comp_FRF(H);

then

plot(ws,abs(Haverage));

or all in one line

plot(ws, comp_FRF(H));

The 'pickpeaks.m' function allows you pick points on the active Matlab plot window and extracts the 'x' and 'y' value of the point that you pick. The function returns the index locations of the x-value of the point you picked. So, suppose my ws vector is [3 5 8 9 10]. If I picked a point on the plot near ws = 8. Then, 'inds' would return 3, the value of the index in ws that corresponds to ws(inds)=8; Then, you can go back into you full H(ω) matrix and extract the rows corresponding to 'inds'. What does one of these rows mean? Well, remember that near a natural frequency (which is a peak in the FRF) the formula

$$\left\{ H\left(\omega \right) \right\} = \sum \frac{\left\{ \phi_r \right\} \phi_{pr}}{\omega_r^2 - \omega^2 + i\omega 2 \zeta_r \omega_r} \tag{1}$$

goes to

$$\left\{ H\left(\omega \right) \right\} = \frac{\left\{ \phi_r \right\} \phi_{pr}}{i\omega 2 \zeta_r \omega_r}, \tag{2}$$

However, we have accelerometer measurements, so our measurements are related to the mode shapes by:

$$\left\{ H\left(\omega \right) \right\}_{accel} = \frac{-\omega^2 \left\{ \phi_r \right\} \phi_{pr}}{i\omega 2 \zeta_r \omega_r}, \tag{3}$$

$\{\phi_r\}$ in Equation 3 is the experimental mode shape that we are after, so **the imaginary part** of the vector of FRF measurements is proportional the true mode vector..

To run AMI and extract modes:

Type the following commands to set up AMI:

```
global AMISET
AMISET = AMIG_def_opts;
AMISET.DVA = 'A';  % this tells AMI that the data is acceleration/force FRFs
AMISET.LM = 'on';  % this makes the file smaller if you save the AMI fit.
ami(H,ws,AMISET) % where H is the Nf by No FRF matrix and ws is the frequency
vector in radians per second.
```

At this point you run through the AMI fitting process. If you are having trouble with this or do not understand it, come in to talk to the instructor or the TA.

After completing the fitting process, run these lines of codes to get to the natural frequencies, damping ratios, and residue vectors.

```
global AMIMODES  % brings AMI's variables into the workspace
fn = AMIMODES.mode_store(:,1)/2/pi;   % These are the natural frequencies in Hz
zt = AMIMODES.mode_store(:,2);        % These are the damping ratios, zeta
% The following gives the residue vectors
res = (-2*real(diag(conj(AMIMODES.mode_store(:,3)))*AMIMODES.A_store)).';
```

'res' is a matrix containing "residue" vectors in the columns, each of which is related to a mode shape by,

$$res(:,r) = \{\phi_r\}\phi_{rp}$$

which is the numerator of Equation 1.

So if your drive point was the first measurement point (p=1), then

$$res(1,r) = \phi_{r1}\phi_{r1} = \phi_{r1}^2, \quad then, \quad \phi_{r1} = \sqrt{res(1,r)}$$

and

$$res(2,r) = \phi_{r2}\phi_{r1}$$

$$\phi_{r2} = \frac{res(2,r)}{\phi_{r1}}$$

so you can continue the process over the rest of that column to estimate that mode shape and then repeat the process for each of the columns of 'res' to obtain all of the mass-normalized mode vectors ϕ_r. Note that the drive point residues will sometimes come out negative leading to non-sense results (imaginary mode shapes). If several drive point residues are negative there is likely a problem with your measurements or you may be using some point other than the true drive point in the equations above. On the other hand, the drive point residue for a mode or two may come out negative even in relatively good measurements, so you may be able to take the absolute value and obtain reasonable results.

