Computing Nonlinear Frequency Response Functions (FRFs) for Systems with Iwan Joints

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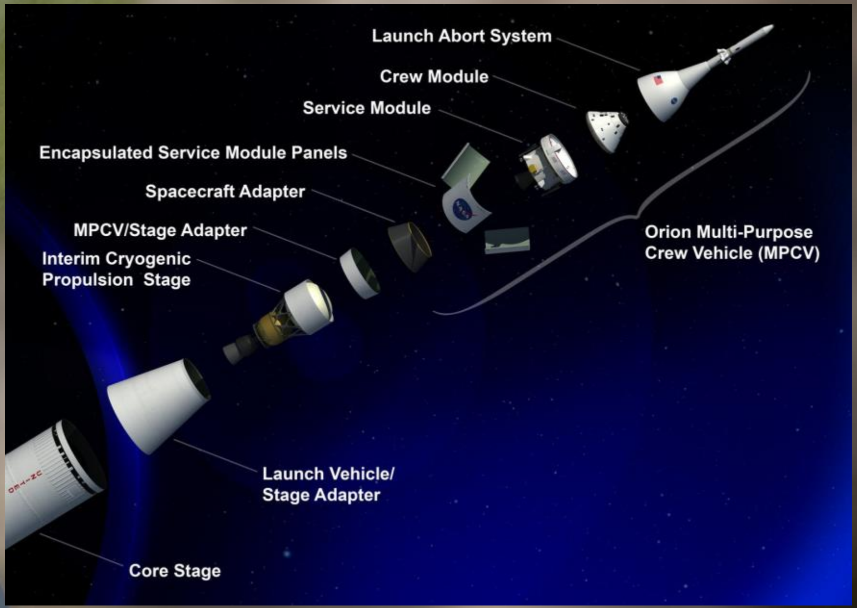
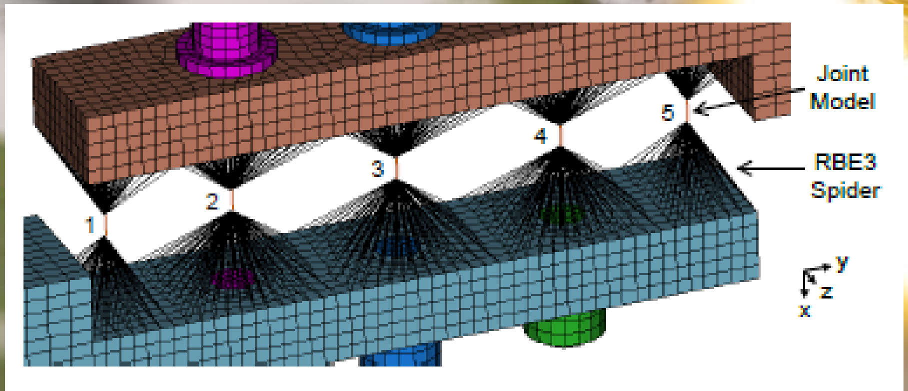
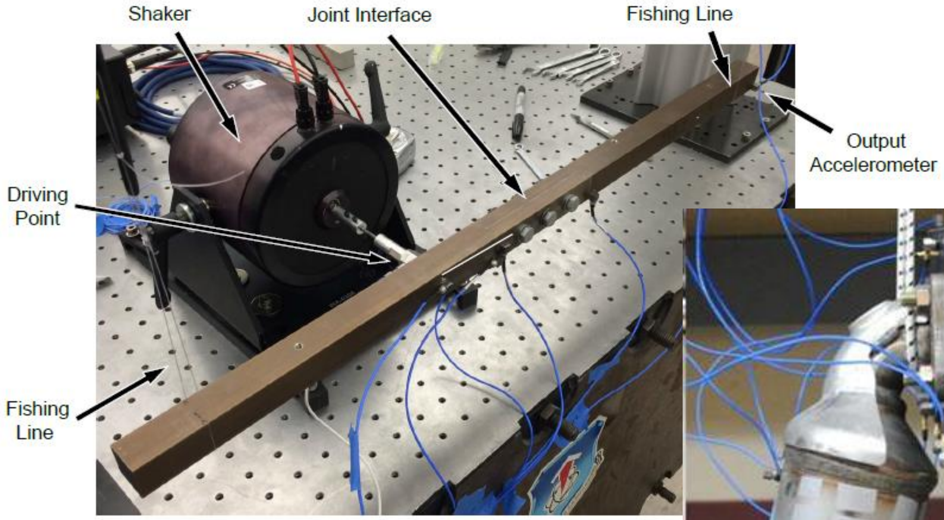
Nonlinearities due to joints are a major source of error in modeling complicated, built-up structures.

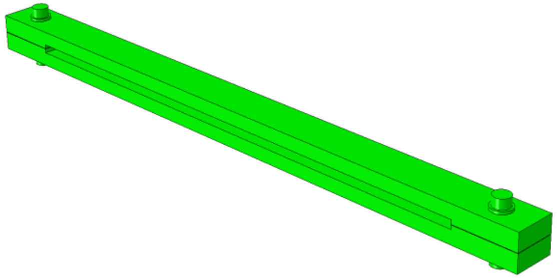


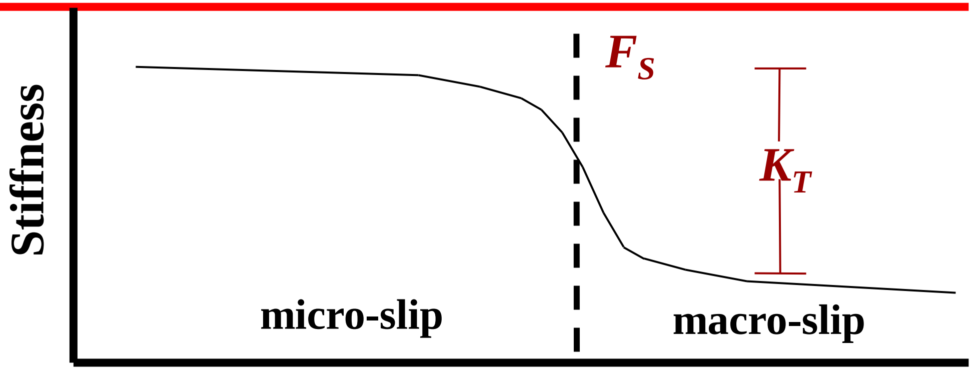


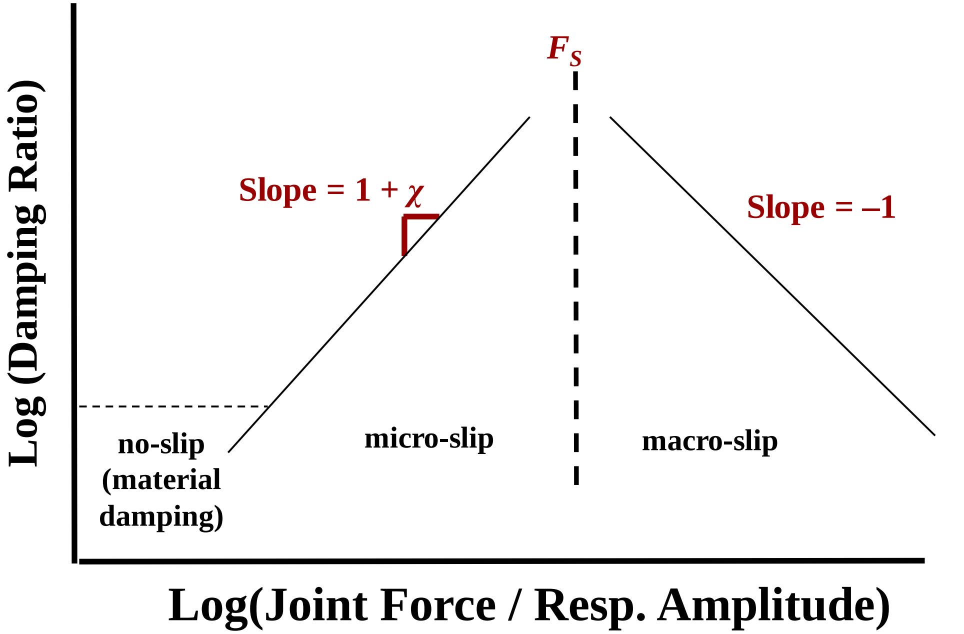
Micro- and Macro-slip in joints introduces nonlinearity

Segalman's 4-parameter Iwan model has proven effective at capturing the nonlinearity observed in joints in a variety of experimental studies.



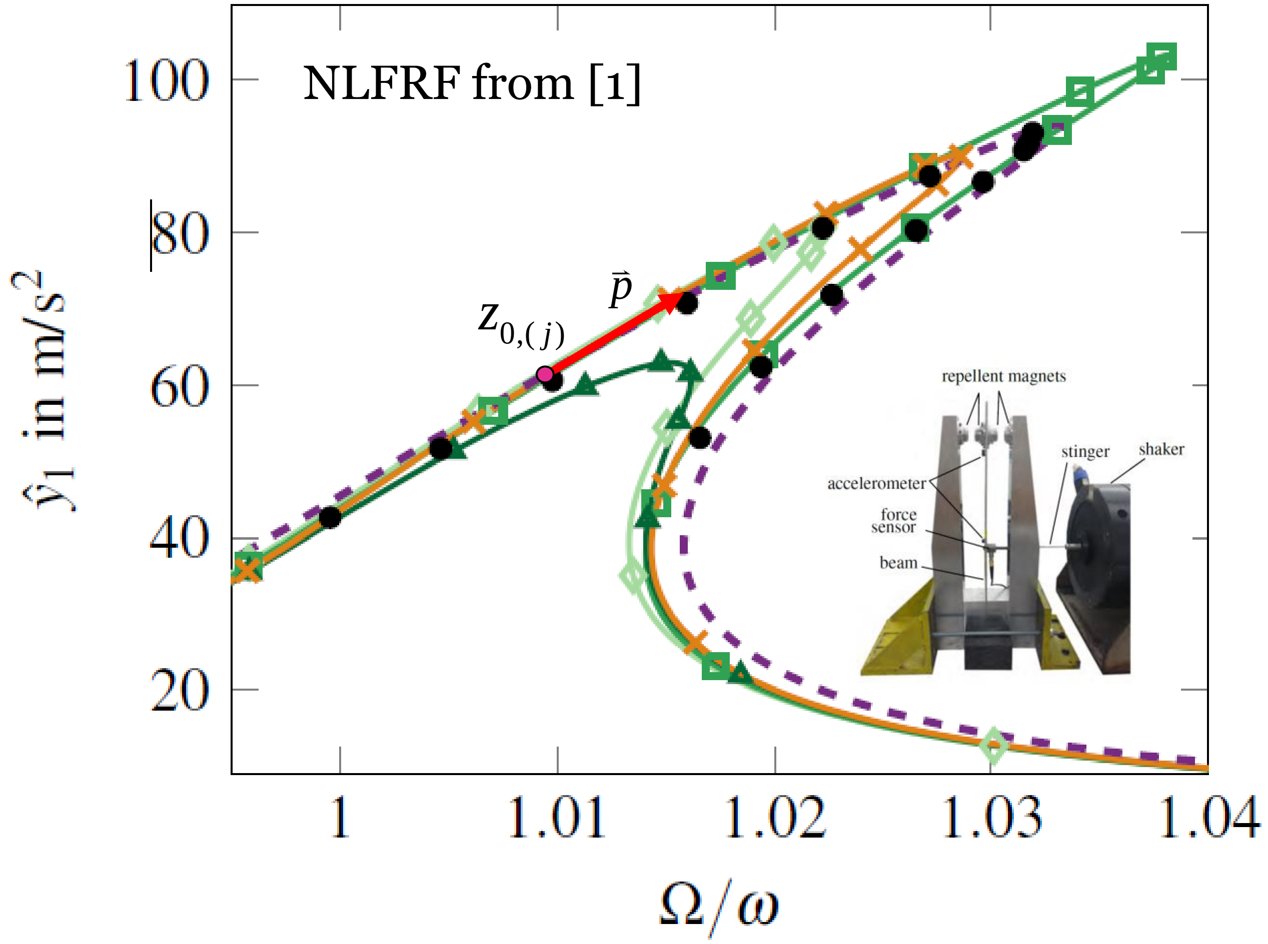






[1] D. J. Segalman et al., "Handbook on Dynamics of Jointed Structures," Sandia National Laboratories, Albuquerque, NM 87185, 2009.
[2] B. Deaner et al., "Application of Viscous and Iwan Modal Damping Models to Experimental Measurements ...," ASME JVA, vol. 137, p. 12, 2015
[3] D. R. Roettgen and M. S. Allen, "Nonlinear characterization of a bolted, industrial structure using a modal framework," MSSP, vol. 84, pp. 152-170, 2017.

The 4-parameter Iwan joint is straightforward to integrate, but no method exists for computing the nonlinear frequency response.



$$\mathbf{H}(T,\mathbf{z}) = \mathbf{z}(T,\mathbf{z}_0) - \mathbf{z}(0,\mathbf{z}_0) = \mathbf{0}$$

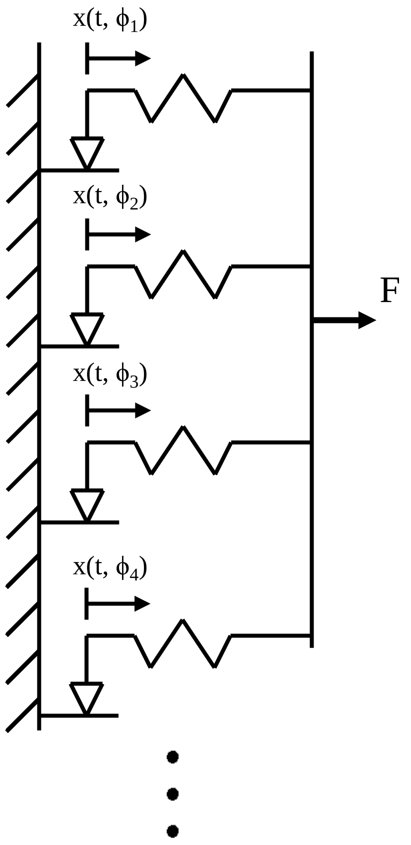
$$\left[\underbrace{\left.\frac{\partial \mathbf{H}}{\partial \mathbf{z}_0}\right|_{\mathbf{z}_0,T}}_{\mathbf{P}_z^T} \quad \underbrace{\left.\frac{\partial \mathbf{H}}{\partial T}\right|_{\mathbf{z}_0,T}}_{P_T}\right] \begin{Bmatrix} \Delta \mathbf{z}_0 \\ \Delta T \end{Bmatrix} = \begin{Bmatrix} -\mathbf{H}(T,\mathbf{z}_0) \\ 0 \end{Bmatrix}$$

$$\left.\frac{\partial \mathbf{H}}{\partial \mathbf{z}_0}\right|_{\mathbf{z}_0,T}$$

?

- Iwan joints modeled with 30-100 sliders
- Discontinuous time histories

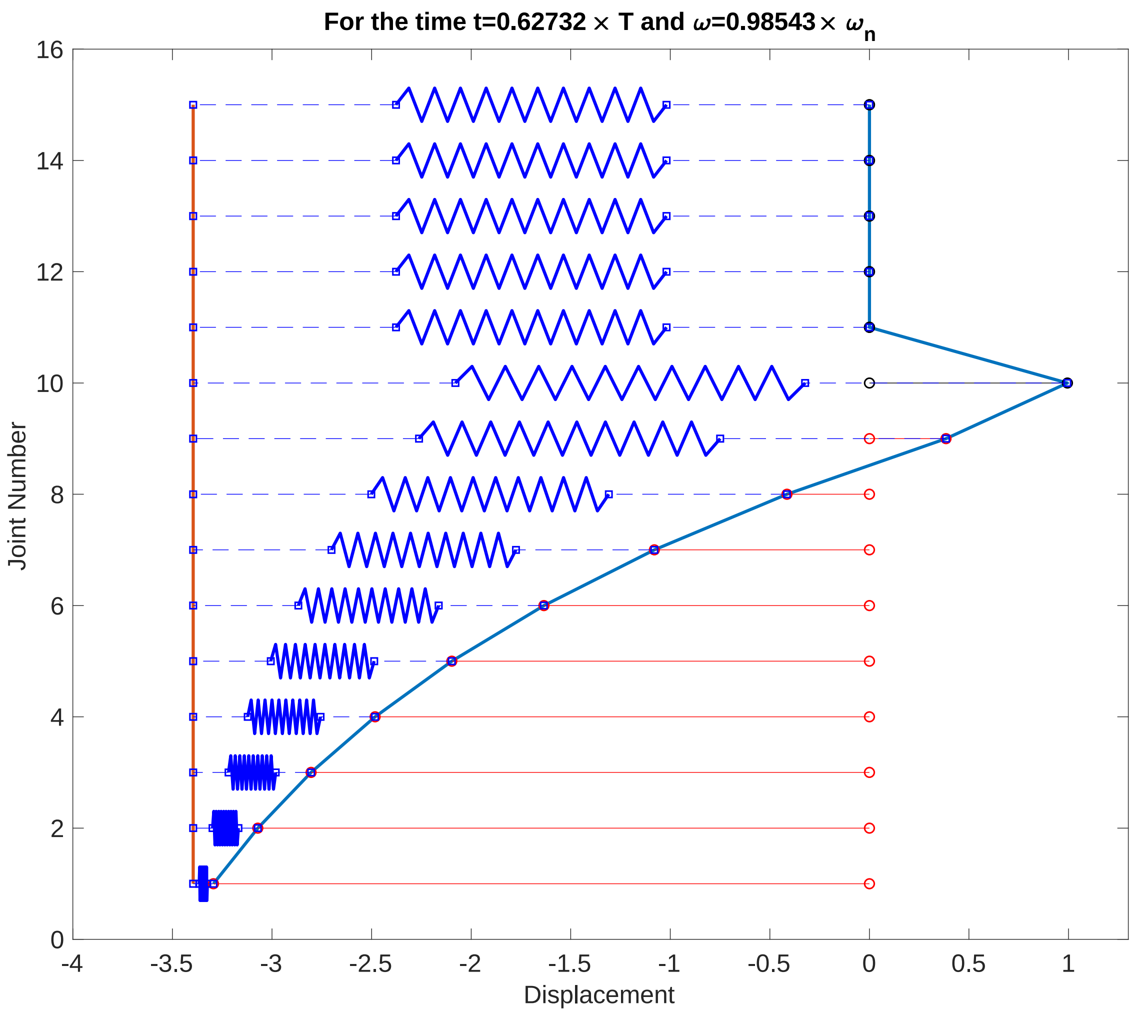
Iwan Model



[1] M. Scheel et al., "Experimental Assessment of Polynomial Nonlinear State-Space and Nonlinear-Mode Models…," MSSP, Submitted 2019.