It is important to consider the units here. We want our mass normalized modes to have units of 1/sqrt(mass). So, the residue vector components should have units of 1/mass. However, if your FRF was measured with different units then this might not be correct. So before mass normalizing, convert your units to metric. The units of 'res' will be the same as those of your FRF, but converted to displacement. So, if you measured the FRFs in units of [g's/lbf], then the units of res will be [g's/lbf]*(sec^2). Make the appropriate conversion to change this to units of (m/sec^2)/N = 1/kg.

LAB 4B: IMPACT MODAL TEST OF FREE-FREE BEAM: STRUCTURAL MODIFICATION

In this part of the lab you will predict the effect of a simple structural modification on the mode shapes and natural frequencies of a beam. Your modification consists of the addition of a mass to the beam. You will use the Ritz method together with your experimentally measured mode shapes and natural frequencies to predict the new natural frequencies and mode shapes of your beam after adding the mass, and compare your predictions with experimental measurements on the beam+mass system.

Measurements:

1.) Attach a steel block to your beam as shown below so that it covers two measurement points, which are at the same position along the axis of the beam. We will denote these points d_1 and d_2. Place the block at the measurement points that are nearest to (1/6)L from the end of the beam (For example, with $N_o = 20$ points, 2 inches between points and one inch between the 10th point and the end of the beam, the block should be at points 8 and 13, which are across from each other, and so $d_1 = 8$ and $d_2 = 13$). The steel blocks measure 0.5 by 1.0 by 2.0 inches, so they should cover the entire width of the beam.
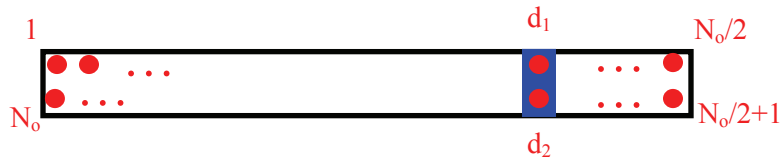

Figure 1: Beam with mass modification.

2.) Measure the frequency response functions of the beam along at least one row of points (e.g. 1, 2… $N_o/2$) and use those to extract the true natural frequencies and mode shapes of the beam with the mass added.

Analysis:

1.) Use your experimentally measured, mass normalized mode vectors and natural frequencies with a one-term Ritz model to compute the natural frequencies of the beam after adding a 144-gram block of steel to your beam. Add half of the beam mass, or 72-grams to each point in your analysis. ***How much has each natural frequency shifted? Can you explain why some modes have shifted more than others?***

2.) Use the mode shapes that you generated analytically (as described in the (4a) handout for this lab) to predict the natural frequencies of the beam with the mass added. This analysis gives the Ritz predictions that you would have obtained from this analysis if your measurements were perfect. ***Based on the comparison of all of these results, what is the likely cause of any discrepancies?*** Think about both measurement errors and the assumptions inherent in the 1-term Ritz model.

Analysis with One-Term Ritz Model

To perform your analysis for parts (1) and (2) you will need to determine an expression for the natural frequency of a mode based on the mass added and the mode shape and natural frequency of the mode. For example, in class we used a one-term Ritz model to derive the following expression for the shift in a natural frequency as a function of an additional stiffness Δk_i added to the ith measurement point.

$$\Delta \omega_r = \frac{1}{2\omega_r}\left(\phi_{ir}\right)^2 \Delta k_i$$

Optional Analysis with Full Ritz Model:

The one-term Ritz model assumes that the mode shapes of the structure do not change with the addition of the mass. This assumption can be relaxed somewhat using an N-term Ritz model (where N is the number of modes identified). This can be created from the modes of the beam without the mass. This model has diagonal

mass and stiffness matrices and uses the measured mode shapes as basis vectors. Then one can determine the new mass matrix after adding the block, and use that to compute the new natural frequencies and mode shapes.

This approach is implemented in the `ritzsmod.m` file in the `EMAfun` folder of the course Matlab tools. You may use this function for the N-term analysis, or to debug your 1-term model, but it cannot be used exclusively. ***Each team must turn in a handwritten derivation and the computer code that they have developed to perform the analysis in parts (1) and (2)***. In case you want to also use the `ritzsmod.m` function, it is described below.

First you will need to generate the two following two rigid body modes of the beam (these could be measured experimentally, but that would require some additional work). You will need to include these in your Ritz model for the beam in order to obtain accurate results:

$$\phi_{rigid,1}(x) = 1/(\rho AL)^{(1/2)}$$

$$\phi_{rigid,2}(x) = (x\text{-}0.5)/(\rho AL/12)^{(1/2)}$$

where x ranges from 0 to 1 (nondimensional) in those equations.

```
% Structural modification using experimentally determined mode vectors as
% a Ritz vector basis set.
%
% Computes the modal parameters of the system modified by adding the masses
% and stiffnesses in the vectors dm and km to nodes ns_m and ns_k;
%
% [wn_mod,zt_mod,phi_mod,M_mod,C_mod,K_mod] =
%     ritzsmod(wn,zt,phi,dm,ns_m,dk,ns_k);
%
% "ritzmod" requires the following necessary inputs:
%   wn - vector of natural frequencies for the un-modified structure.
%   zt - (optional) vector of modal damping ratios.
%   phi - matrix of mass normalized mode vectors where each column
%       corresponds to a natural frequency in wn.
%   dm - vector of masses to be added to node numbers listed in ns_m.
%   dk - vector of stiffnesses to be added to node numbers listed in ns_k.
%       Each spring is between the node listed and ground.
```

For example, suppose that you measured the beam's mode shapes at 20 points and that your mass attaches to the beam at nodes 8 and 13. Then you would use the following to call ritzsmod:

```
[wn_mod,zt_mod,phi_mod] = ritzsmod(wn, zt, phi, dm, ns_m, [], []);
```

where `wn` and `zt` are N by 1 vectors of the natural frequencies and damping ratios, `phi` is a 20 by N vector of mass normalized mode vectors (each vector is known at all 20 measurement points).

```
dm = [0.072, 0.072]
ns_m = [8,13]
```

Notice that we're passing in empty vectors for `dk` and `ns_k` since we haven't attached any springs to the system.

LAB 5: OUTPUT ONLY (NATURAL EXCITATION) IDENTIFICATION OF A DOWNHILL SKI

The performance of downhill skis has been shown to depend on the relationship between the natural frequencies of bending and torsion modes [2]. However, in its natural environment a ski's boundary condition is very difficult to model (e.g. the interaction between the ski and the snow), so tests are needed to determine the in-operation dynamics. Since the forces that the snow applies to a ski cannot be easily

measured, one must rely on output only modal analysis techniques. In this lab your task is to identify the natural frequencies of the first seven modes of a downhill ski and the corresponding mode shapes in order to determine whether each is a torsion or bending mode. An unknown excitation is simulated in the lab by tapping the ski at random with a hammer. You will use the spectra of the response alone to identify the modes of the ski. (I know, it would have been more fun to do this on the mountain, but what can we do?)

DERIVATION (BEFORE CLASS):

Suppose a system is excited by forces f_1 and f_2 at two different points p_1 and p_2. The cross spectrum between each of the forces (and each force and itself) is constant with frequency and denoted $S_{f_1 f_1} = \text{const}$, $S_{f_1 f_2} = \text{const}$, etc… Based on what we have covered in class, the frequency response at the jth location (this is the FFT of $x_j(t)$) is,

$$X_j(\omega) = H_{j,p1}(\omega)F_1(\omega) + H_{j,p2}(\omega)F_2(\omega)$$

where the accelerance FRF is defined as follows in terms of the modal parameters.

$$H_{j,p1}(\omega) = \sum_{r=1}^{N} \frac{-\omega^2 \phi_{j,r} \phi_{p1,r}}{\omega_r^2 - \omega^2 + i\omega 2\zeta_r \omega_r}$$

One definition of the cross spectrum between measurements x_j and x_n is,

$$S_{x_j x_n}(\omega) = \frac{1}{N_{\text{avg}}} \sum_{m=1}^{N_{\text{avg}}} X_j(\omega)_m X_n(\omega)^*_m .$$

This equation also gives $S_{f_1 f_1}(\omega)$, $S_{f_1 f_2}(\omega)$, etc… with the appropriate substitutions. The index m denotes the mth estimate of the spectra, obtained by taking the FFT of the mth time series. There are assumed to be a total of N_{avg} different measurements that are averaged to estimate the spectra.