Outline

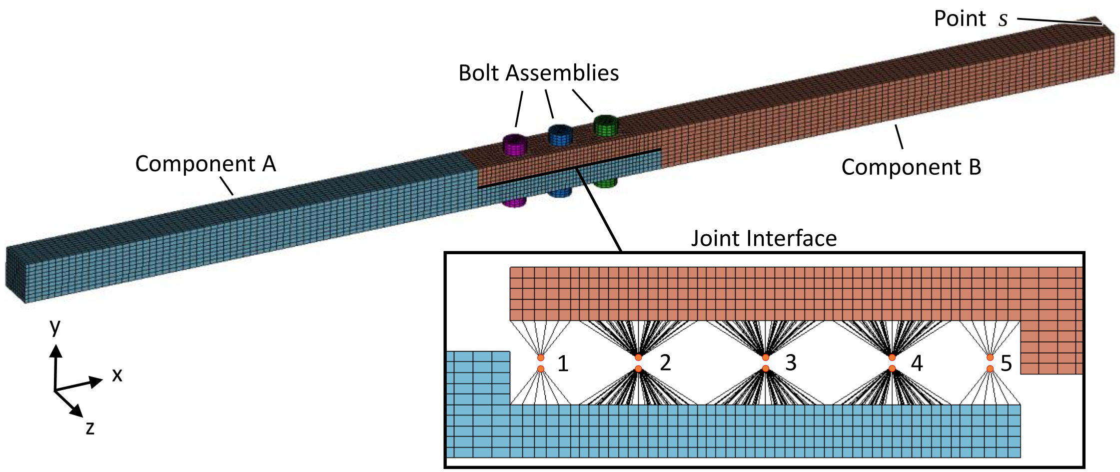
- Computing Iwan slider states from history of reversal points.
- SDOF Case:
  - Reversals / Maximum Displacement
  - Implementation
- Extension to MDOF Systems
- Conclusions

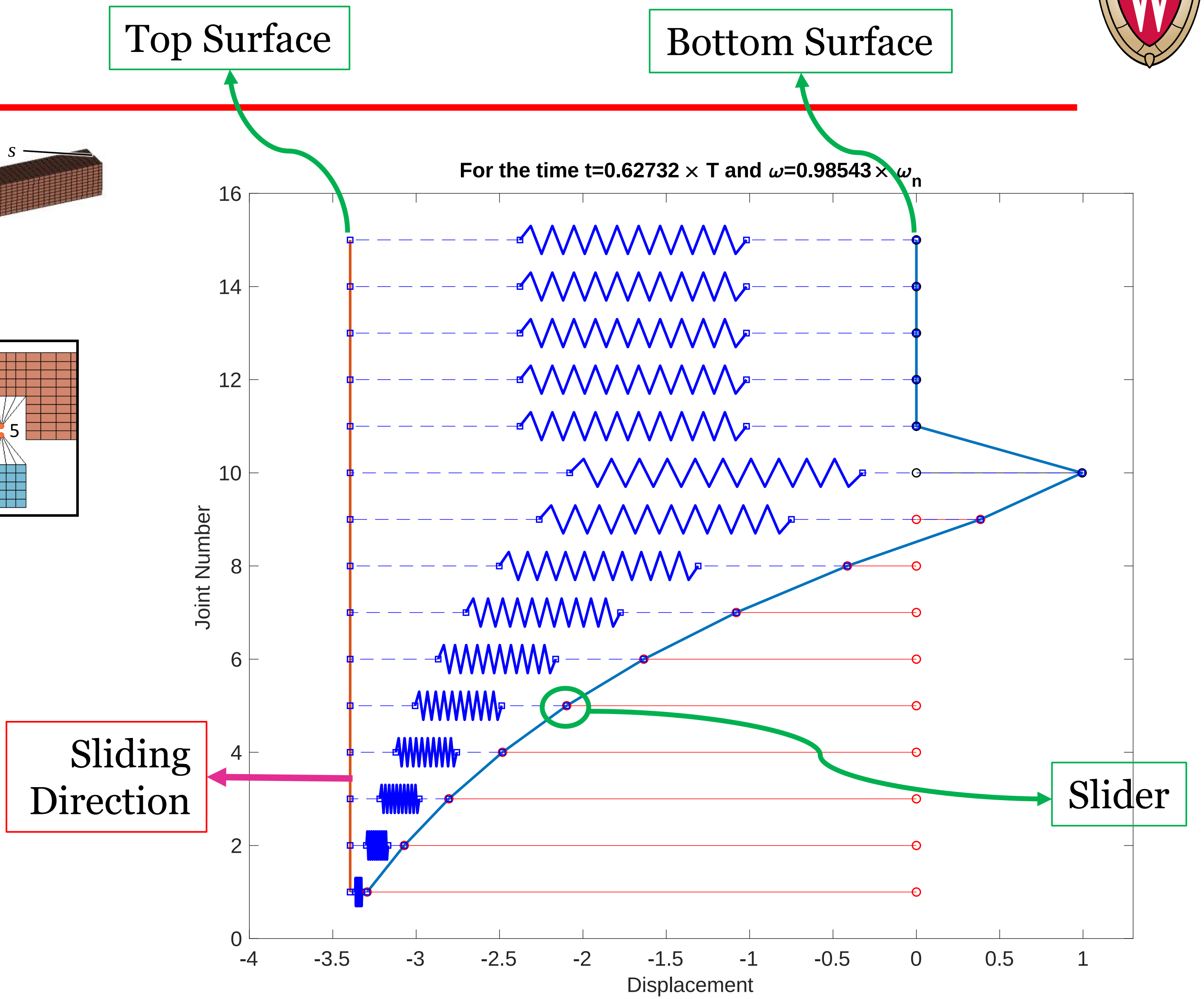


Iwan Joint

Determining the positions of the Iwan sliders from the past history of load reversals

Iwan Joint





- An Iwan element is a one-dimensional, parallel arrangement of Jenkins elements.
- Each Jenkins element consists of a linear spring in series with a friction slider.

Iwan Joint: Nonlinear Force

- Force-deflection relation for an element [1]:

$$f_i = kx; \qquad \dot{x} > 0,\ 0 \le x \le f_i^\star/k$$

$$f_i = f_i^\star; \qquad \dot{x} > 0,\ x \ge f_i^\star/k$$

$$f_i = \left[kx - (kA - f_i^\star)\right]; \qquad \dot{x} > 0,\ A - 2f_i^\star/k \le x \le A$$

$$f_i = -f_i^\star; \qquad \dot{x} < 0,\ x \le A - 2f_i^\star/k$$

- $f^\star$: Strength of the element
- $\rho(f^\star)$: Distribution function
- f_i : Force of the element "i"
- A : Maximum displacement

- Nonlinear force in the joint [1]:

$$f_{nl} = \int_0^{kx} f^\star \rho(f^\star)df^\star + kx\int_{kx}^{\infty} \rho(f^\star)df^\star$$

[1] Iwan, Wilfred D. "A distributed-element model for hysteresis and its steady-state dynamic response." (1966): 893-900.

Iwan Joint: Distribution function

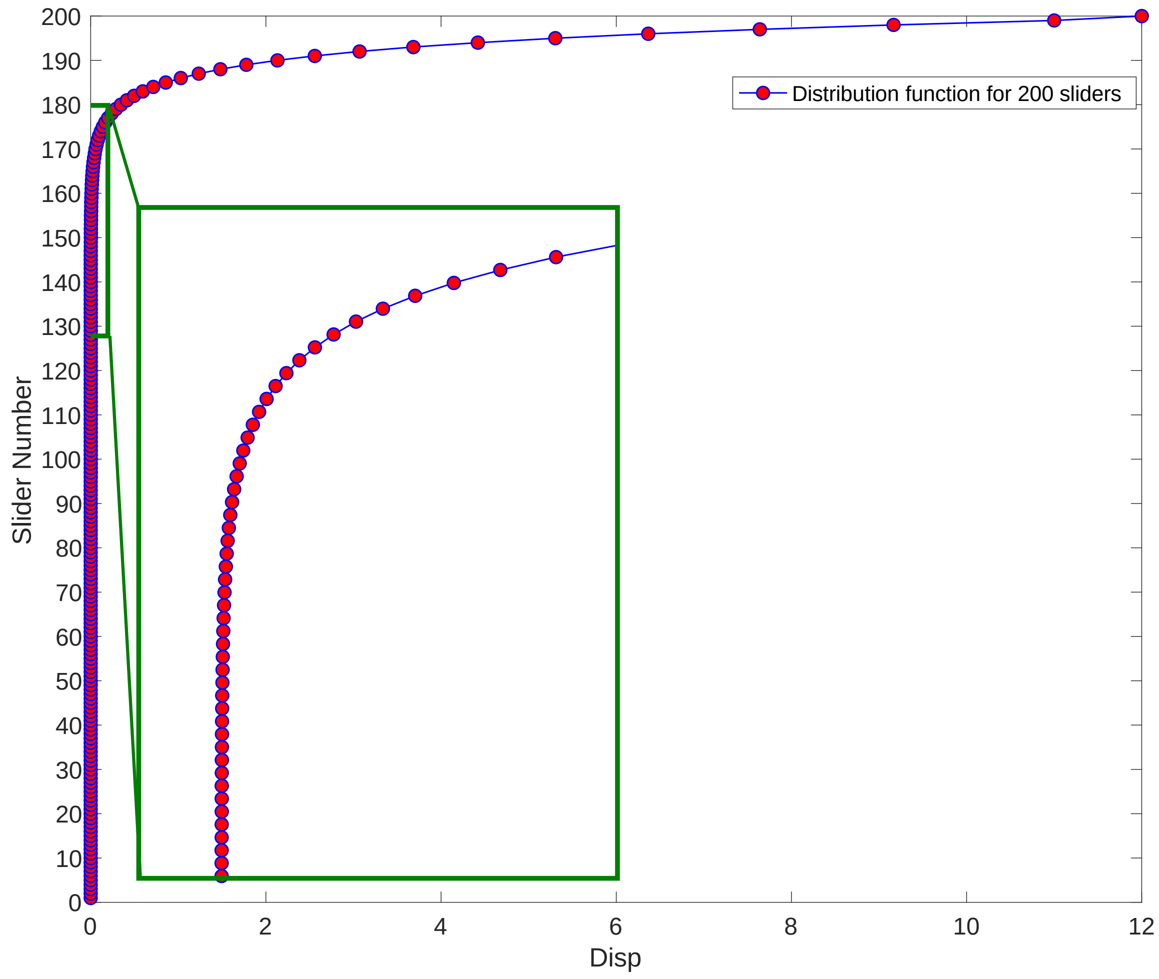
- The maximum extension of the ith spring can be defined as [1]:

$$\phi_i = f_i^\star / k$$

- Distribution function defined based on the power-law distribution [1]:

$$\rho(\phi) = R\phi^\chi \left[H(\phi) - H(\phi - \phi_{\max})\right] + S\delta(\phi - \phi_{\max})$$

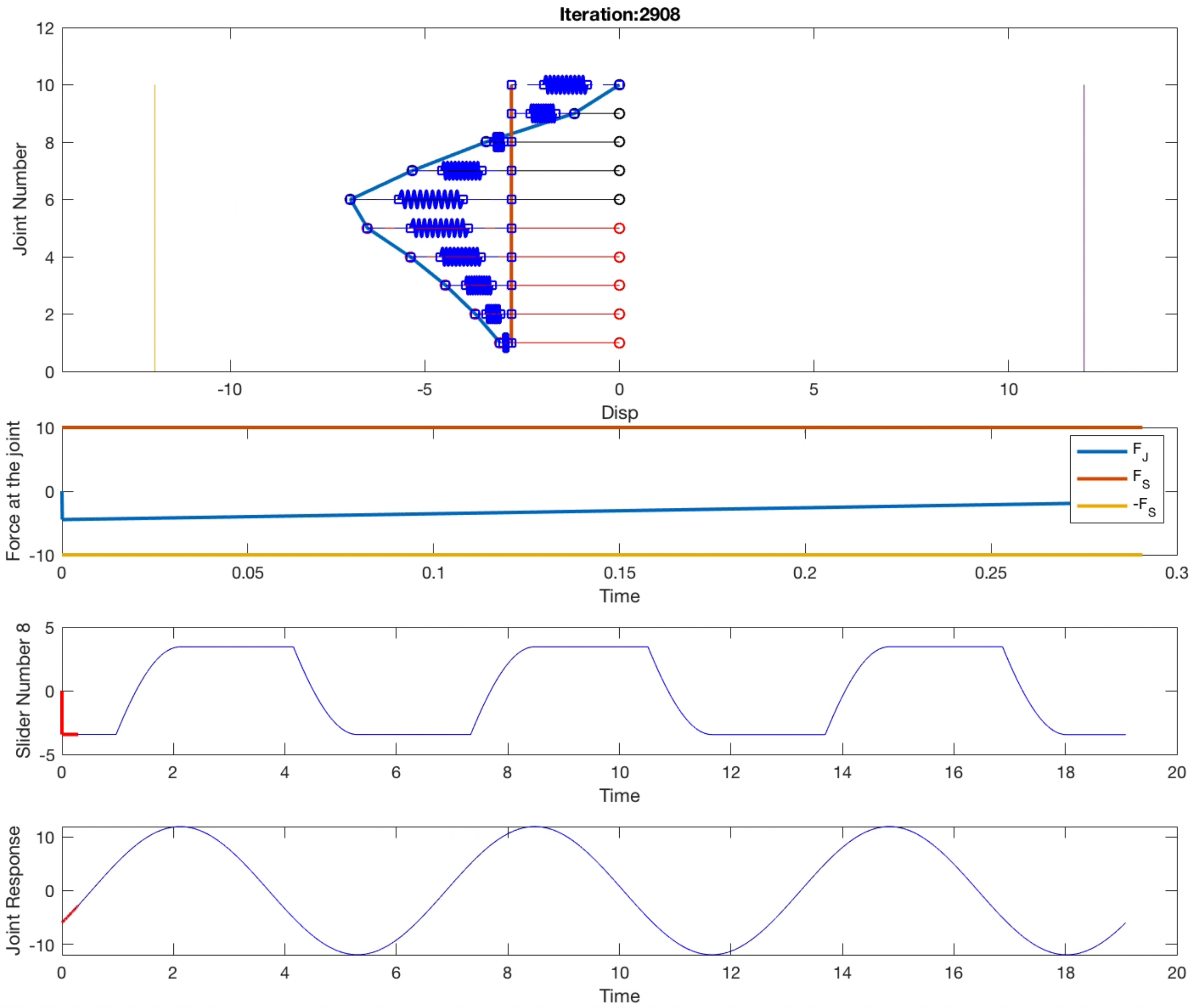
- The nonlinear force depends on the state of the sliders.



[1] Segalman, Daniel J. "A four-parameter Iwan model for lap-type joints." Journal of Applied Mechanics 72.5 (2005): 752-760.

Visualizing the State of the Sliders

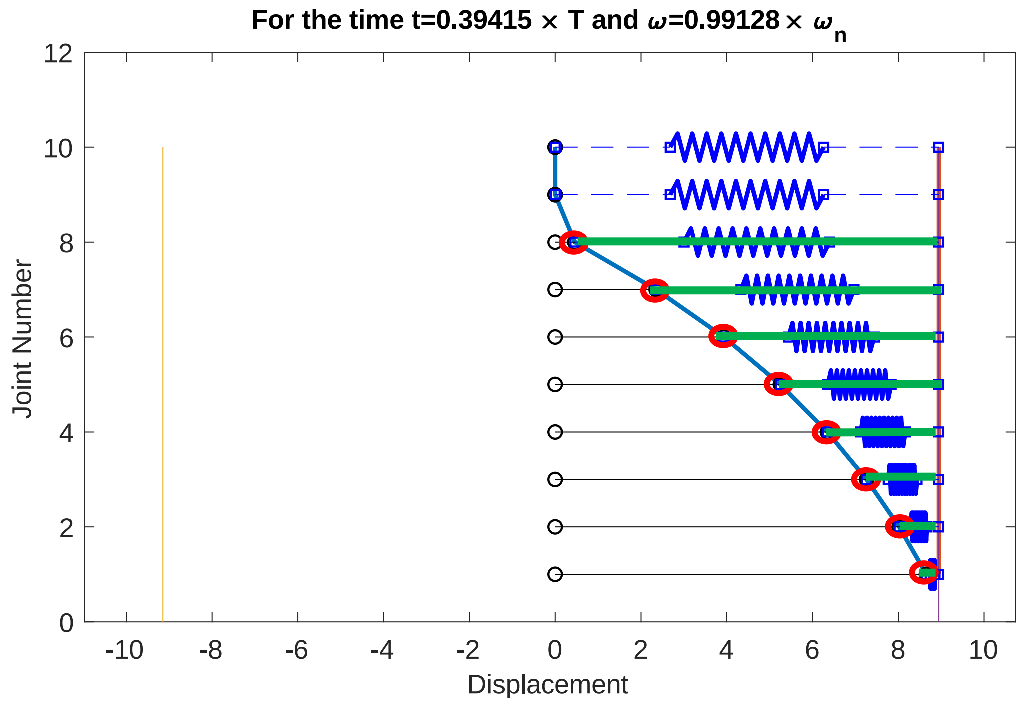
- The sliders don't have any internal dynamics.
- If the response is quasi-periodic, then the state of all of the sliders can be determined from x_{max} : the reversals (or the maximum displacement in past history).



Reversal Points / Maximum Displacement

- The state of sliders can be specified:
    - Some of them are at $x_{max} - \phi_i$
        - ✓ Green lines show ϕ_i
    - Some of them are at Zero. (Never slipped)
- Reversal points can be used as a *benchmark.*

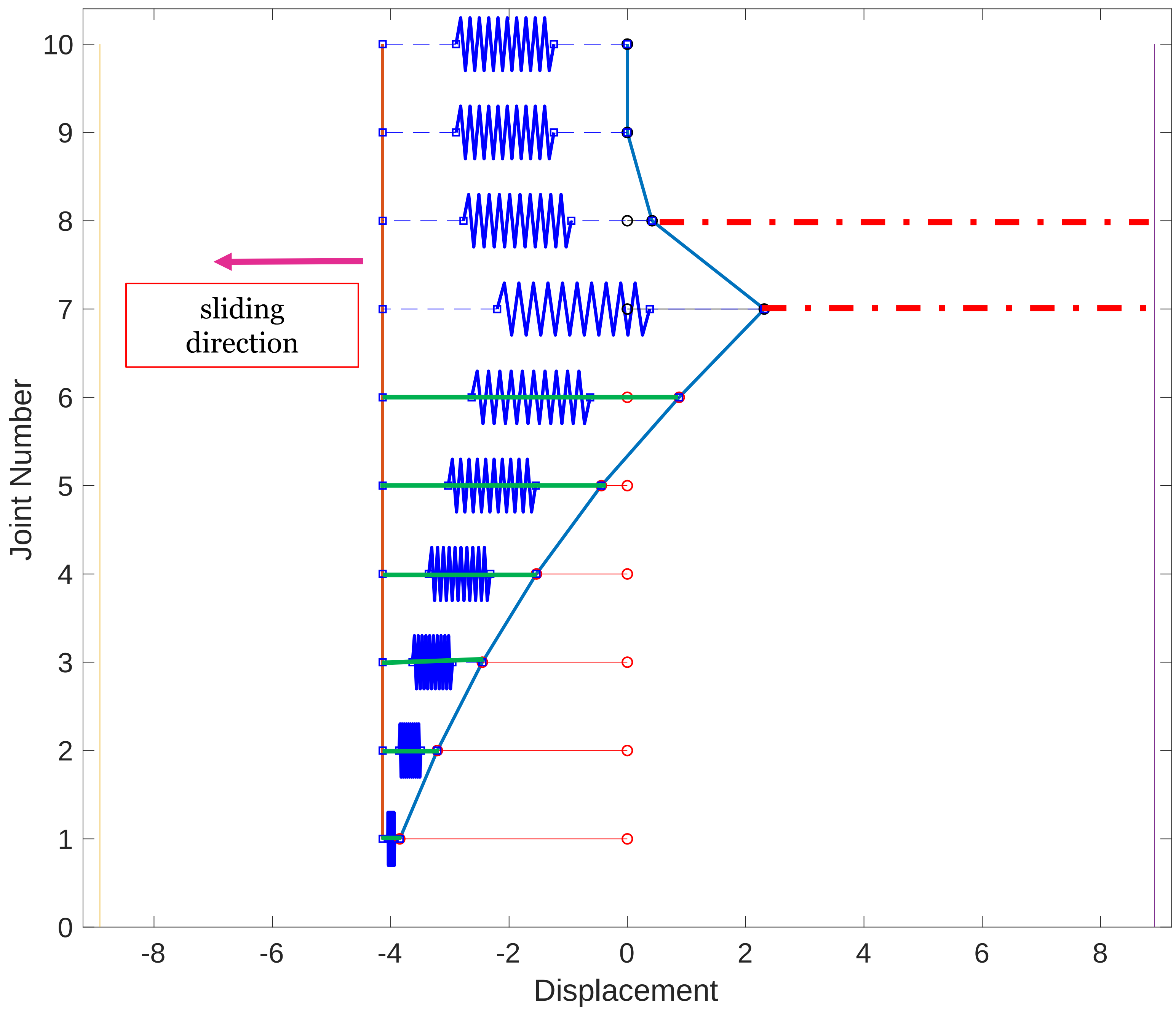
- One moment after max:



Joint is at a reversal point: $\dot{x} = 0$

State of Sliders

- Sliders #1~6 are sliding (red)
- Sliders #1~6 are following the joint by the distance of ϕ_i (Green lines).
- Slider #7 and #8 are still stuck at $x_{max} - \phi_i$
- Sliders #9 and 10 never slipped.



Knowing this, we can use the relations defining the Iwan joint to compute the force at any instant.

- Force-deflection relation for an element [1]:

$$f_i = kx; \qquad \dot{x} > 0,\ 0 \leq x \leq f_i^\star/k$$

$$f_i = f_i^\star; \qquad \dot{x} > 0,\ x \geq f_i^\star/k$$

$$f_i = \left[kx - (kA - f_i^\star)\right]; \qquad \dot{x} > 0,\ A - 2f_i^\star/k \leq x \leq A$$

$$f_i = -f_i^\star; \qquad \dot{x} < 0,\ x \leq A - 2f_i^\star/k$$

$f^\star$: Strength of the element

$\rho(f^\star)$: Distribution function

f_i : Force of the element "i"

A : Maximum displacement

- Sum over the joints to compute the total force
- Total stiffness is the sum of the stiffnesses of the stuck springs.

[1] Iwan, Wilfred D. "A distributed-element model for hysteresis and its steady-state dynamic response." (1966): 893-900.

Modified Continuation Procedure

Modified Continuation Procedure

- One additional state variable is added to the state vector:

$$z_0 = \begin{bmatrix} x_0 \\ \dot{x}_0 \\ x_{max} \end{bmatrix}$$

- Shooting function:

$$H_{z_0}(z_0, t, T, x_{max}) = z_T(z_0, t, T, x_{max}) - z_0$$

$$H_{x_{max}}(z_0, t, T, x_{max}) = max(x(t))_{0 \leq t \leq T} - x_{max_0}$$

- Convergence criteria:

$$\frac{||H_{z_0}(z_0, T, x_{max})||}{||z_0||} < \epsilon$$

$$\frac{||H_{x_{max}}(z_0, T, x_{max})||}{||x_{max}||} < \epsilon$$

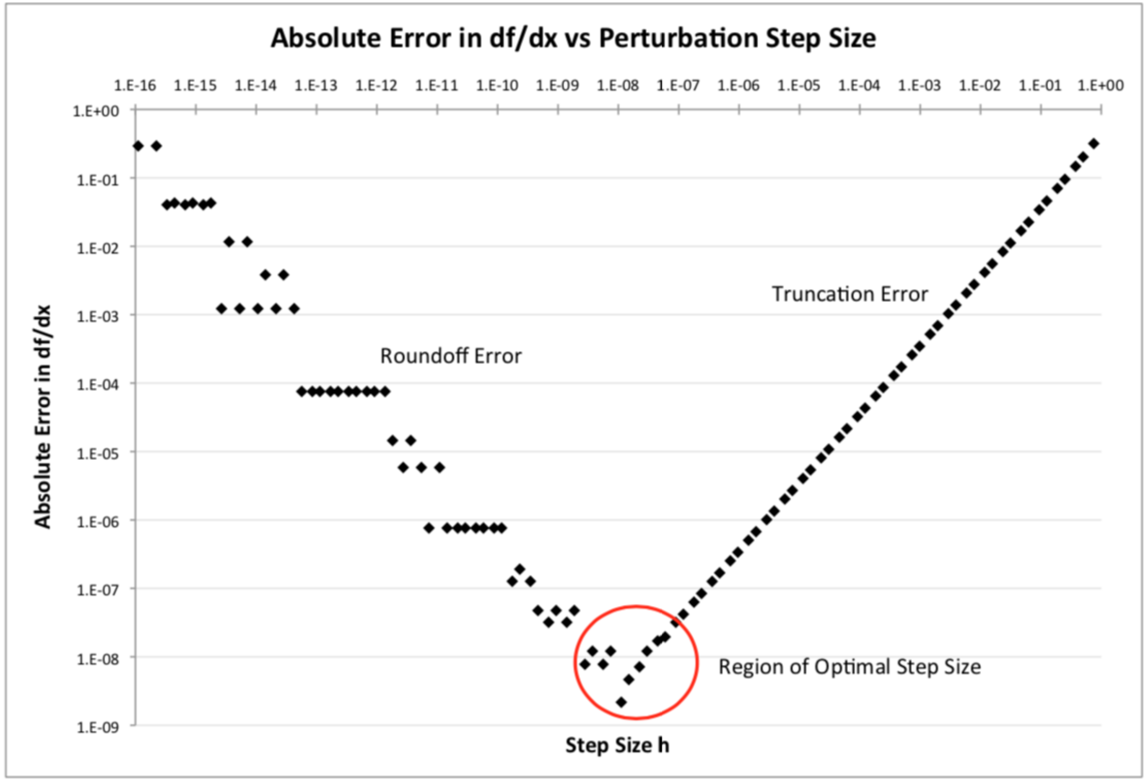
The Jacobians needed for the continuation algorithm are computed using finite differences.

- Correction (Newton-Raphson):

$$\begin{bmatrix} \frac{\partial x}{\partial x_0} - 1 & \frac{\partial x}{\partial \dot{x}_0} & \frac{\partial x}{\partial x_{max}} & \frac{\partial x}{\partial T} \\ \frac{\partial \dot{x}}{\partial x_0} & \frac{\partial \dot{x}}{\partial \dot{x}_0} - 1 & \frac{\partial \dot{x}}{\partial x_{max}} & \frac{\partial \dot{x}}{\partial T} \\ \frac{\partial x_{max}}{\partial x_0} & \frac{\partial x_{max}}{\partial \dot{x}_0} & \frac{\partial x_{max}}{\partial x_{max}} - 1 & \frac{\partial x_{max}}{\partial T} \end{bmatrix} \begin{bmatrix} \Delta x_0 \\ \Delta \dot{x}_0 \\ \Delta x_{max} \\ \Delta T \end{bmatrix} = \begin{bmatrix} x_0(T, x_0, \dot{x}_0, x_{max}) - x_0 \\ \dot{x}_0(T, x_0, \dot{x}_0, x_{max}) - \dot{x}_0 \\ x_{max}(T, x_0, \dot{x}_0, x_{max}) - x_{max} \end{bmatrix}$$

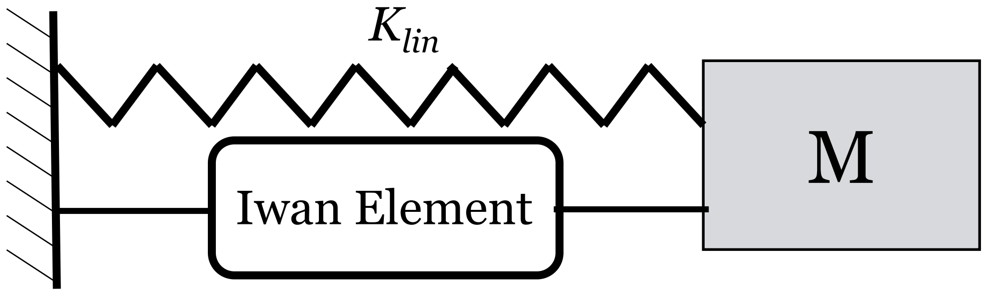
$$\begin{cases} z_{0,(j+1)}^{(k+1)} = \Delta z_{0,(j+1)}^{(k)} + z_{0,(j+1)}^{(k)} \\ x_{max_{j+1}}^{k+1} = \Delta x_{max_{j+1}}^{k} + x_{max_{j+1}}^{k} \\ T_{0,(j+1)}^{(k+1)} = \Delta T_{0,(j+1)}^{(k)} + T_{0,(j+1)}^{(k)} \end{cases}$$

- The step-size dilemma for sin(x) [1]:



[1] Mathur, Ravishankar. *An analytical approach to computing step sizes for finite-difference derivatives*. Diss. 2012.

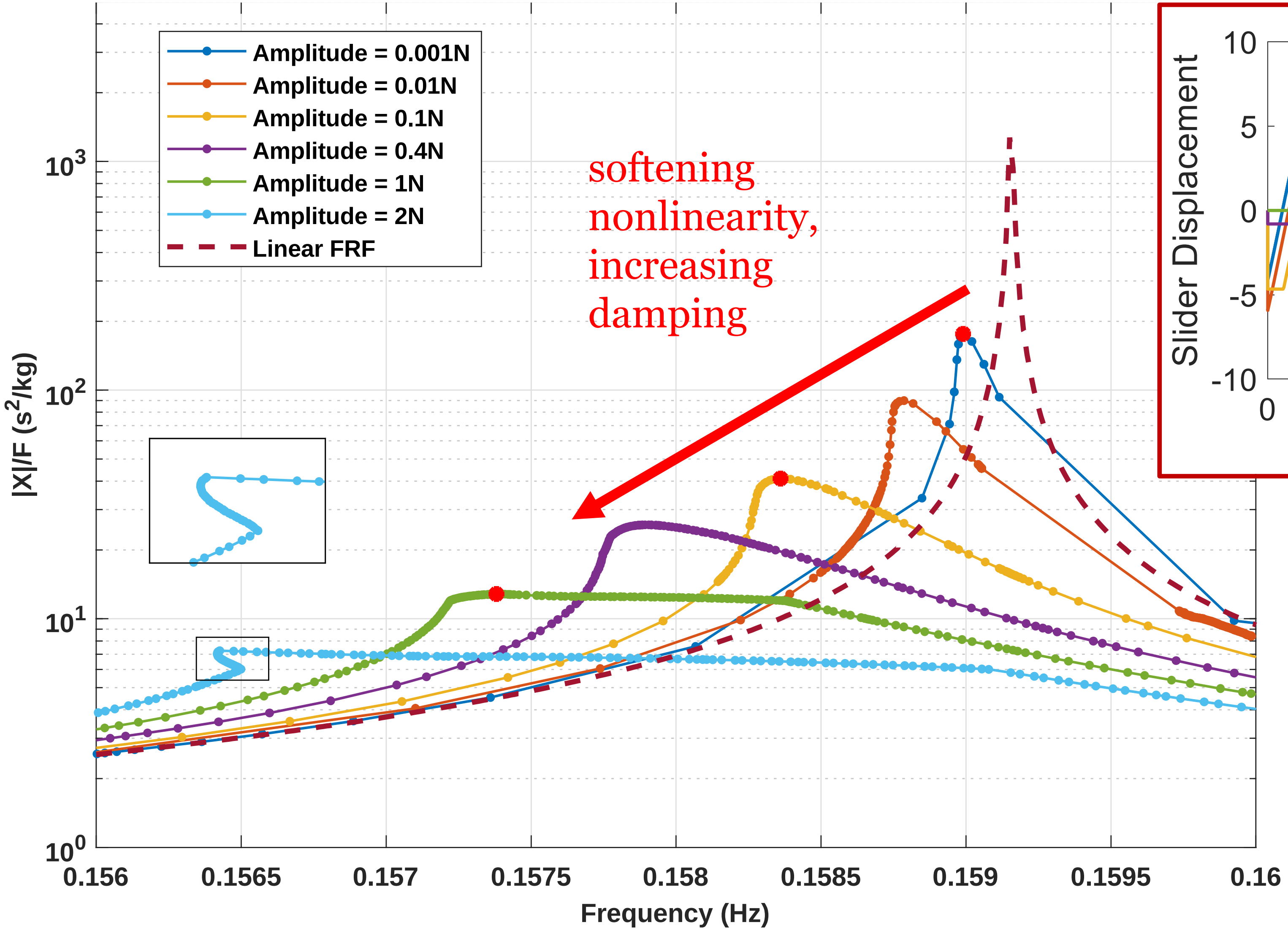
Application to an SDOF System

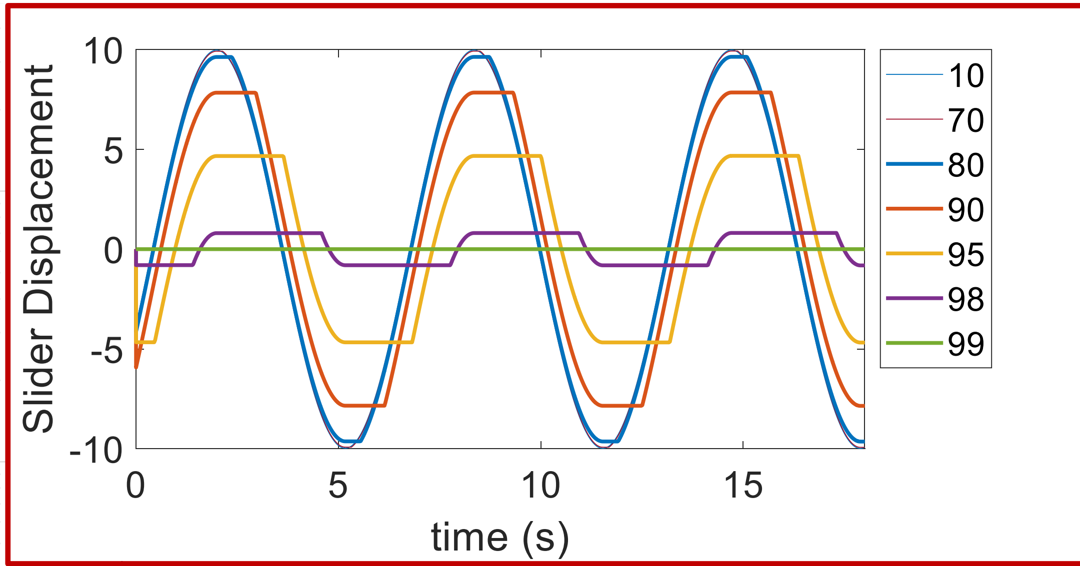


Parameter	Value
$K_{lin}(N/m)$	9
ζ_{lin}	2.78×10^{-5}
$M(kg)$	10
$F_s(N)$	10.0
$K_t(N/m)$	1
χ	-0.5
β	3.0

- Continuation state augmented with $x_{\max}$ (only one reversal considered)
- 100 sliders used to model the Iwan element

SDOF System NLFRFs

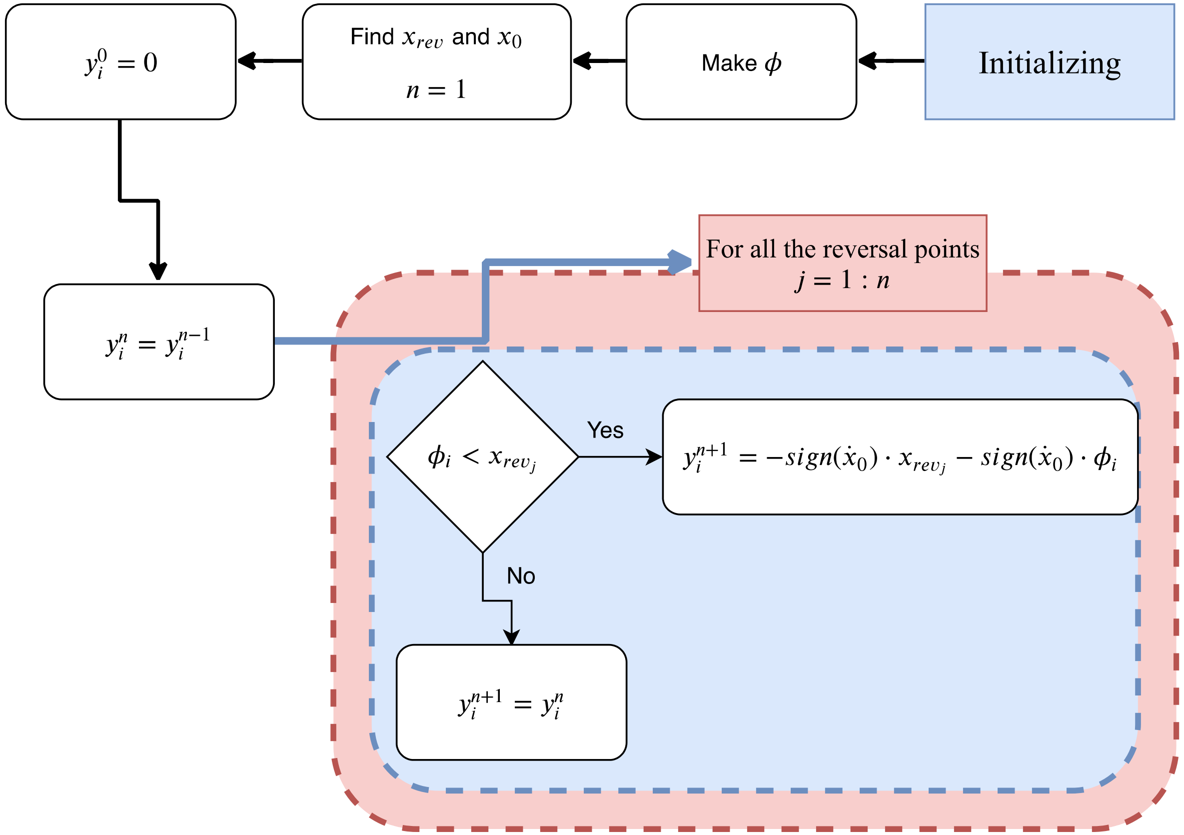


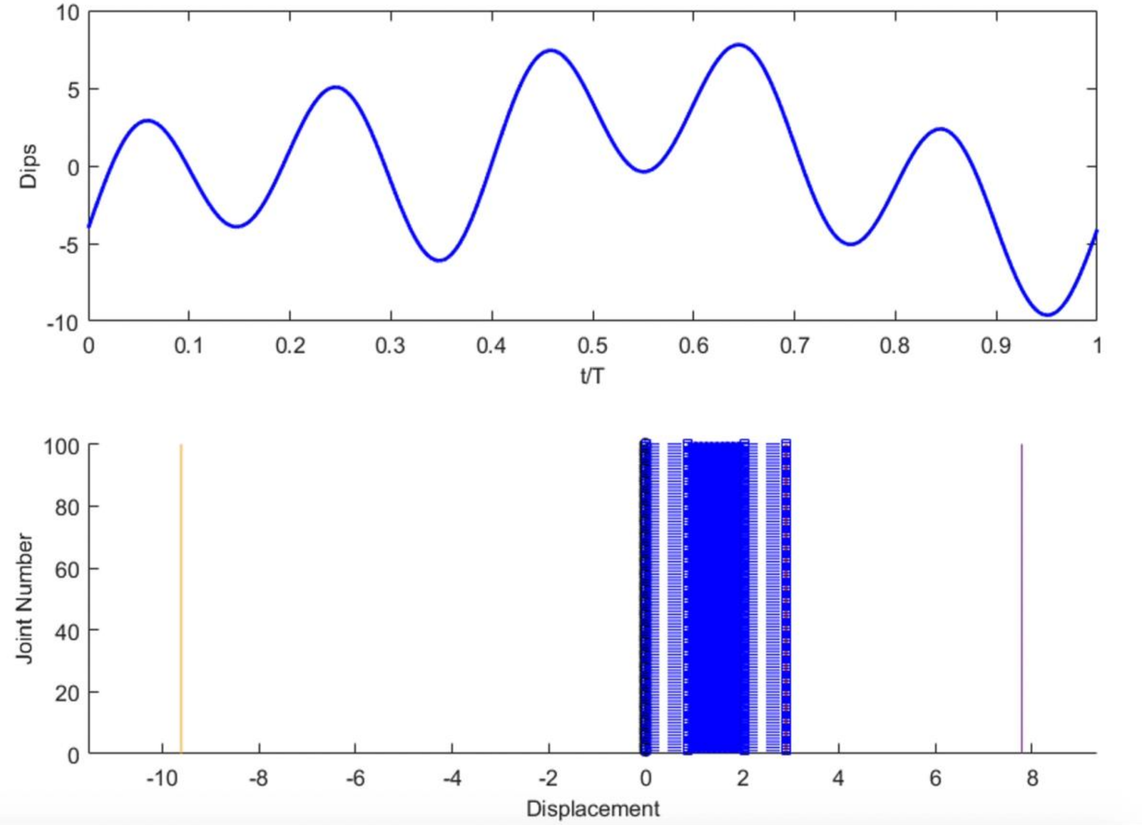


Slider displacements for Amp=1 N show complicated, nonlinear discontinuous response.

Extension for MDOF Systems

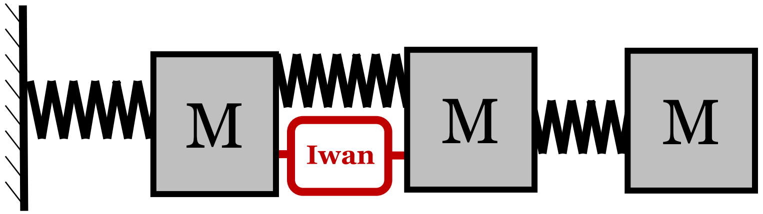
MDOF Algorithm





- Algorithm is similar, but there may be many reversals to keep track of.

Results for a 3-DOF system with an Iwan Element for different force levels



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$K_t(N/m)$	1
χ	-0.5
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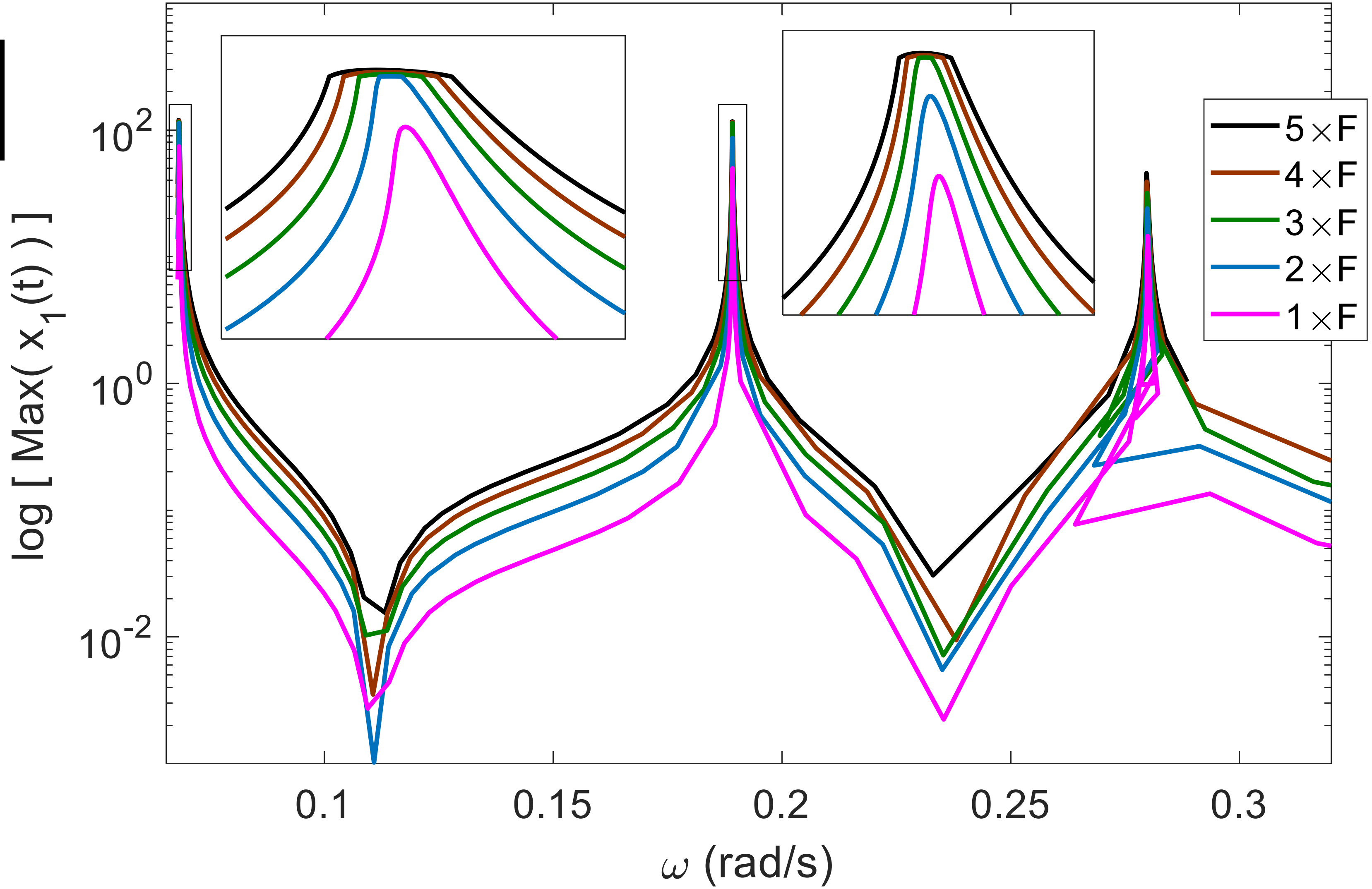

Conclusions

- While Iwan joints are complicated, the state of any Iwan joint can be determined algebraically if the displacement is known at enough reversal points.
- The Nonlinear Frequency Response (NLFRFs) can then be computed using continuation.
- With further development, this could be an effective tool for simulation or model updating using measurements.