Use this expression to show how the mode shape of the rth mode of vibration is manifest in the cross spectra $S_{xj,xn}$ for $j = 1…N_o$ and n being a fixed reference point. You will find that you must assume that the output measurements are all acquired simultaneously so that the input spectrum is the same for each. ***Turn your handwritten derivation in with your lab report.***

IN THE LAB:

PART 1

For this experiment, you will have four (identical) accelerometers mounted along half of the length of the ski. The accelerometer closest to the tip will be set as the reference (since none of the mode shapes has a node at the tip) and called x_1. The software will be set to compute the autospectrum of the tip response and the cross spectrum between the tip and all other points. (This lab was set up using B&K's OMA template: "Operational Modal Analysis.plt".)

Your task is to determine the following parameters and to acquire an OMA measurement that captures the first seven modes of the ski:

- Number of Averages
- Frequency Bandwidth
- Frequency Resolution (governed by the length of the time window that is processed by the FFT, *T*, or the FFT blocksize)

The data acquisition system will already be set to run in free-run mode (no trigger) with a Hanning window.

Choose a sample rate that allows you to capture at least the first 7 bending modes and choose an FFT blocksize that gives reasonable frequency resolution near each peak. You may find that the optimal blocksize is greatly

different for the first mode than for the seventh, so feel free to record a few different data sets if time permits to experiment with the settings. ***How does the blocksize and number of averages affect the quality of your measurements for a fixed total measurement time?*** Tap the ski with the hammer at random times and at random places simulating a random input until the number of averages is reached. You should see the spectra converge as the number of averages increases.

Hints: Recall that the frequency spacing in the FFT is $\omega_1 = 2\pi/T$, or inversely proportional to the sample length, T. The frequency spacing must be smaller than the half power bandwidth $2\zeta_r \omega_r$ or the peaks will be under-resolved and the damping cannot be accurately estimated. You do not know the damping initially, but when setting up your measurements you can assume that the ski's damping ratios will be at least 0.5% for the skis without bindings and at least 1% for the skis with bindings.

See the Appendix for notes regarding how to save your measurements.

These steps can be competed in Matlab: Once you obtain your measurements, use peak picking to determine the natural frequencies of the first 7 modes and plot the corresponding mode shapes using the fact that your derivation has shown that the cross spectrum near each natural frequency, $S_{x_j x_1}(\omega)$, is proportional to the mode shape at the

jth measurement point.

PART 2

This second part of the lab simulates a more realistic operational modal test where the response sensors must be roved to capture the mode shape at a large enough number of points. In the interest of time the TA has acquired measurements and will provide them to you. The measurements were acquired in seven setups using the measurement grid shown below. (For those who are interested, the ski that was tested is a K2 Mt. Baker Superlight ski, size 174 (yellow), donated in 2007 by K2.)
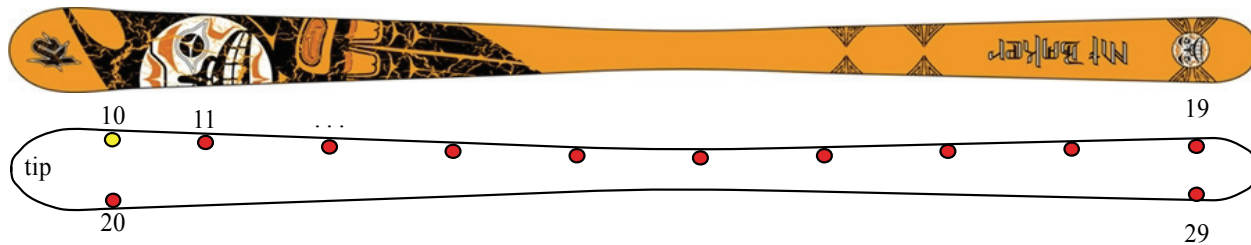

Figure 1: Schematic of measurement grid used when testing the downhill ski.