Options for Computing NLFRFs

Time Integrate until Steady-State	~100 hours
Shooting & Continuation (this talk)	10-15 minutes
Harmonic Balance (ongoing work)	~2-5 minutes

(computation times for SDOF system)

Acknowledgements

Iman Zare

- Iman Zare (who did ~~most~~ all of the work!)
- The National Science Foundation

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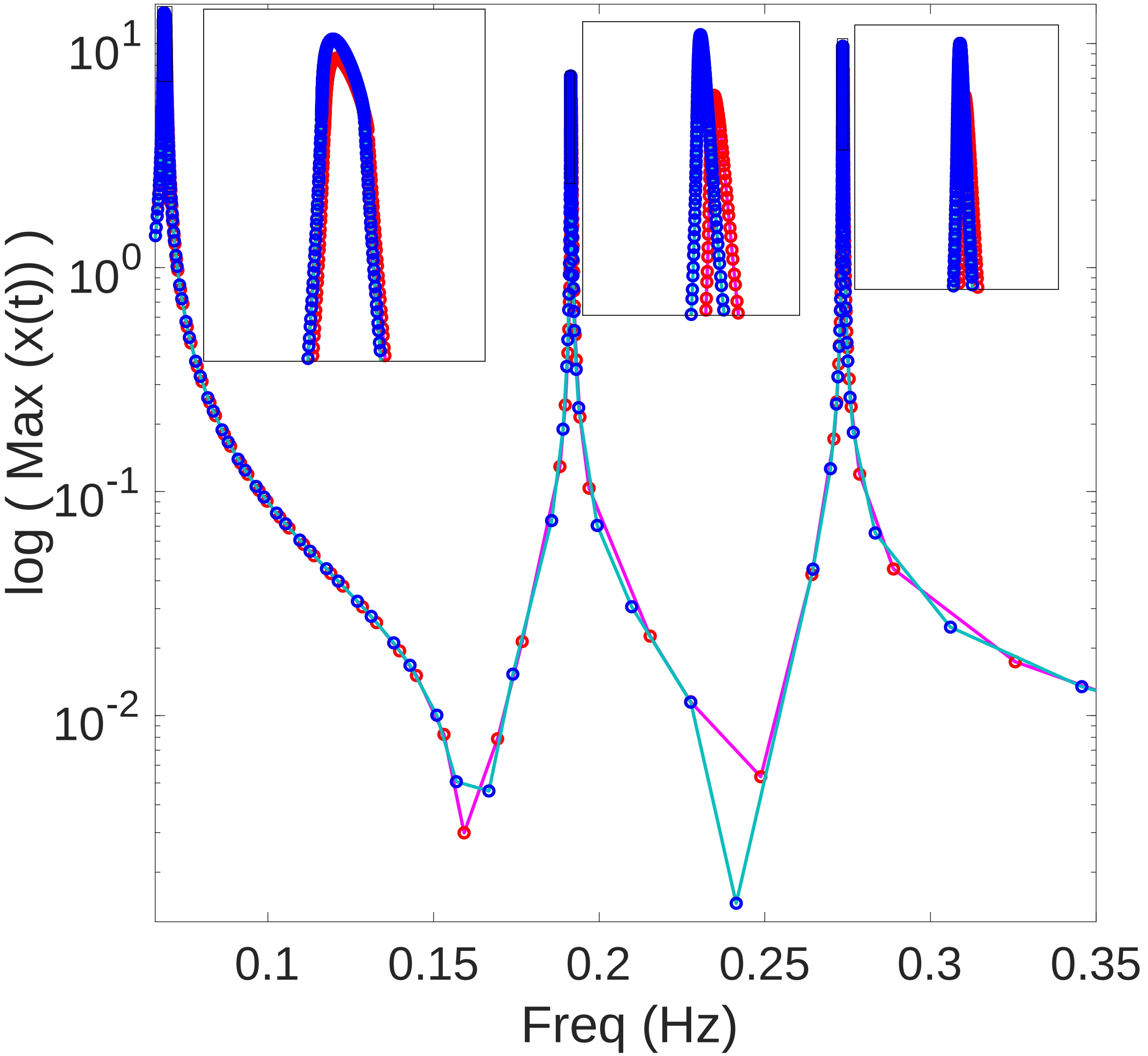
Any Questions?

Results for a 3-DOF system with 3 Iwan Elements for different nonlinearities

Blue: $\chi = -0.5$

Red: $\chi = -0.8$

$$F = \begin{bmatrix} 0.4 \\ 0.3 \\ 0.5 \end{bmatrix}$$



Iwan FRF

- A common numerical model for bolted or riveted joints is the Iwan model.
- In many applications it is desirable to predict the nonlinear Frequency Response Functions (FRFs) of a structure that contains joints.
- The steady-state response must be estimated over a range of frequencies.

$$m\ddot{x} + C_{lin}\dot{x} + K_{lin}x + f_{nl}(x, t, y(t, \phi)) = F_{ext}$$

Continuation Method

A numerical method to obtain the FRF

Continuation Procedure

- Numerical continuation can be used to obtain the FRFs of a nonlinear system.
- For a response to be considered steady-state, the displacement and velocity must be periodic.

$$H(z_0, T) = z_T(z_0, T) - z_0$$

$$\frac{||H(z_0, T)||}{||z_0||} < \epsilon$$

$$\begin{cases} z_0 = \begin{bmatrix} x_0 \\ \dot{x}_0 \end{bmatrix} \\ z_T = z(z_0, t = T) \end{cases}$$

Numerical Integration

- The differential equation is solved using implicit integration methods: Newmark-Beta.

- The state variables are sent to the integration function as the initial conditions.

- All the history dependent variables should be specified as the initial conditions to start the integration.

Continuation Procedure

- Prediction (pseudo-arclength continuation):

$$\begin{bmatrix} \frac{\partial H}{\partial z_0}\big|_{z_{0,(j)},T_{(j)}} & \frac{\partial H}{\partial T}\big|_{z_{0,(j)},T_{(j)}} \end{bmatrix} \{P_{(j)}\} = \begin{Bmatrix} \{0\} \\ 0 \end{Bmatrix}$$

$$\{P_{(j)}\} = [P_{s,(j)}^T \quad P_{T,(j)}]^T$$

$$\begin{cases} z_{0,pr,(j+1)} = z_{0,j} + s_j P_{z,(j)} \\ T_{pr,(j+1)} = T_j + s_j P_{T,(j)} \end{cases}$$

- Correction (Newton-Raphson):

$$\begin{bmatrix} \frac{\partial H}{\partial z_0}\big|_{z_{0,(j)},T_{(j)}} & \frac{\partial H}{\partial T}\big|_{z_{0,(j)},T_{(j)}} \\ P_{s,(j)}^T & P_{T,(j)} \end{bmatrix} \begin{bmatrix} \triangle z_{0,(j+1)}^{(k)} \\ \triangle T_{(j+1)}^{(k)} \end{bmatrix} = \begin{Bmatrix} -H(z_{0,(j)}, T_{(j)}) \\ 0 \\ 0 \end{Bmatrix}$$

$$\begin{cases} z_{0,(j+1)}^{(k+1)} = \triangle z_{0,(j+1)}^{(k)} + z_{0,(j+1)}^{(k)} \\ T_{0,(j+1)}^{(k+1)} = \triangle T_{0,(j+1)}^{(k)} + T_{0,(j+1)}^{(k)} \end{cases}$$

Iwan FRF: Challenges

The implicit nature of the state variables makes it non-trivial to use continuation to compute the frequency response using already established techniques such as the shooting method.

State of sliders

- It specifies if a slider is slipping or not:

$$\begin{cases} \Delta y_i = x_i - y_{i-1} < \phi_i \Rightarrow \ y_i = y_{i-1} \\ \Delta y_i = x_i - y_{i-1} \geq \phi_i \Rightarrow \ y_i = x_i - \phi_i \end{cases}$$

The non-linear force of Iwan joint is *history dependent*.

- For the case of steady-state response, the state of sliders should be periodic.

$$Y(t) = \begin{bmatrix} y_1(t) & y_2(t) & \dots & y_n(t) \end{bmatrix}^T$$

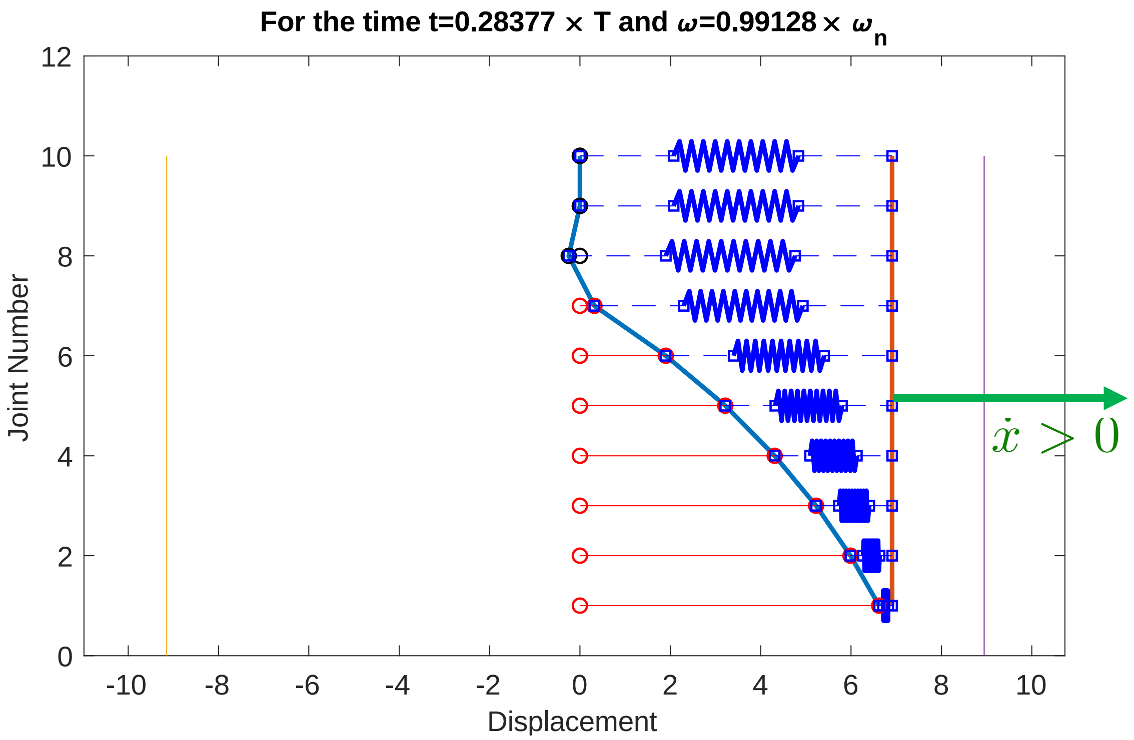
$$Y(t) = Y(t + nT)$$

- The state of sliders should be considered as the initial condition as well.

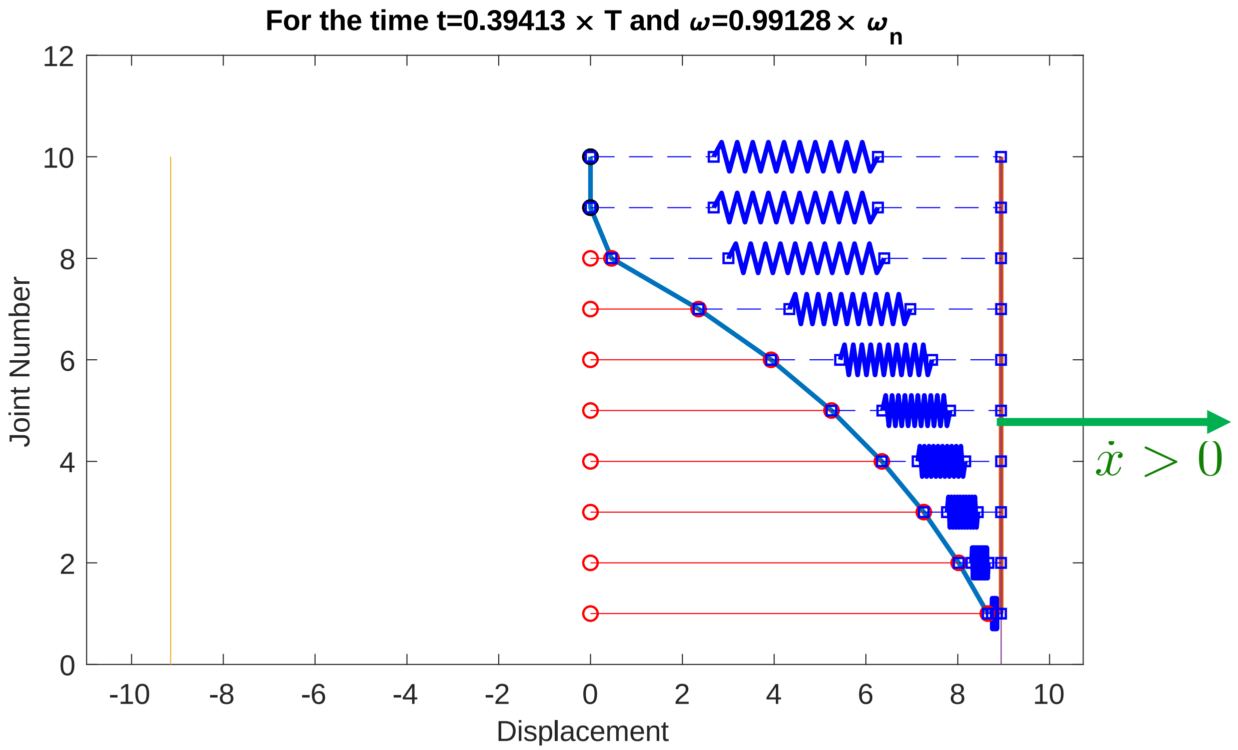
$$Y(0) = Y(0 + nT)$$

Reversal Points

- Before max:

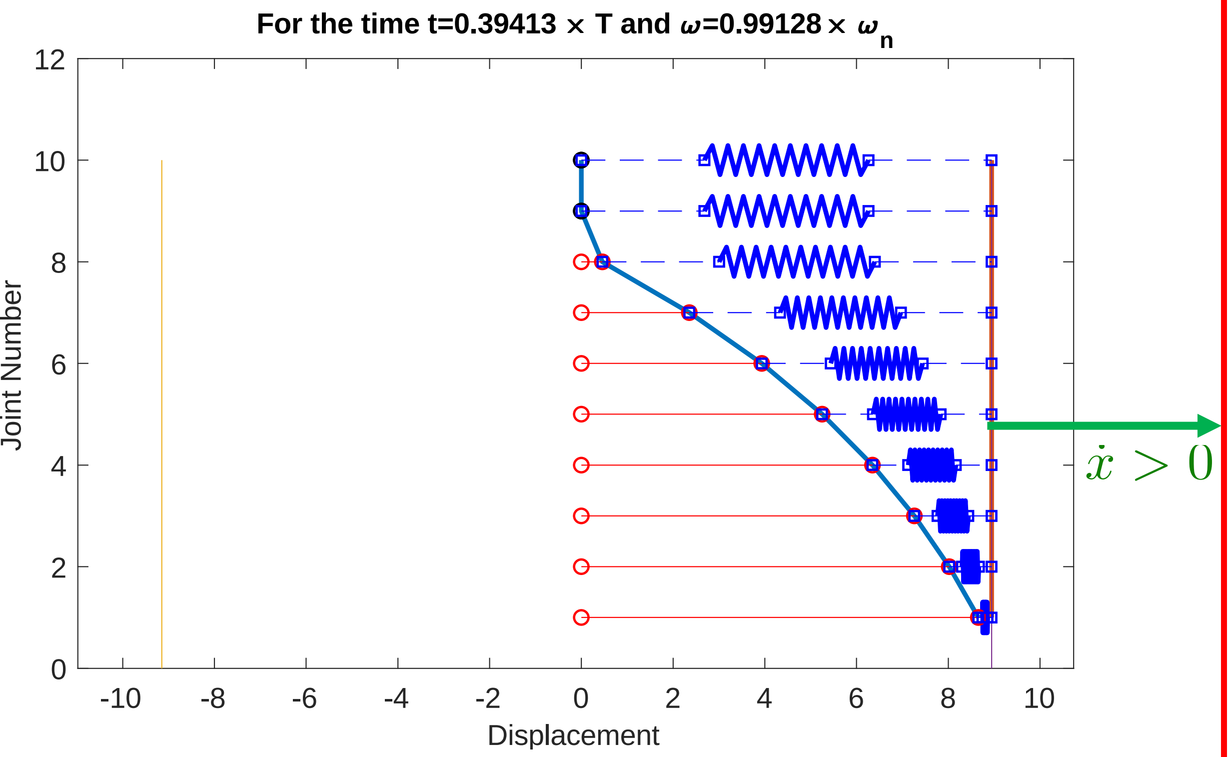


- At max:

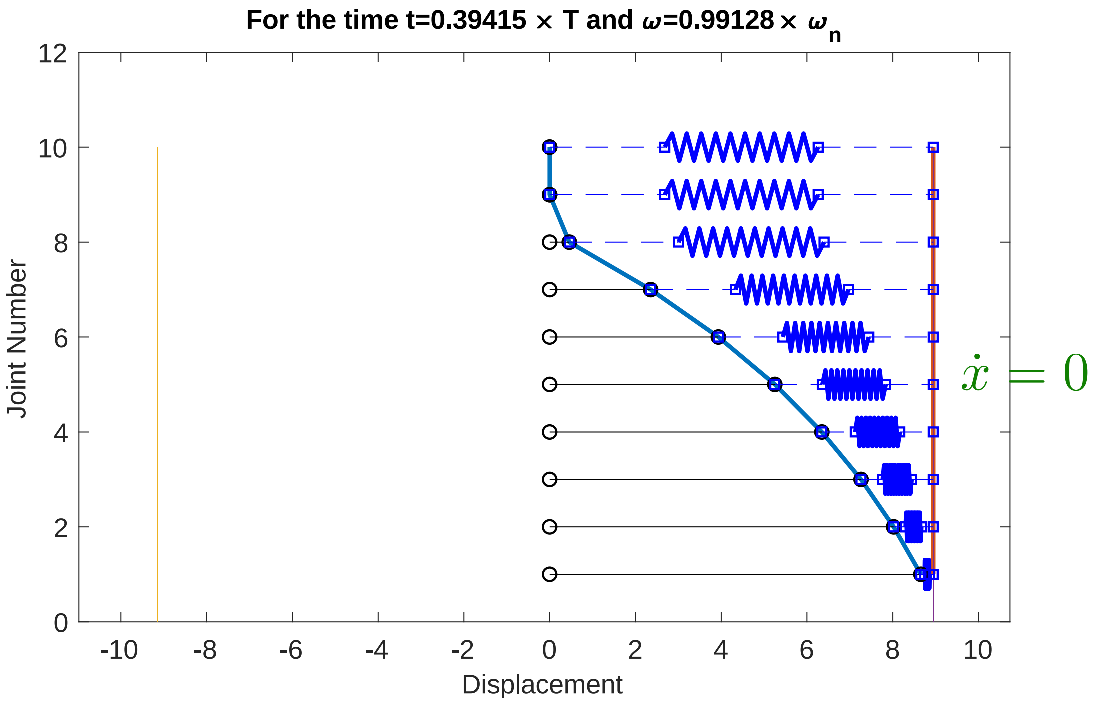


Reversal Points

- At max:



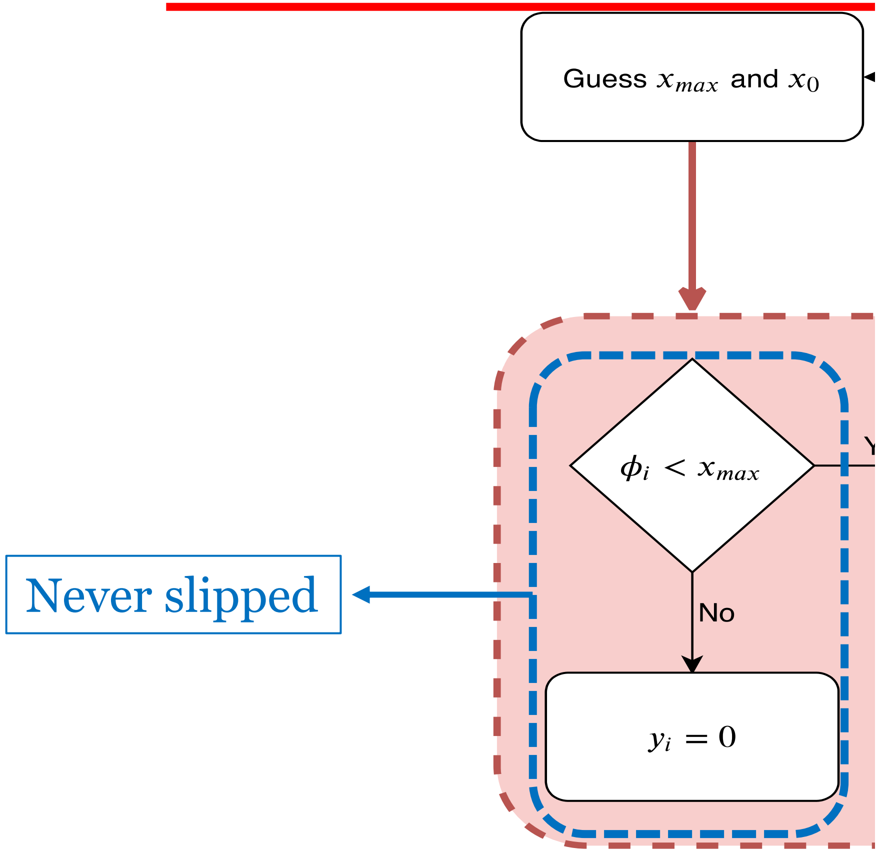
- One moment after max:

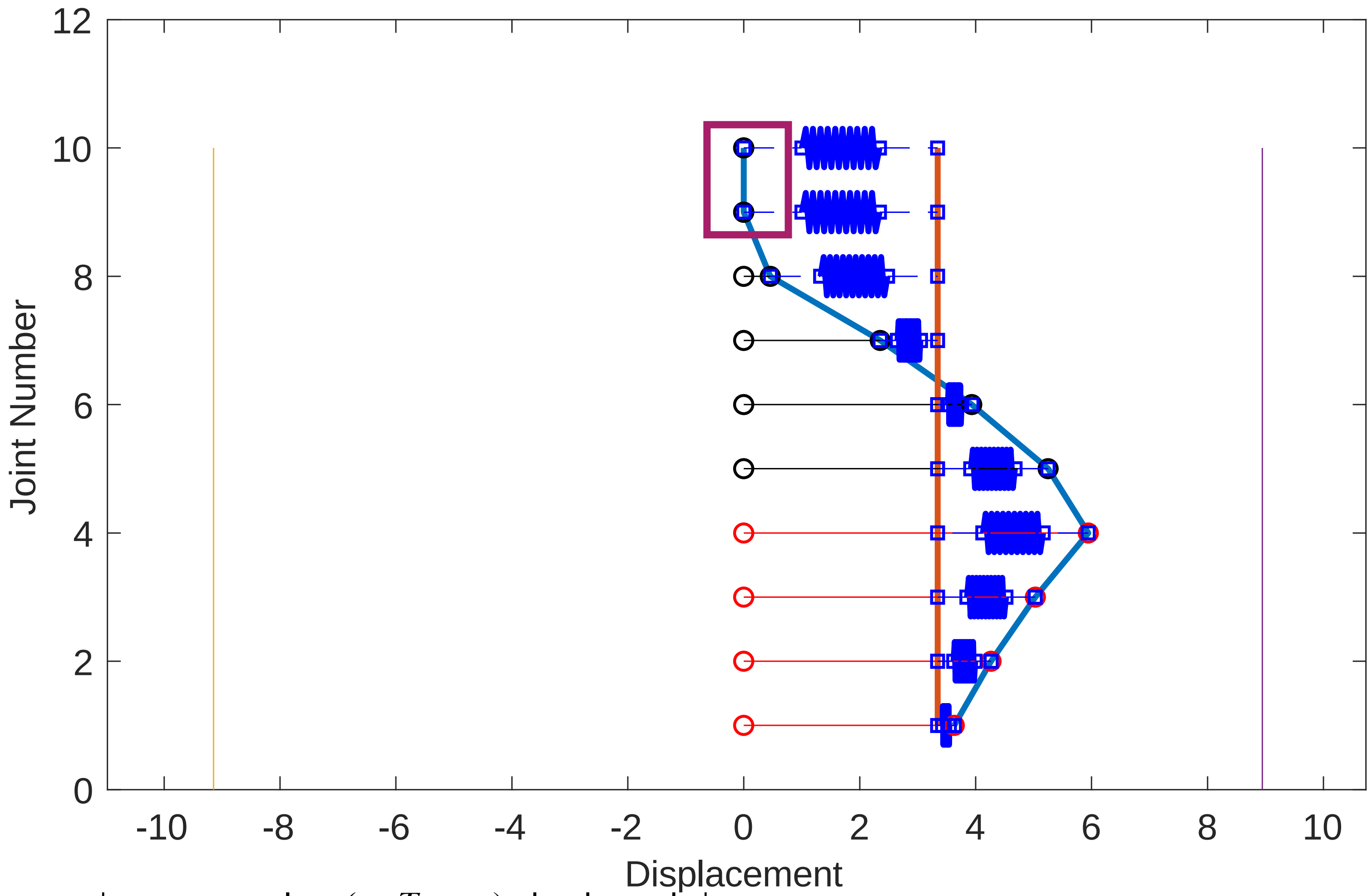


Joint is at a reversal point

Algorithm

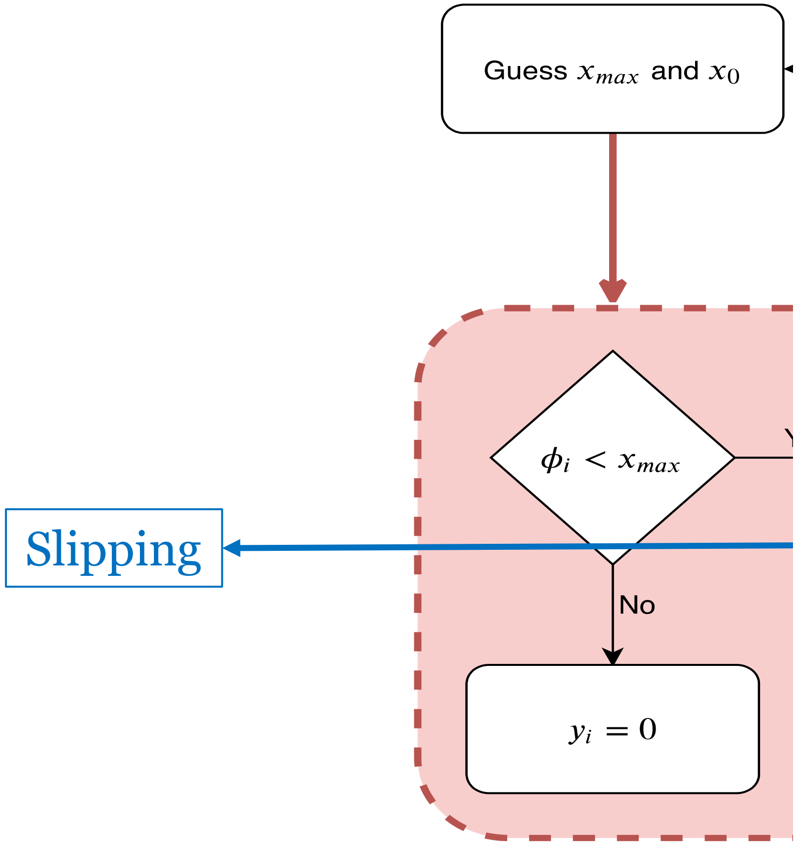
Algorithm



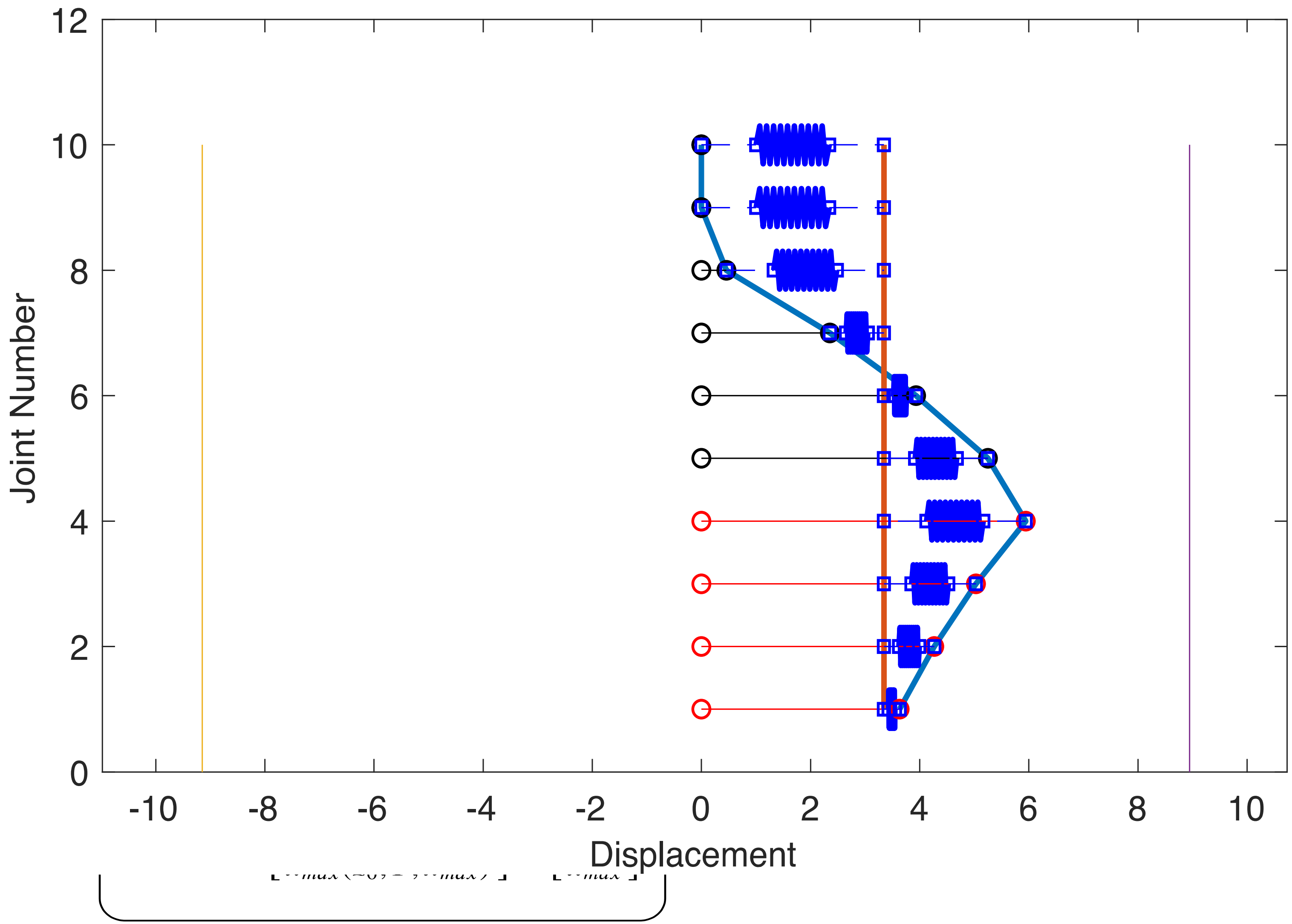


$$h = \begin{bmatrix} z_0(z_0, T, x_{max}) \\ x_{max}(z_0, T, x_{max}) \end{bmatrix} - \begin{bmatrix} z_0 \\ x_{max} \end{bmatrix}$$

Algorithm

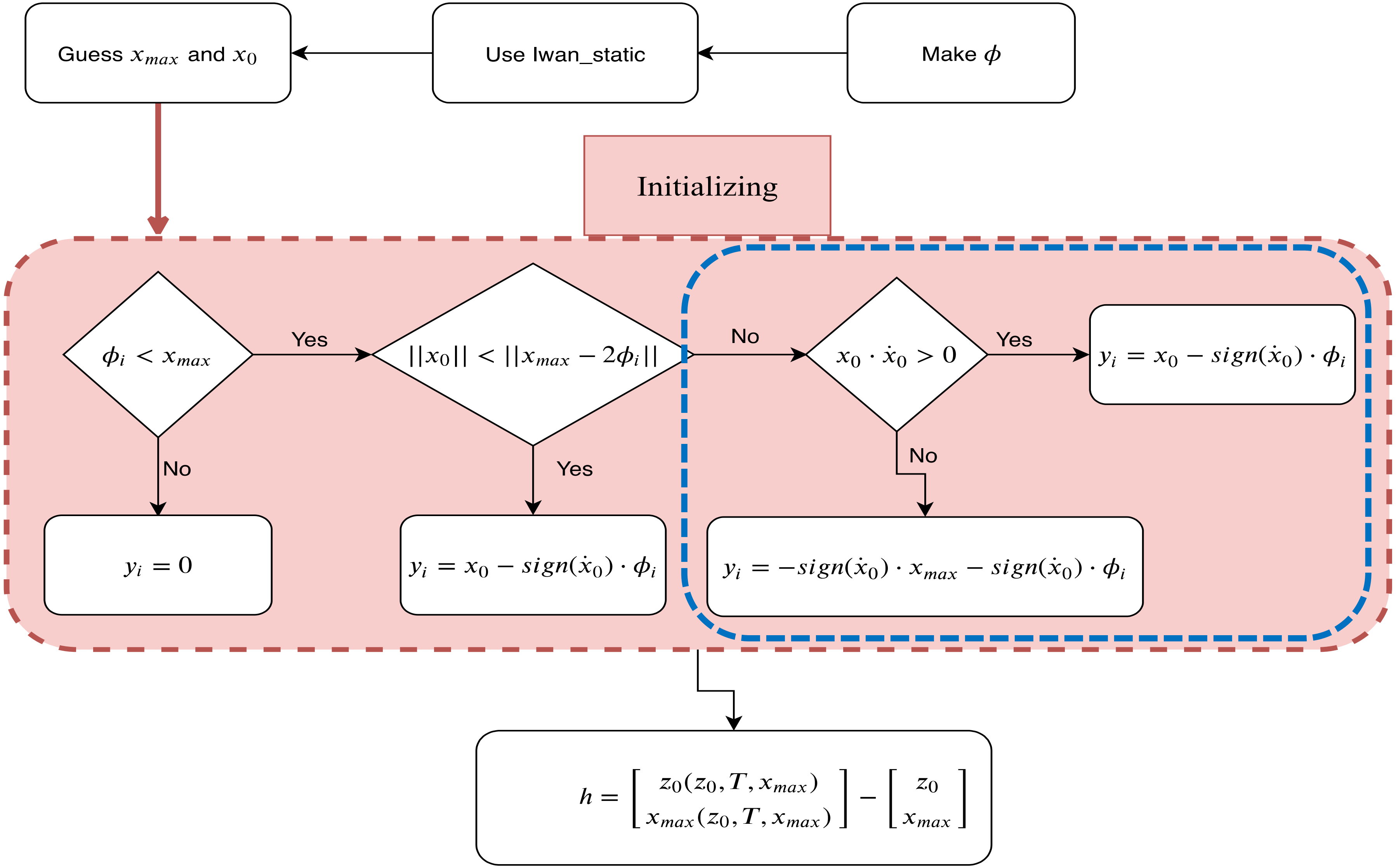


Guess x_{max} and x_0
$\phi_i < x_{max}$
No
$y_i = 0$
Slipping



Joint Number
Displacement

Algorithm



Guess x_{max} and x_0
Use Iwan_static
Make ϕ
Initializing
$\phi_i < x_{max}$
Yes
No
$||x_0|| < ||x_{max} - 2\phi_i||$
Yes
No
$x_0 \cdot \dot{x}_0 > 0$
Yes
No
$y_i = 0$
$y_i = x_0 - sign(\dot{x}_0) \cdot \phi_i$
$y_i = -sign(\dot{x}_0) \cdot x_{max} - sign(\dot{x}_0) \cdot \phi_i$
$y_i = x_0 - sign(\dot{x}_0) \cdot \phi_i$
$h = \begin{bmatrix} z_0(z_0, T, x_{max}) \\ x_{max}(z_0, T, x_{max}) \end{bmatrix} - \begin{bmatrix} z_0 \\ x_{max} \end{bmatrix}$

Finite Difference

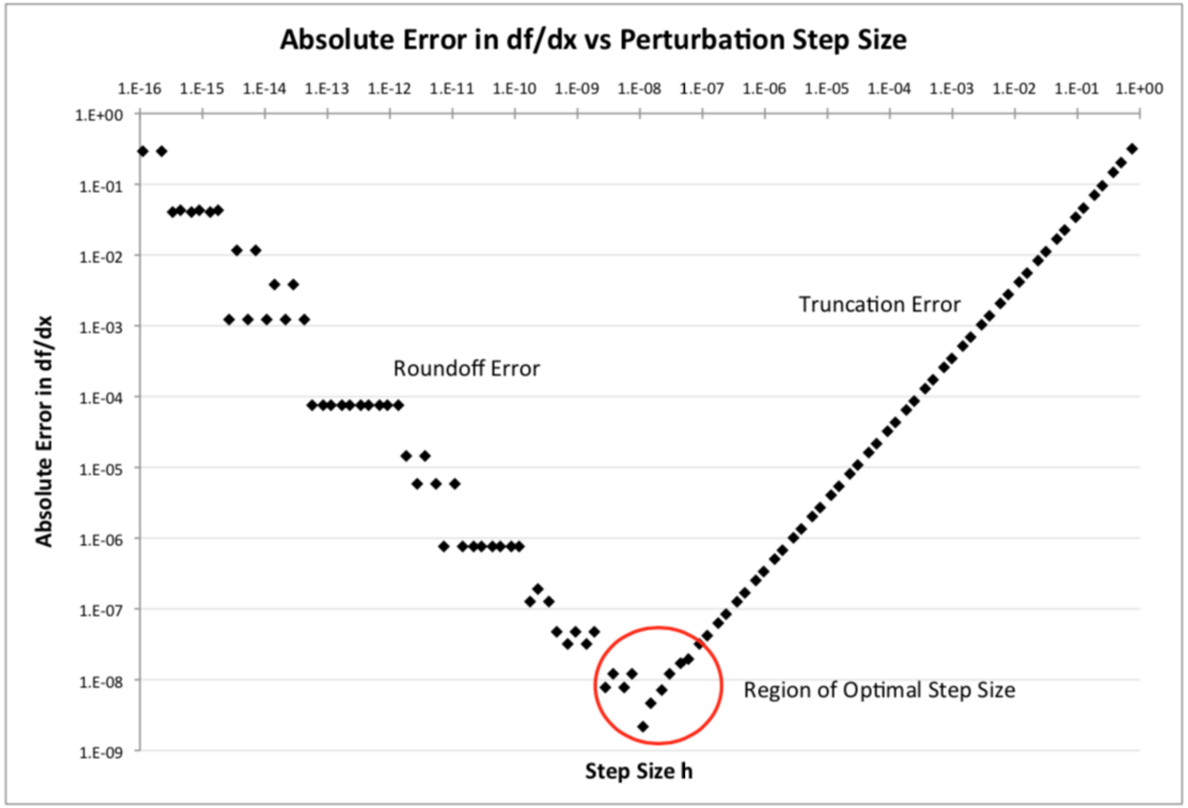
Finite Difference

- Finite difference is used:

$$\frac{\partial H}{\partial z} = \frac{z_p - z_0}{\epsilon_z} \qquad \frac{\partial H}{\partial t} = \frac{z_p - z_0}{\epsilon_t}$$

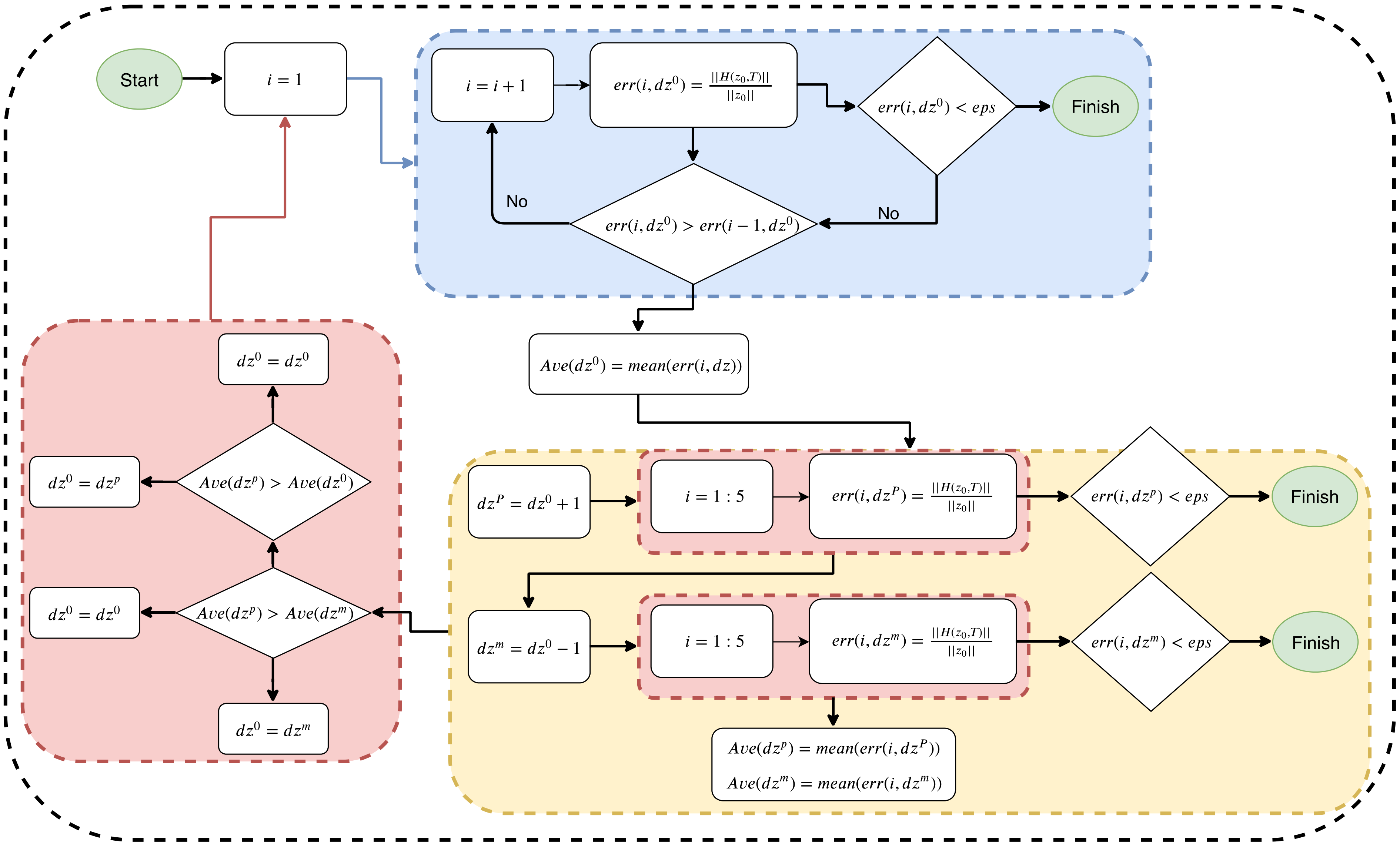
- Step-size makes the differences:
  - Number of iteration
  - Next predictions

- The step-size dilemma for sin(x) [1]:



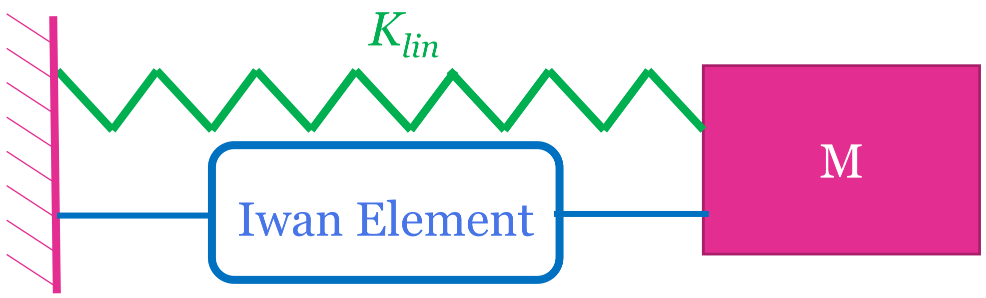
[1] Mathur, Ravishankar. *An analytical approach to computing step sizes for finite-difference derivatives*. Diss. 2012.

Step-Size Algorithm



Results

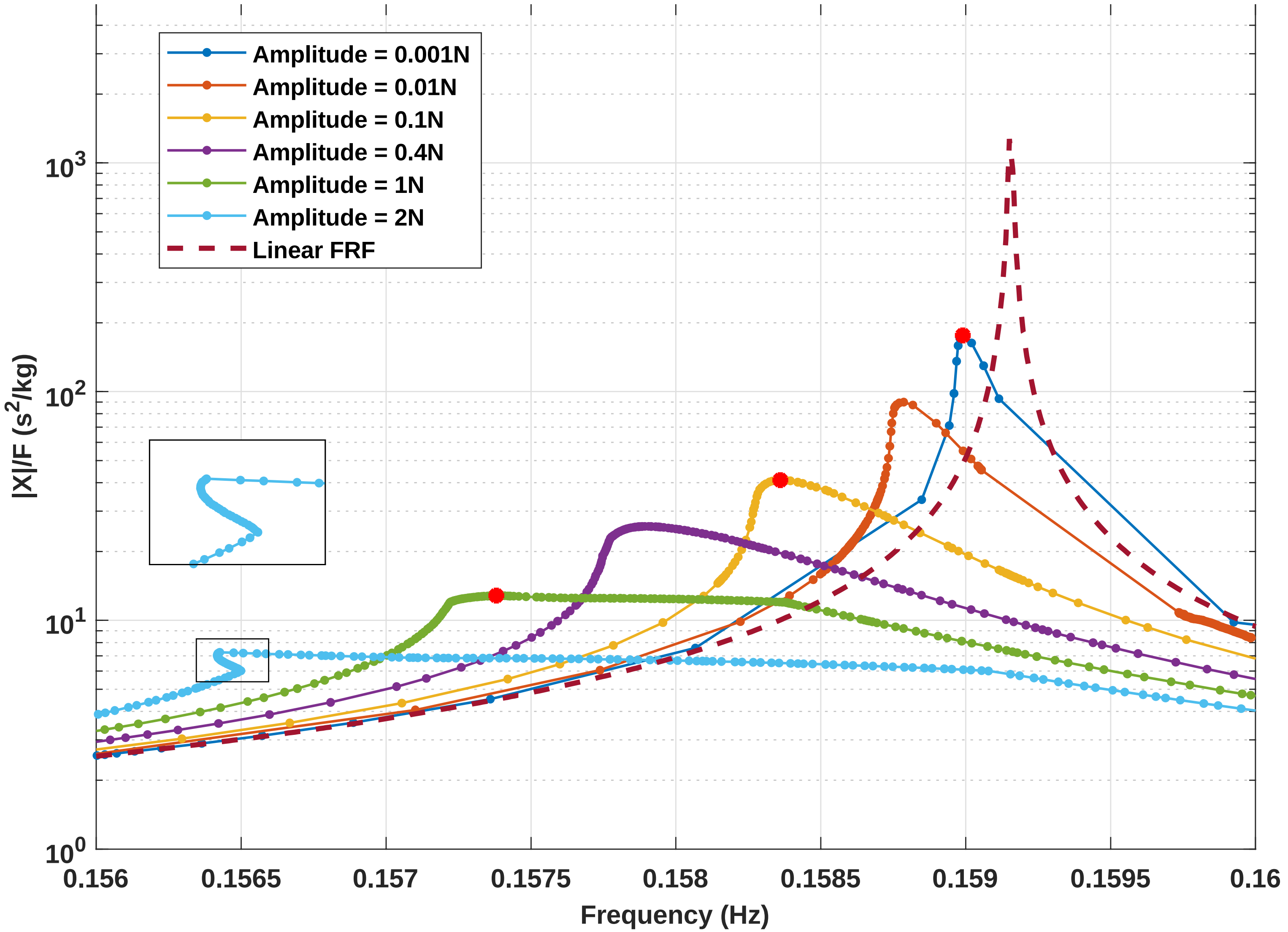
SDOF System



Parameter	Value
$K_{lin}(N/m)$	9
ζ_{lin}	2.78×10^{-5}
$M(kg)$	10
$F_s(N)$	10.0
$K_t(N/m)$	1
χ	-0.5
β	3.0

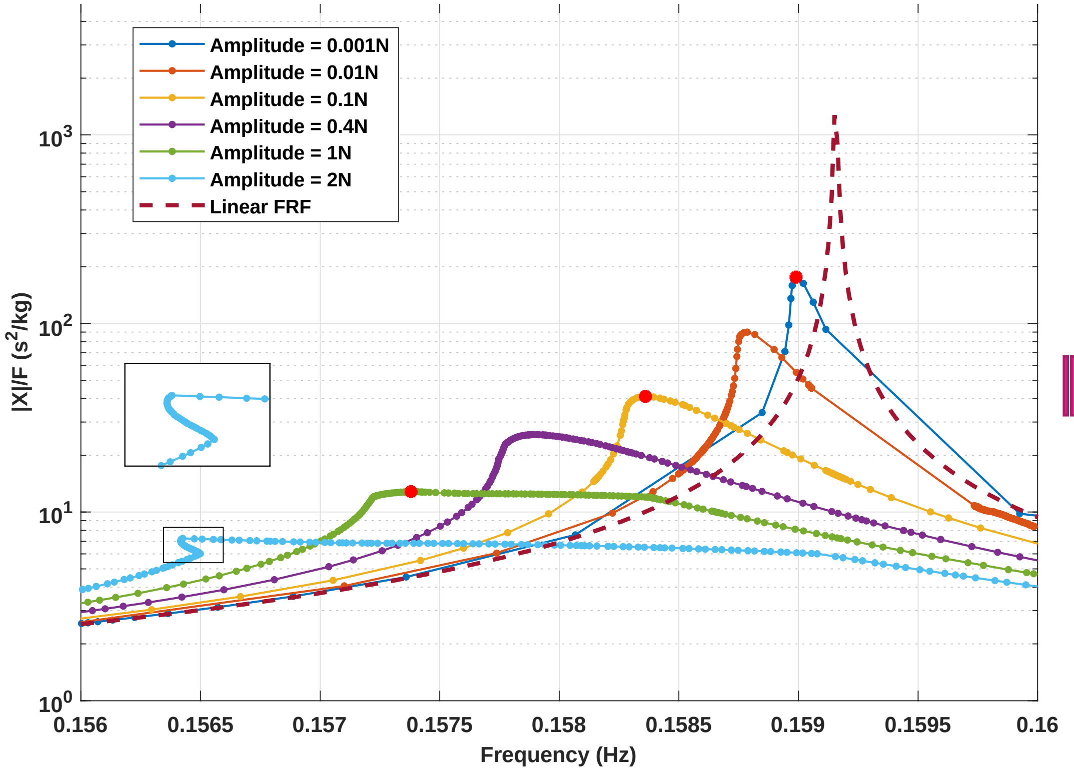
- Just one reversal point is considered, and it seems that is enough.
- 100 sliders are used.
- Different Load cases is considered.
- For MDOF system more reversal points should be considered.

SDOF System

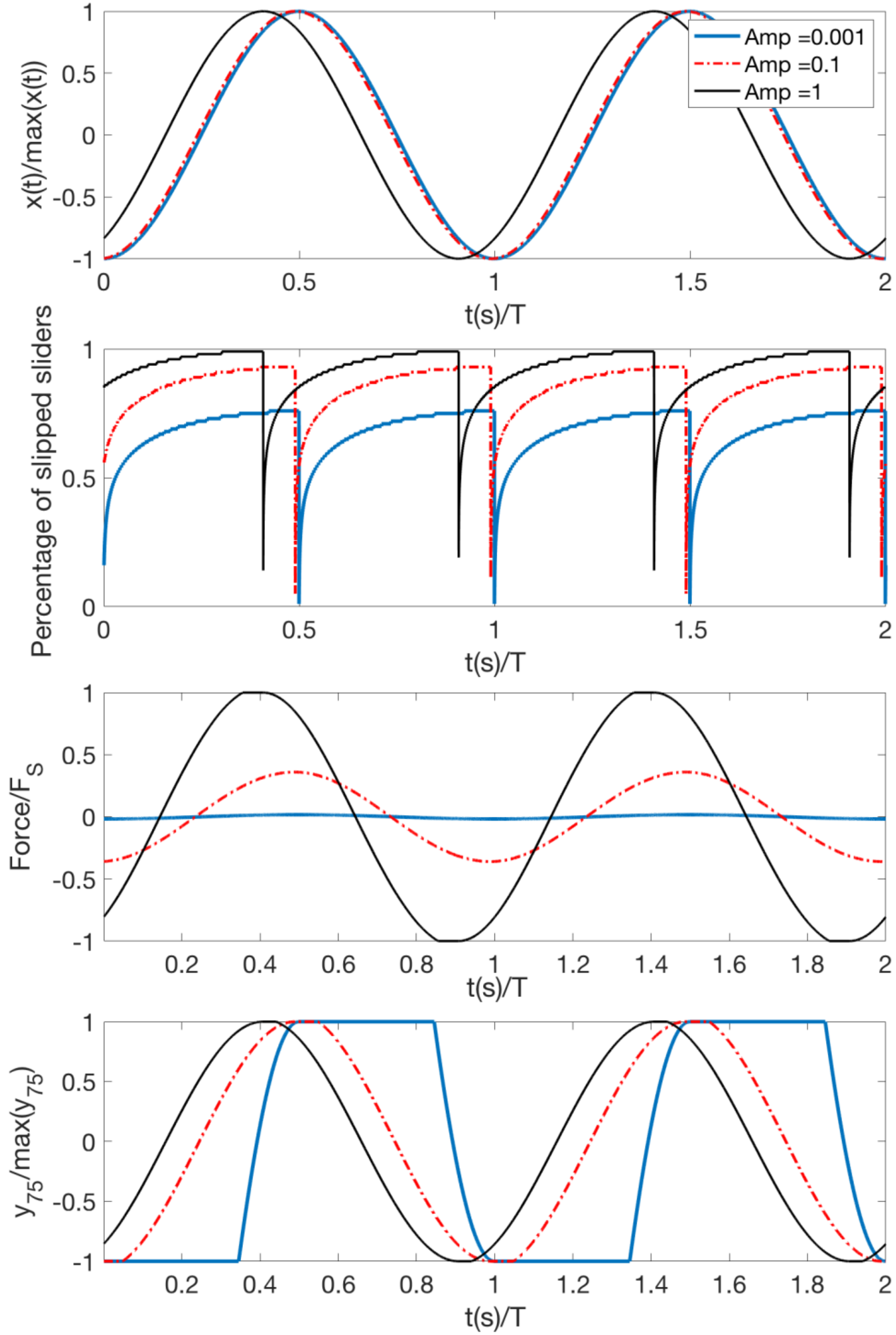


Amplitude = 0.001N
Amplitude = 0.01N
Amplitude = 0.1N
Amplitude = 0.4N
Amplitude = 1N
Amplitude = 2N
Linear FRF
|X|/F (s^2/kg)
10^3
10^2
10^1
10^0
0.156
0.1565
0.157
0.1575
0.158
0.1585
0.159
0.1595
0.16
Frequency (Hz)

SDOF System

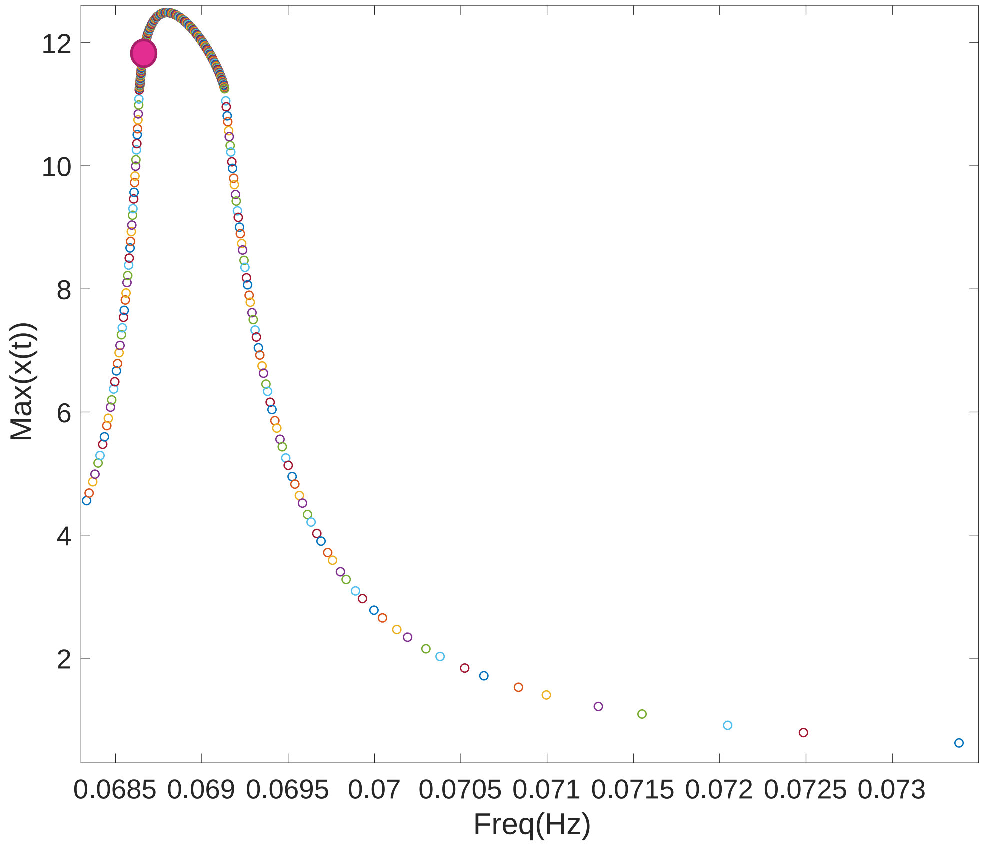


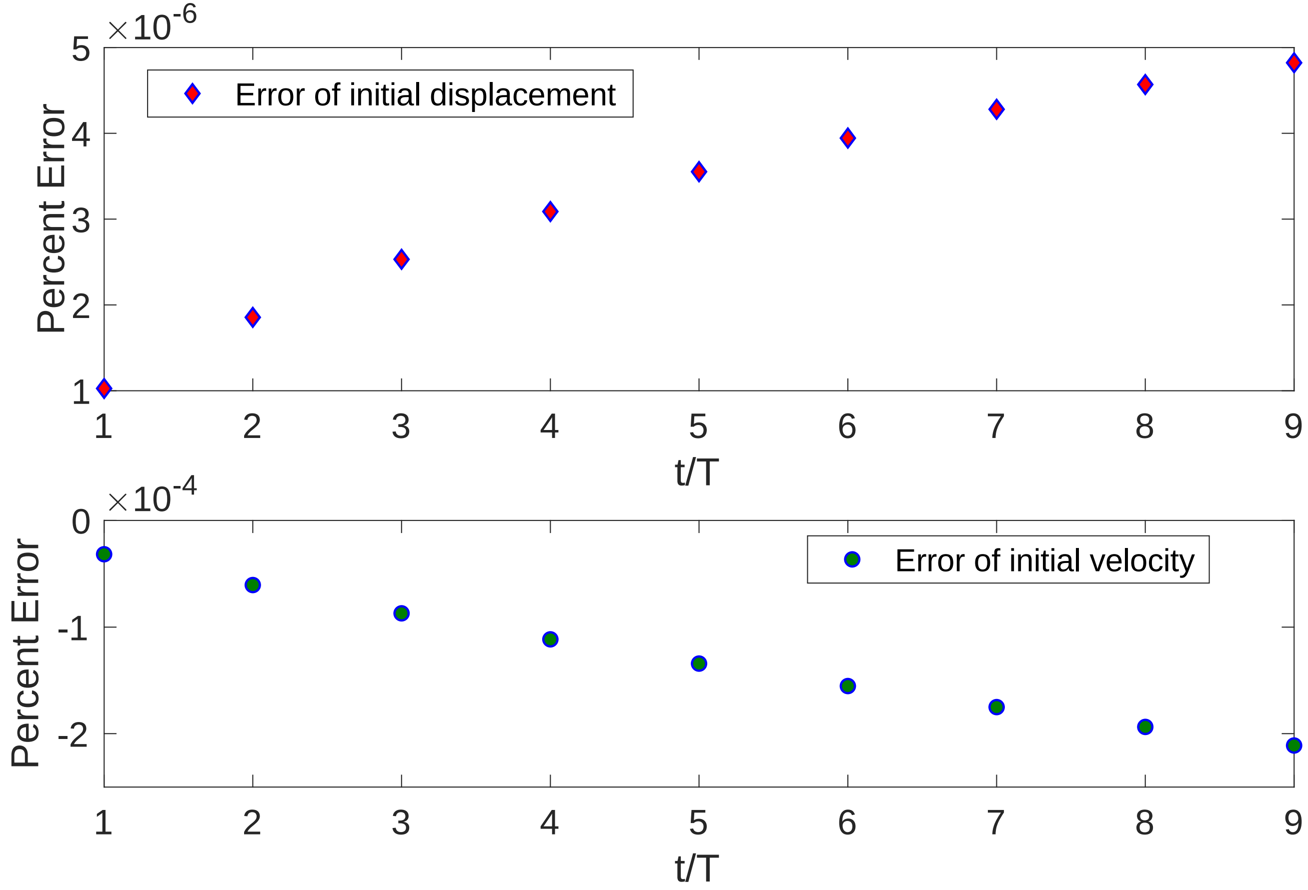
Amplitude = 0.001N
Amplitude = 0.01N
Amplitude = 0.1N
Amplitude = 0.4N
Amplitude = 1N
Amplitude = 2N
Linear FRF
|X|/F (s^2/kg)
Frequency (Hz)
0.156
0.1565
0.157
0.1575
0.158
0.1585
0.159
0.1595
0.16



Amp =0.001
Amp =0.1
Amp =1
x(t)/max(x(t))
t(s)/T
Percentage of slipped sliders
t(s)/T
Force/F_S
t(s)/T
y_75/max(y_75)
t(s)/T