In all of the measurements, **point 10 near the tip of the ski** was used as a reference, so the cross spectra of all of the other points were computed relative to this point. Since the number of data acquisition channels was limited, the measurements were acquired in seven setups, which are described below. This type of approach is often used when testing large structures to limit the length of the cables that must be used, or when many sensors are desired.

Setup	Sensor 2	Sensor 3	Sensor 4
A	20	21	11
B	12	22	13
C	14	24	23
D	25	26	15
E	27	17	16
F	29	28	18

G	29	20	19

This information is also included in the *.mat file of the data that you will receive.

In each setup, the accelerometers are located between ¼" and 3/8" from the edge of the ski. The positions of each measurement point are given below. (For your information, the width of the ski at certain positions is also given, and the taper is approximately linear between those.)

Grid Point #	10, 20	11, 21	12, 22	13, 23	14, 24	15, 25	16, 26	17, 27	18, 28	19, 29
Axial Pos (in)	2.0	8.75	15.5	22.0	28.75	35.25	42.0	48.5	55.25	62.0
Ski Width (in)	4.25	-	-	-	3.5	3.5	-	-	-	4.75

Use the auto and cross spectra provided to estimate the first 10 modes of the ski with the peak picking method. Then use the methods described in class to estimate the mode shapes of the system over the four sensors for each measurement setup. Then, normalize the mode shapes such that the mode shape at the reference sensor is 1.0, in order to stitch the measurements together, and plot the modes. Create a table showing the natural frequencies of each mode and whether it is primarily bending or torsion.

LAB REPORT:

Include the following items in your lab report:

- Plot of the spectra that you measured for part 1, a list of natural frequencies that you identified and a plot of the corresponding mode shapes.
- Plot each of the mode shapes that you identified from the data provided for part 2 and a table showing the natural frequencies and whether each mode is primarily bending or torsion
- Attach your derivations (one per team member) from the "Before Class" section above as an appendix.
- Briefly discuss your results pointing out any interesting features. ***How did this procedure compare with the hammer modal analysis that you performed in Lab 4? Which was easier to perform? Was there any degradation in the quality of the measurements when the OMA technique was used instead of the conventional EMA approach in Lab 4?***

NOTES REGARDING DATA:

The data for this lab is contained in the *.mat file "Lab5_ref10_Spectra_ABCDEFG.mat" on the course website. An N_f x 4 matrix of auto- and cross-spectral data is supplied for each setup, denoted SA, SB, etc…, where N_f is the number of frequencies and the four columns each correspond to a sensor. The sensor locations for each setup are given in ">>rspsA.name"; these should agree with the information in the table above. (Specifically, SA(:,1) contains the autospectra for point 10 in setup A, SA(:,2) contains the cross spectrum between points 10 and 20, SA(:,3) the cross spectrum between points 10 and 21, SB(:,2) the cross spectrum between 10 and 12 for the B setup, etc…)

APPENDIX

Use the following procedure to export your measurements to a UFF file. You should already have the autospectrum of your reference accelerometer and the cross spectra of the other accelerometers in your function group as shown below.
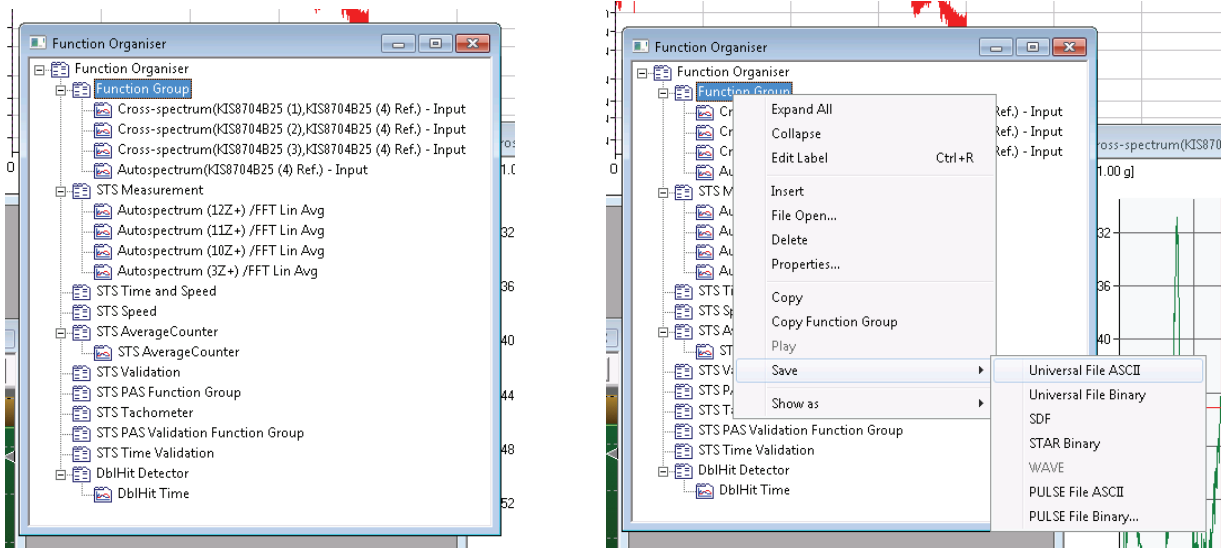
Right click on the function group (not the individual measurements) and select "Save >> Universal File ASCII"

The following dialog box comes up with options for the UFF file, such as the units or the ability to edit the point numbers. You should be able to simply select "OK" and the UFF file will be written to the current directory.
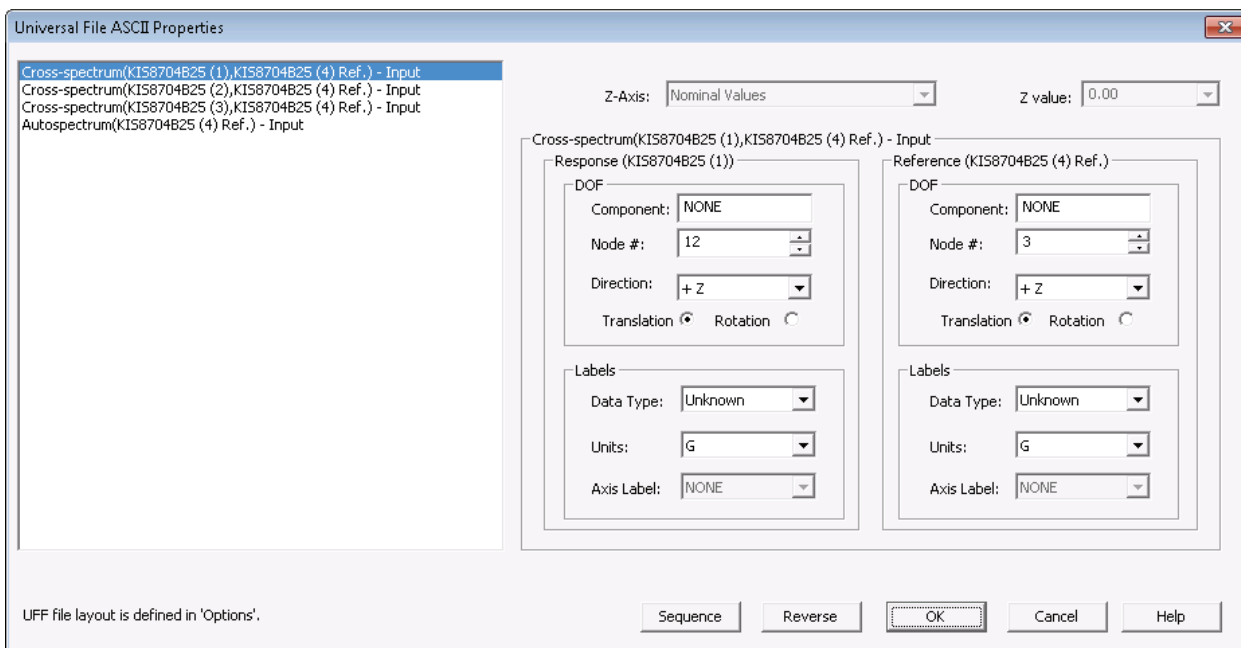
FINAL PROJECT ASSIGNMENT