Preiodicity



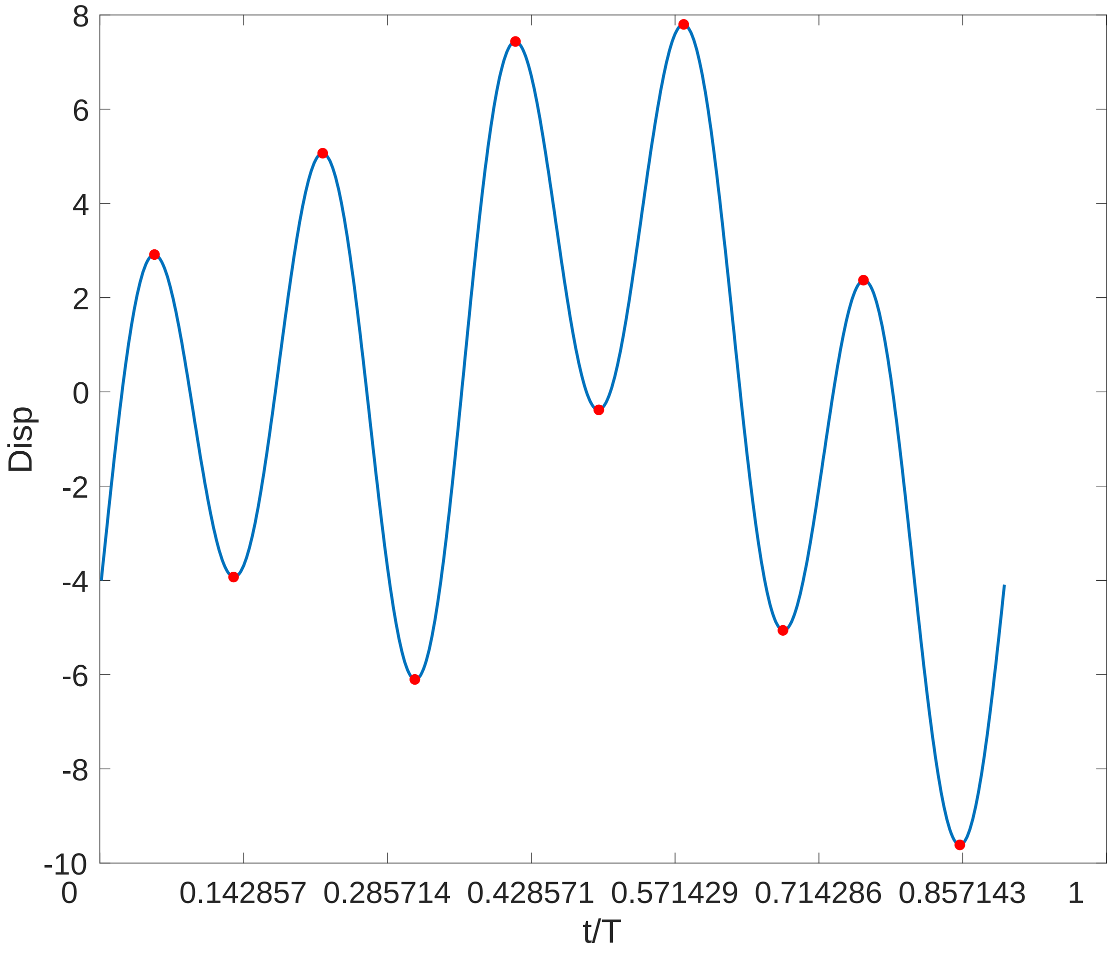


MDOF System

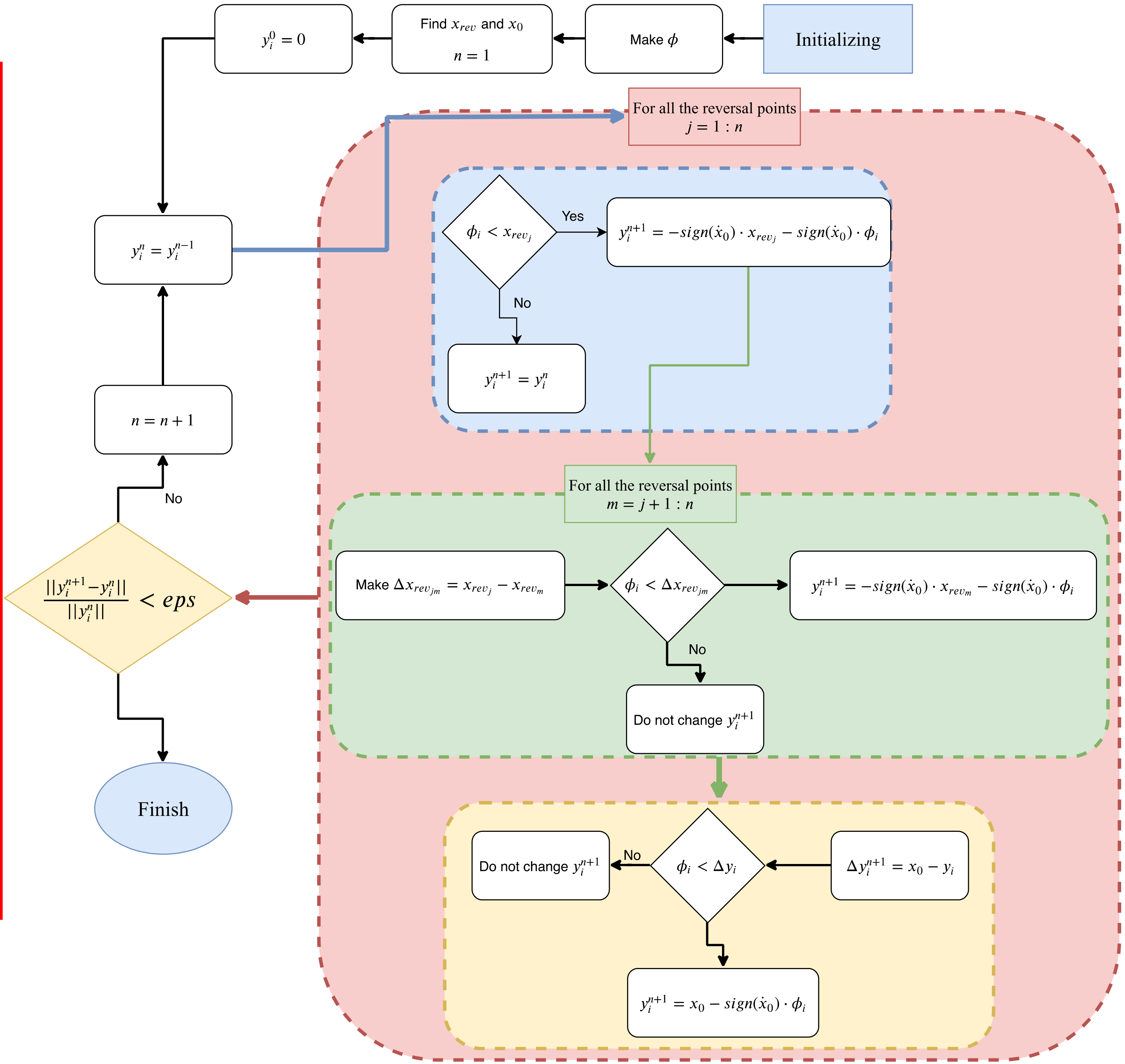
The challenges and a need to a new algorithm

MDOF System

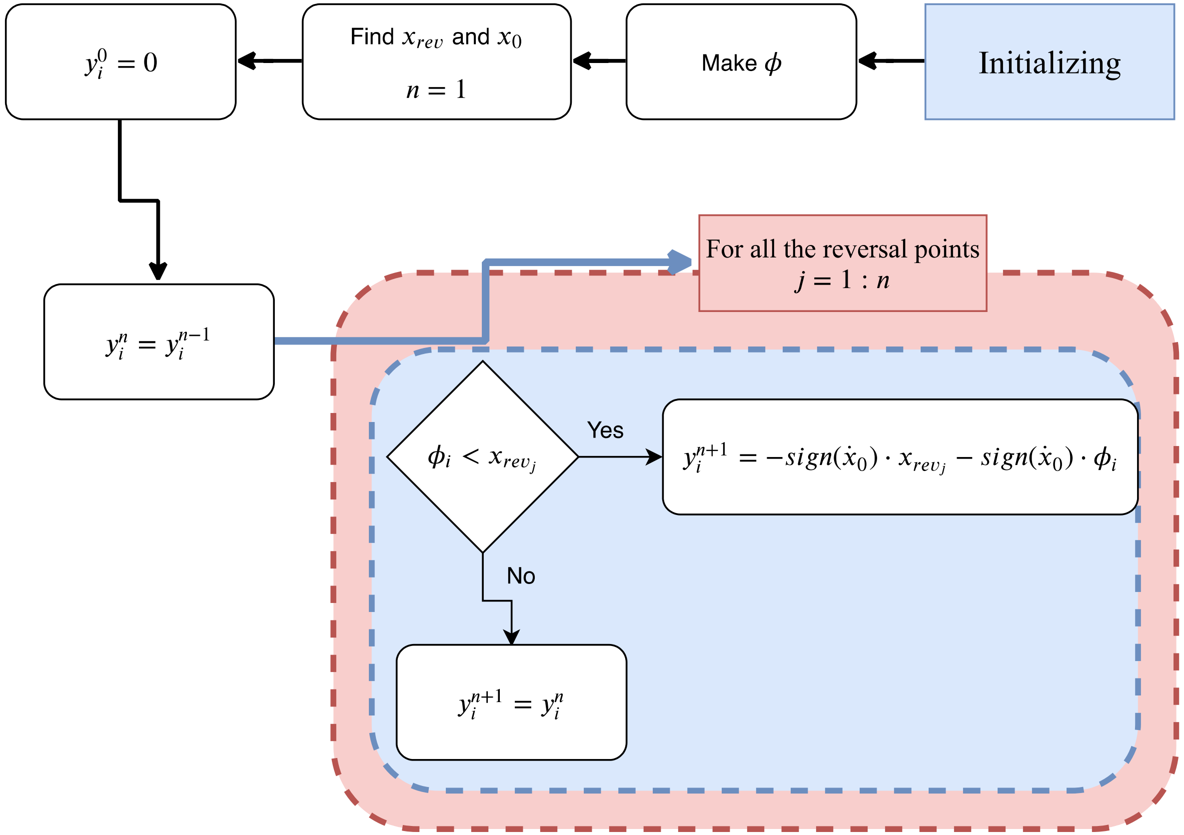
- More harmonics because of more non-linearity in the system.
- More than two reversal points, all of them should be considered.
- The number of reversal points may change at each iteration. That causes the size of Jacobians to be Dynamic.

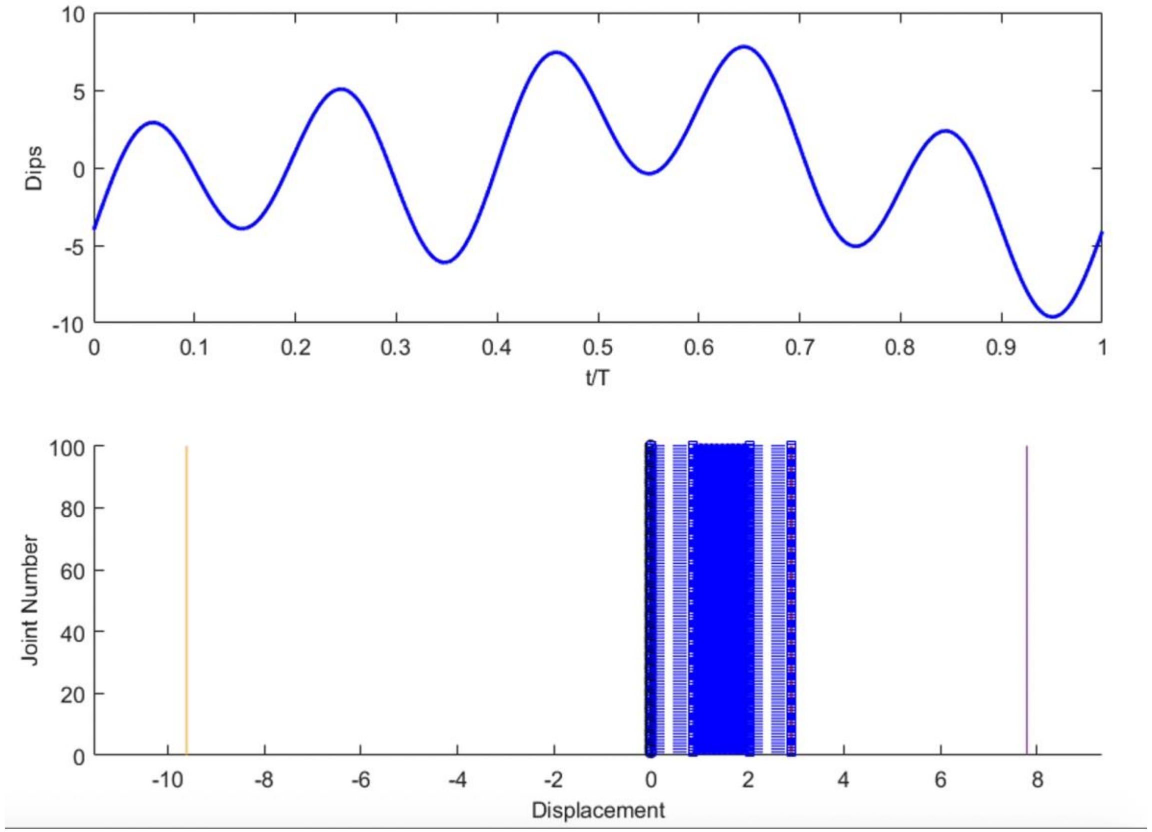


MDOF Algorithm

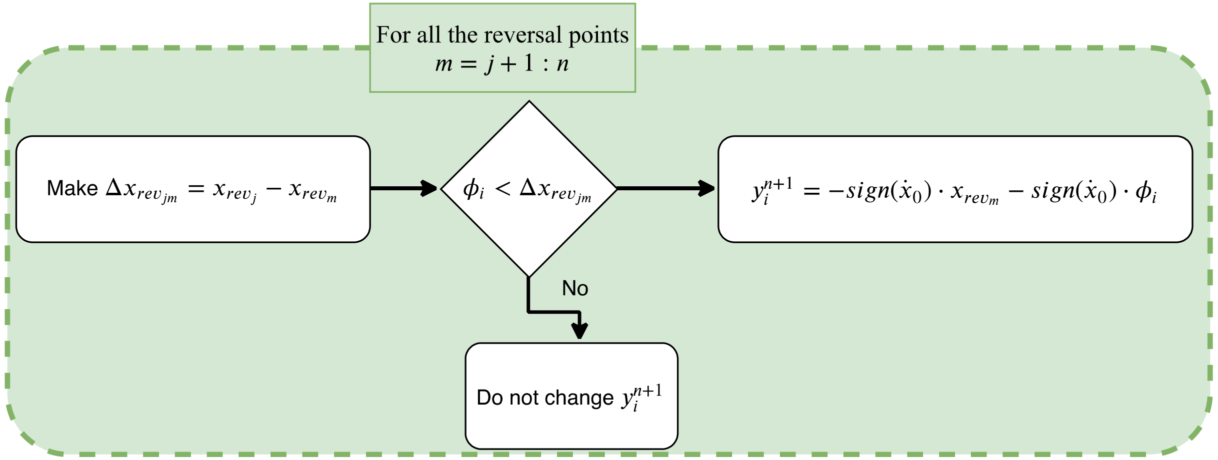


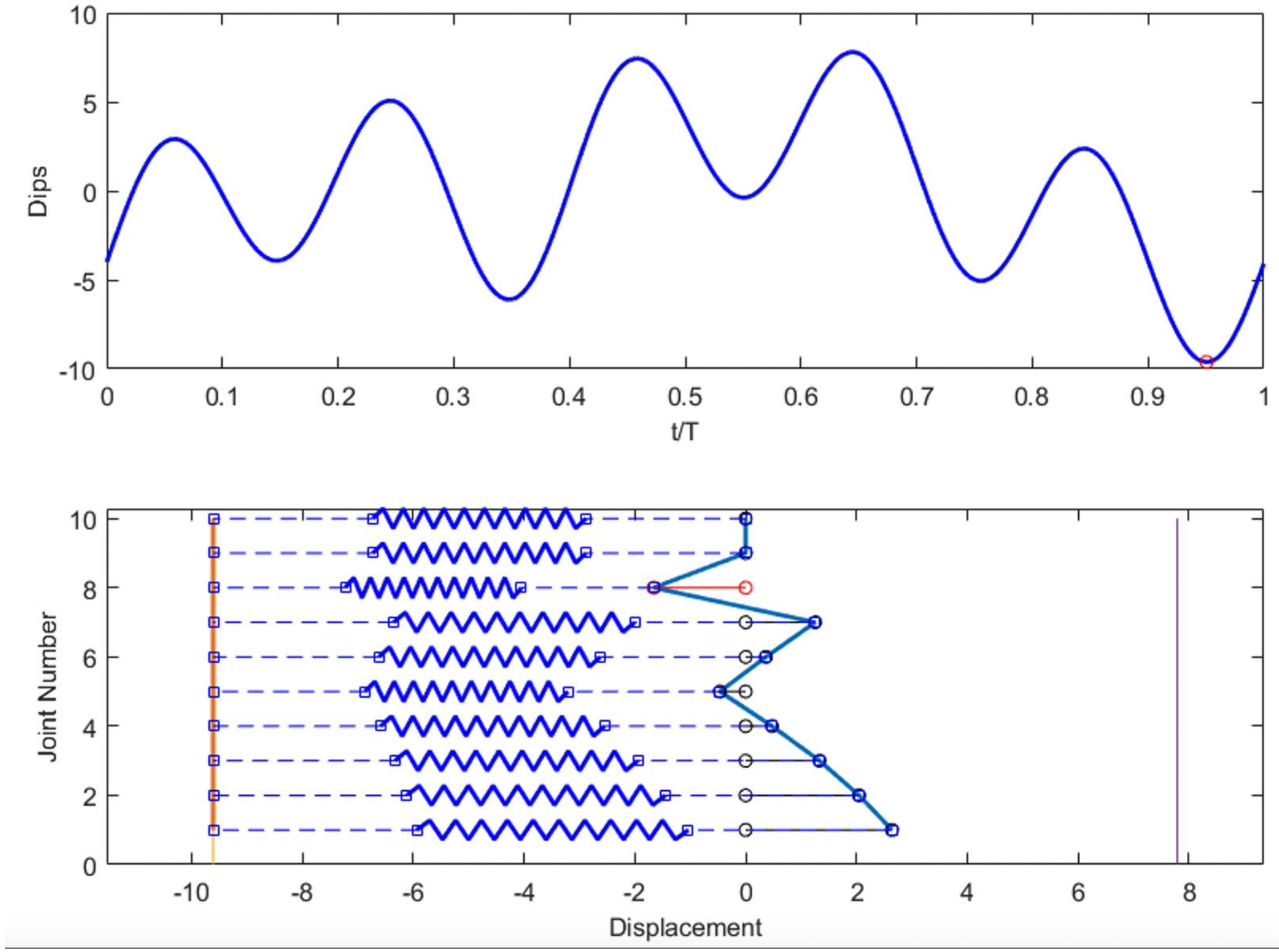
MDOF Algorithm : Part I



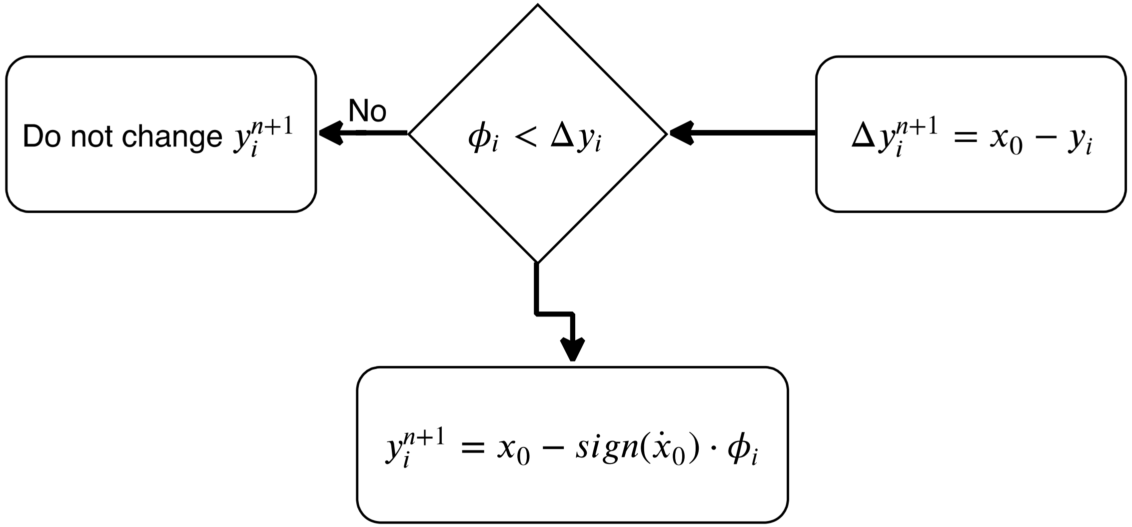


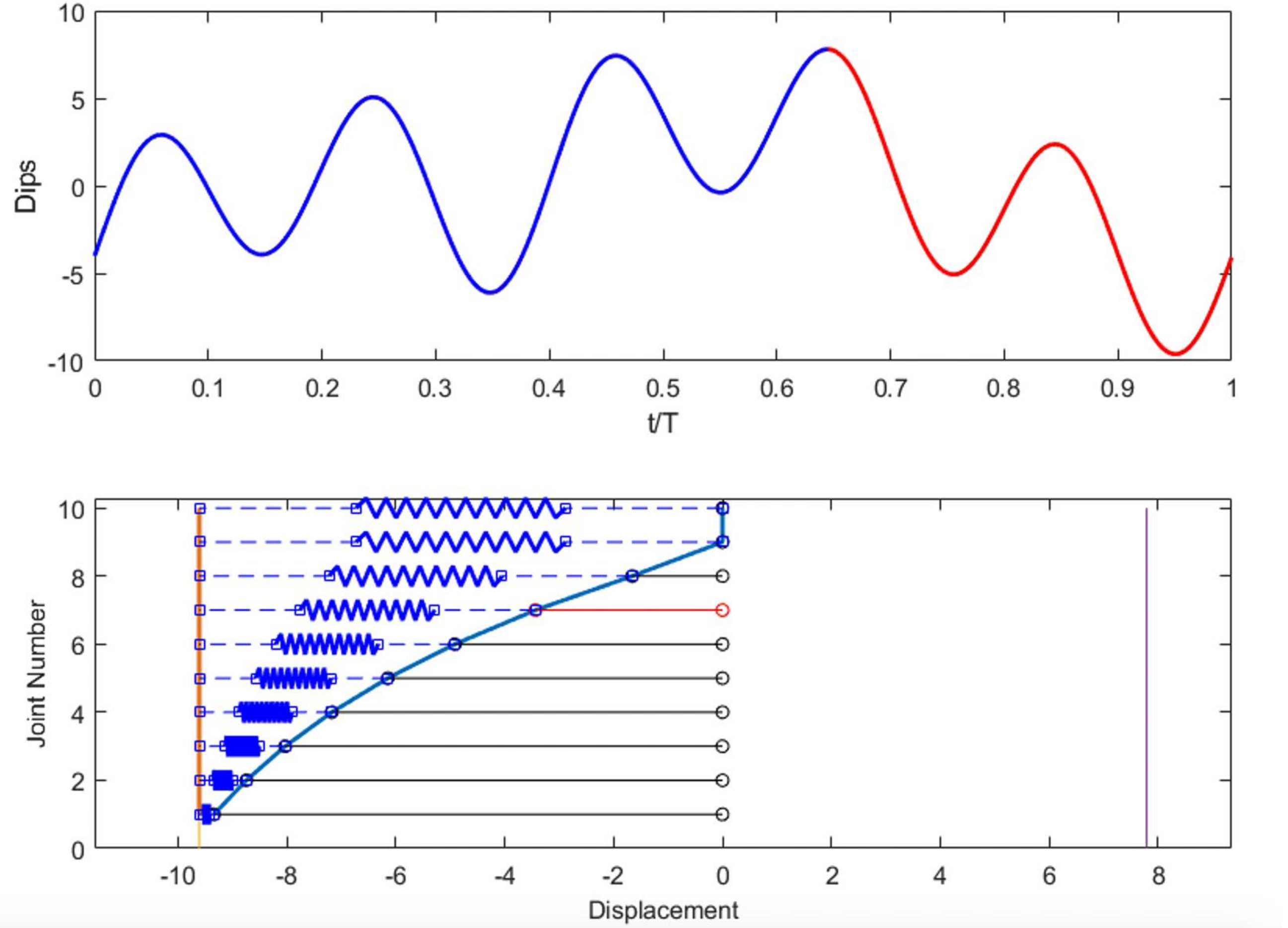
MDOF Algorithm : Part II



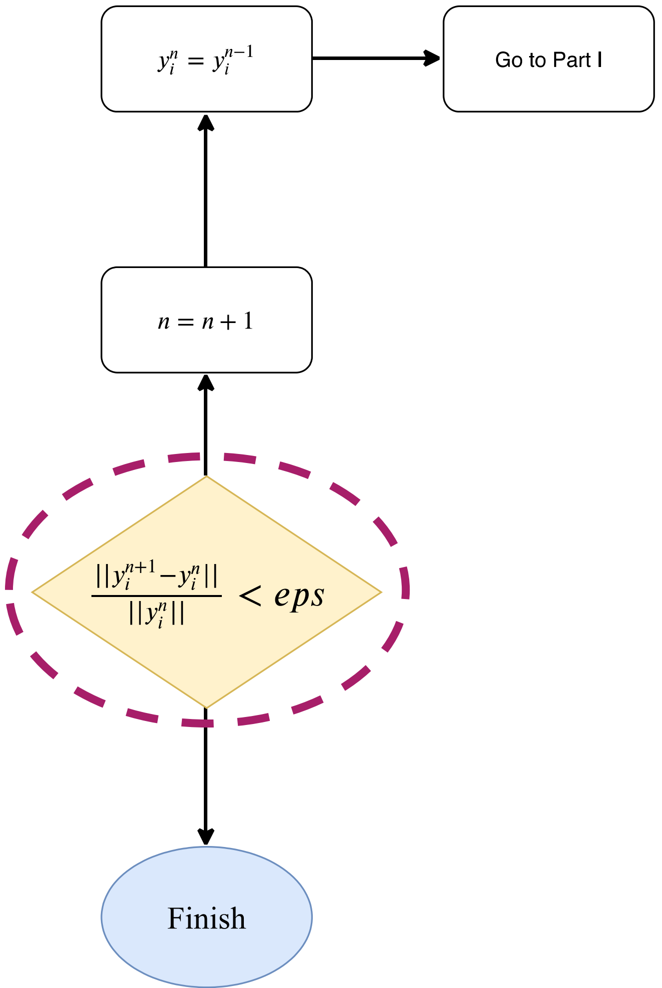


MDOF Algorithm : Part III

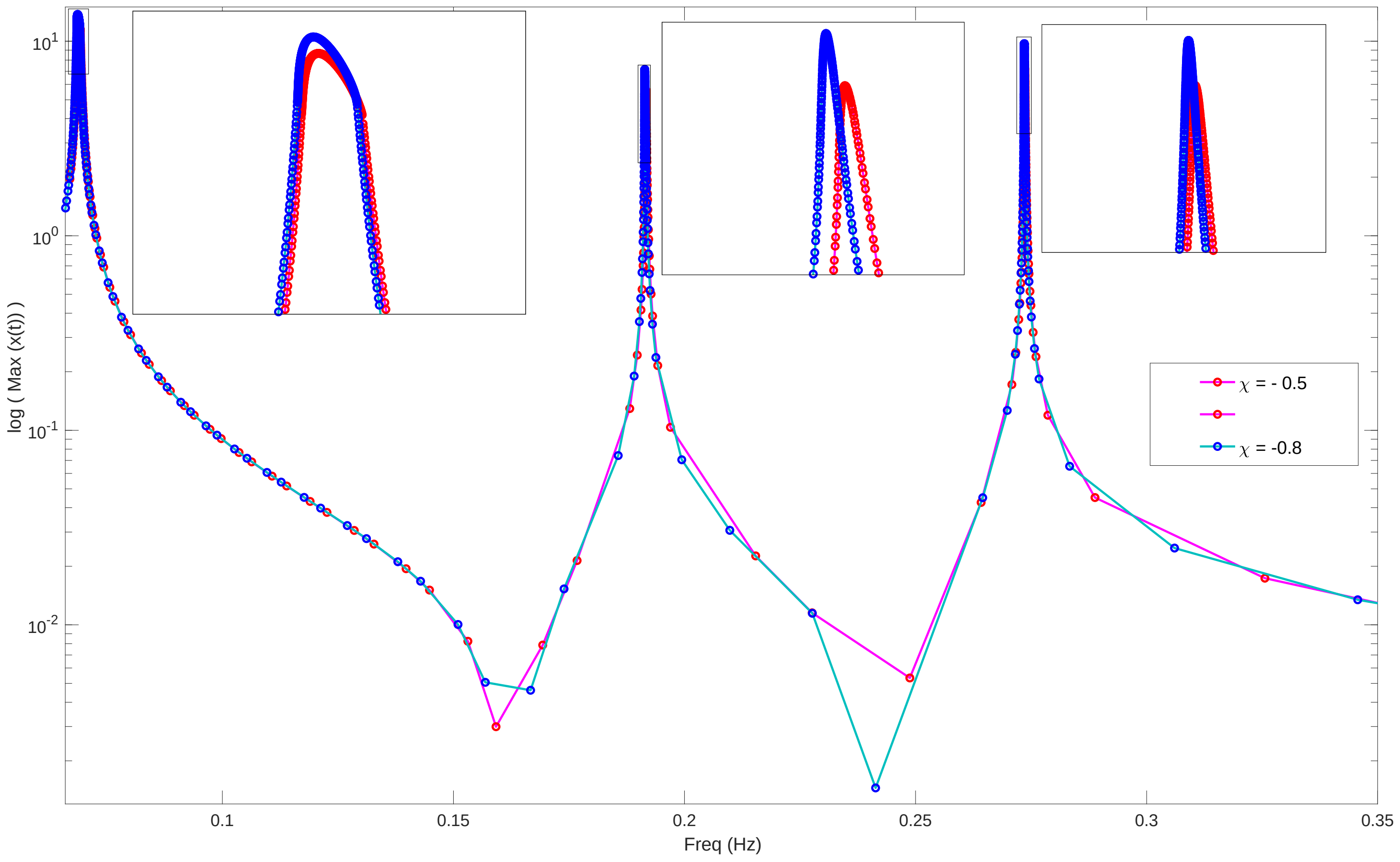




MDOF Algorithm : Part IV



- It continues until we get the state of sliders are periodic.



10^{1}
10^{0}
10^{-1}
10^{-2}
log (Max (x(t)))
0.1
0.15
0.2
0.25
0.3
0.35
Freq (Hz)
χ = - 0.5
χ = -0.8

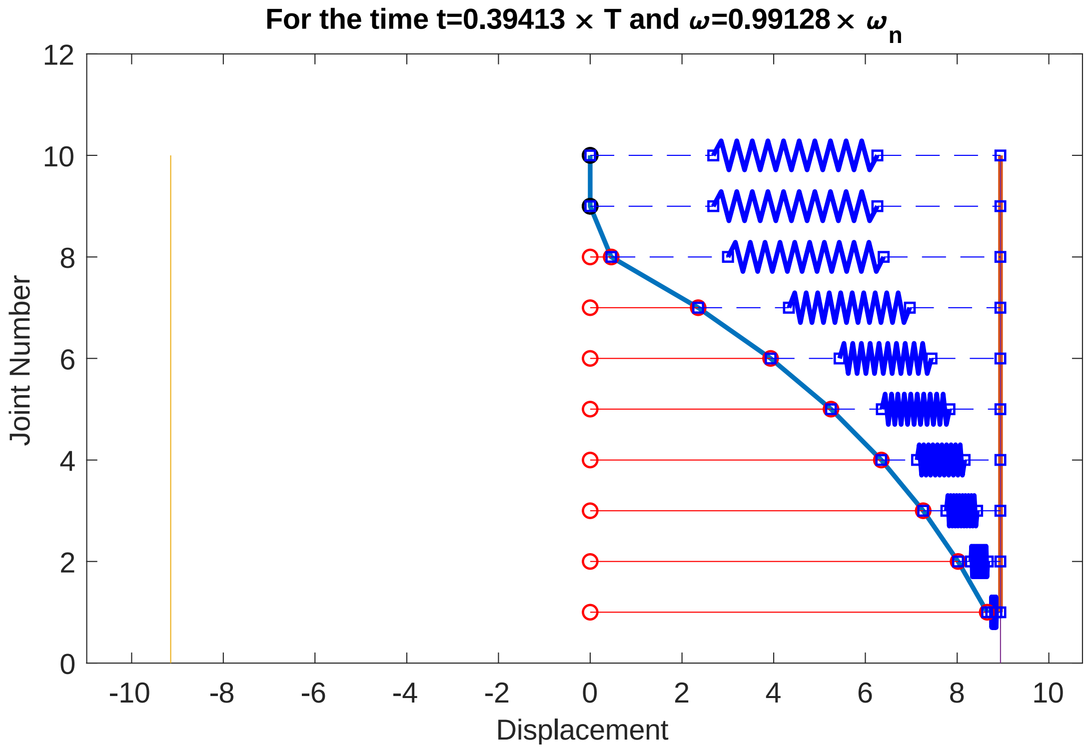
Appendix

Iwan FRF: Challenges

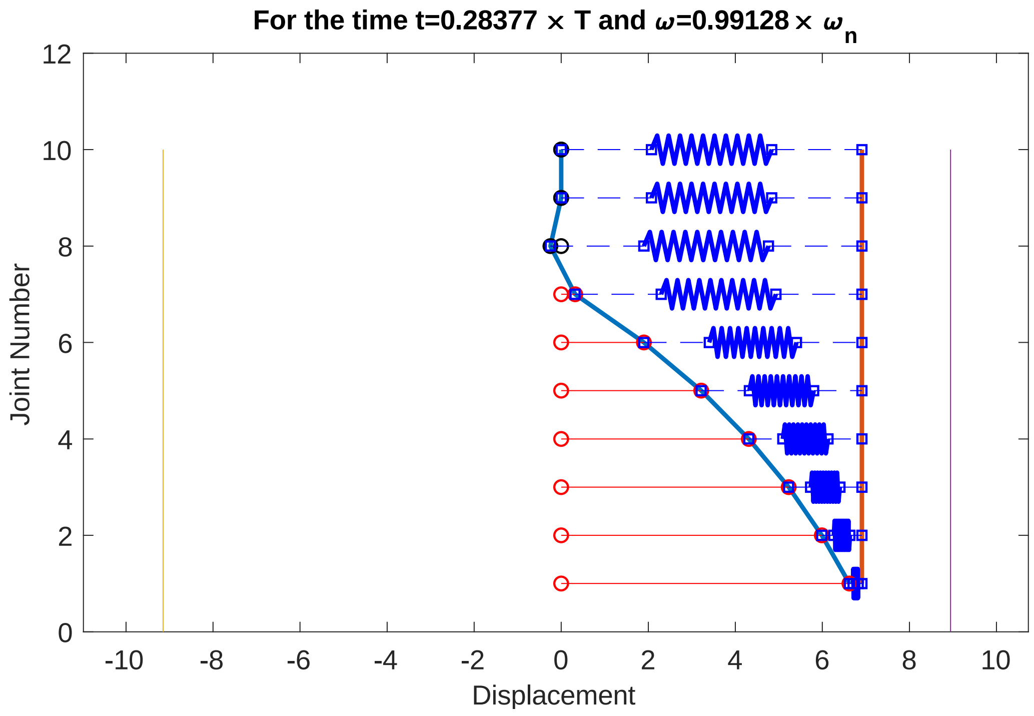
- The implicit nature of the state variables makes it non-trivial to use continuation to compute the frequency response using already established techniques such as the shooting method.
- A novel method to numerically compute the non-linear FRFs of a system with an Iwan element,
- The reversal points over the response period is included as a state variable.
- The shooting method is modified to account for the added state variable.

Reversal Points

- One moment after max:



Reversal Points

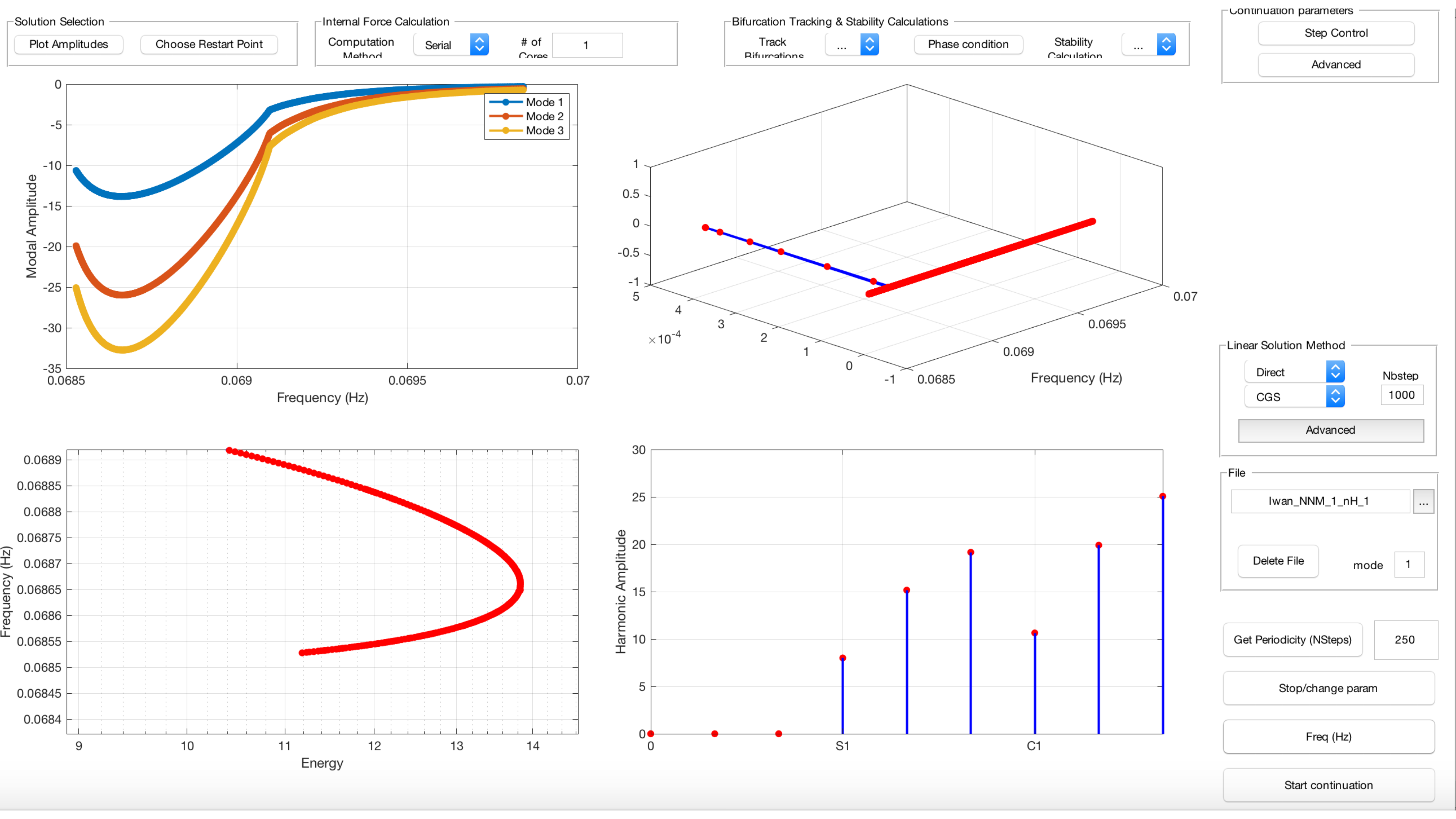


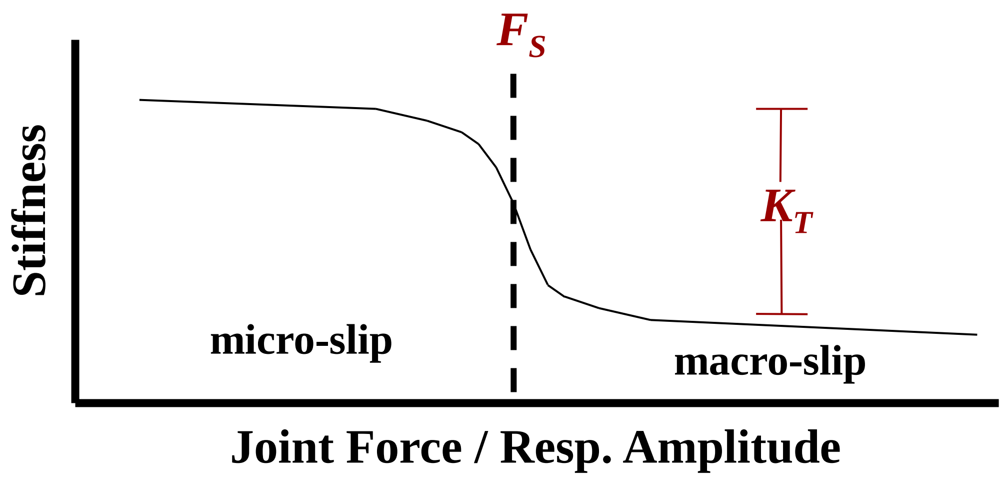
- When $\dot{x} > 0$ and $0 < x < x_{max}$ the 1th to 7th sliders are sliding, the 8th slider is stuck at some initial location. The 9th and 10th sliders are stuck at equilibrium.

- When and the 1th to 7th sliders are sliding, the 8th slider is stuck at some initial location. The 9th and 10th sliders are stuck at equilibrium.

Iwan_FRF/MHB/3DOF/my-try-captured-all-weeks.fig

Iwan_FRF/MHB/3DOF/my-try-captured-all-the-peak-max(xdt).fig


Solution Selection
Plot Amplitudes
Choose Restart Point
Internal Force Calculation
Computation Method
Serial
# of Cores
1
Bifurcation Tracking & Stability Calculations
Track Bifurcations
...
Phase condition
Stability Calculation
...
Continuation parameters
Step Control
Advanced
Mode 1
Mode 2
Mode 3
Modal Amplitude
Frequency (Hz)
$\times 10^{-4}$
Frequency (Hz)
Linear Solution Method
Direct
CGS
Nbstep
1000
Advanced
File
Iwan_NNM_1_nH_1
...
Delete File
mode
1
Frequency (Hz)
Energy
Harmonic Amplitude
S1
C1
Get Periodicity (NSteps)
250
Stop/change param
Freq (Hz)
Start continuation



F_S
K_T
Stiffness
micro-slip
macro-slip
Joint Force / Resp. Amplitude