DEADLINES:

- **Nov. 22th** – Select final project group (3 students per group) and turn in 1-page summary of project plan. (There can be two groups of 2, email me immediately if you want to be in a group of 2.)
- **Dec. 1st** - Must have all hardware ready (and in 2163 if it is portable) to begin testing.

- **Thursday Dec.15th** – presentation due and must be uploaded to course website in a *.zip file along with all of your measurements and your post-processing scripts, Matlab files, etc… Scan any hand calculations and any other supporting materials and include them in the *.zip file.
- **Saturday Dec. 17th** – Each group gives a 12-15 minute presentation summarizing their results during the final exam period, 2:45PM - 4:45PM on Saturday Dec. 17.

DESCRIPTION: *Your objective is to perform an accurate dynamic test in order to understand some aspect of the dynamic response of your system. <u>You must not only perform the test but also do calculations to explain how the modal parameters that you identify affect the performance of the system.</u>* The list below gives a sampling of ideas for your final project with the associated analysis underlined. You are certainly welcome to propose other ideas as well. Discuss your ideas with the instructor prior to the Nov. 22nd deadline if you are unsure whether they would be feasible.

- Modal test of mass spectrometer hardware (available in lab). Objective is to control the position of the sensor to within one micron (10^{-6} meters). What forcing frequencies should be avoided to accomplish this? <u>What if the system is inadvertently forced at resonance? How large can the forcing be in that case?</u>
- Sporting equipment such as a tennis racquet (or similarly for baseball, hockey, etc…): Compare the dynamic properties of two racquets (or one racquet in multiple configurations, e.g. with a vibration absorber.) <u>Simulate the impact of a ball with the racquet using the modal model that you've developed and use that result to compare how the two will feel to the user, or which will hit the ball farthest?</u>
- Test a crystal glass to identify the shape of the mode(s) that contribute to sound generation when a finger is rubbed along the top of the glass. <u>What other modes are present in the frequency range that do not generate sound? Why might that be?</u>
- Hard Drive: Perform modal survey of the case. <u>How do modes compare with the drive frequency and its harmonics? At what spindle speed would you expect to encounter vibration problems with the hard drive? Remove damping treatments and compare.</u>
- For RC enthusiasts: Perform a Modal test of an RC plane to identify the first few bending and torsional modes. Compare the natural frequencies and <u>predict which bending-torsion mode combination is most likely to lead to flutter at high airspeeds</u>.
- Operational Modal Analysis of Mech. Eng. Building (or ERB): Use OMA techniques to perform a roving sensor modal test of the ME building. Estimate the first few modes of vibration of the 2nd floor walkway surrounding the atrium. <u>Estimate how many people would have to jump in unison on the walkway to obtain three inches of peak displacement</u>.
- Cross Axis Sensitivity: All accelerometers are designed to sense acceleration in one direction, but they are inevitably sensitive to acceleration in other directions as well. Design an experiment on a simple structure, such as a thick beam, to evaluate this. Place several accelerometers perpendicularly on the structure and see to what degree the bending motion is detected by an axially oriented sensor, etc… <u>Look for other anomalies that might help one to detect cross-axis sensitivity (e.g. strange phasing, etc…)</u>.

References

[1]   J. H. Ginsberg, *Mechanical and Structural Vibrations*, First ed. New York: John Wiley and Sons, 2001.
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