Effect of Model Order Ambiguity in Experimental Modal Analysis on Substructuring Predictions

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Abstract:

Many structures are built up from a number of subsystems, yet the performance of the assembled system is often the primary concern. In some applications there are compelling reasons for replacing the finite element models of some of these substructures with models that are derived through test. For example, the subsystem may have complex geometry, uncertain material properties, or perhaps it is supplied by an outside source and so one has little interest in modeling it because its design cannot be changed. A number of researchers over the past few decades have investigated techniques that combine experimental models of substructures with analytical ones to predict the built up system's response, but most of these efforts have produced unsatisfactory results because the problem can be highly sensitive to uncertainties in the experimental model and in the interface characteristics. Some of these works have found that much more accurate results are obtained if one applies modal parameter identification to the measurements, and then uses the identified model in the substructuring, because the measurements themselves may have small inconsistencies that cause large errors in the substructuring predictions. In that process, one may erroneously fit more modes to the measurements than are actually represented. This work explores the effect of model order ambiguity on substructuring predictions, especially the case where more modes are fit to a peak or cluster of peaks than actually reside there. This work shows some cases where this can lead to severe errors in the substructuring predictions, although it appears that many of the difficulties can be minimized if MIMO identification is employed and if an adequate set of input points is used. Simulated measurements are used to explore some of these ideas, and experimental measurements are used to illustrate the sensitivity that can be encountered.

1. Introduction

Substructuring, whether based on the measured frequency response functions (Frequency Based Substructuring or FBS) or experimentally identified modes of a structure (Modal Substructuring or MS or CMS) has attracted attention since the beginning of experimental modal analysis [1-4]. Sometimes this is referred to as structural modification [5], although that term usually refers to the scenario where one wishes to study the effect of adding pure masses or springs to a system whereas FBS and MS encompass joining two flexible dynamic systems, or even subtracting one substructure from another [6-9]. There is tremendous motivation to develop methods for MS, because it could allow one to predict the response of a structure in many configurations based on one test, and without creating a finite element model. There are numerous applications in the Automotive [10] and Aerospace industries, because systems there are often built up from subcomponents that are created by different suppliers or groups of researchers. For example, an auto manufacturer has little interest in creating detailed finite element models of a muffler that is designed and supplied by an outside source, but it is important to know how it will affect the noise and vibration of the assembled vehicle. In other cases, the subcomponents

are simply too difficult to model effectively, containing intricate geometry, material properties, or potential for nonlinearity. In either case, it may be much easier to perform tests on a subcomponent and develop a test based model rather than model the subcomponent analytically, even more so if one considers that tests are often needed anyway to validate the analysis models.

The FBS method operates on the measured frequency response functions (FRFs) of the subcomponents directly, to estimate the FRFs of the coupled system. Modal substructuring operates on the modal model, and is preferable in many applications because

- MS is simple to integrate with existing finite element analysis codes.
- MS is readily extended to nonlinear systems, such as structures with joints [11-13].
- The database required to describe a substructure using a modal model is much smaller than a FBS model (unless the substructure has more than a few hundred modes.)
- The modal parameters are easier to interrogate and perhaps lend themselves to physical interpretation more easily than frequency response functions.

Indeed, the only disadvantage of MS is that it requires one to identify an accurate modal model of the substructure from the response measurements. Modal parameter identification can be difficult on many systems of interest, especially if the system is highly complicated, as tends to be the case in the most compelling applications of MS. The most important challenge is determining the number of modes active in the system of interest and assuring that their parameters are obtained with reasonable accuracy. When a structure has modes with close natural frequencies, their individual character diminishes and various descriptions may adequately reproduce the measured response [14]. When the response is contaminated by small inconsistencies [15], signal processing errors [16], nonlinearities or noise, as most real measurements are to some extent, one can usually obtain closer agreement between the measured and reconstructed response by fitting additional modes to the response.

This work explores the potential for over-fitting, or fitting more modes to a response than are actually there, and seeks to characterize its effect on substructuring predictions. It has been suggested that over-fitting should not affect the results drastically, that any modal model will suffice, so long as it reproduces the frequency response functions of the system accurately. On the other hand, slightly inconsistent measurements sometimes wreak havoc on substructuring predictions, as shown in [17], and one possible explanation is that the measurements actually describe a system that with many close and localized modes instead of a few global modes due to small inconsistencies. This work presents a few scenarios in which dramatic sensitivity can occur due to over-fitting, depending on how the shape of a mode is apportioned between two fit modes and where modifications are applied. A number of scenarios are considered, with a focus on the physics behind each. This work also shows that multi-input-multi-output (MIMO) identification can be used to avoid some of these problems, but perhaps only if the MIMO scheme assures that reciprocity is enforced, and most schemes do not provide a convenient way of doing this. A simulation example involving a two-degree-of-freedom (DOF) system is used to illustrate the concepts. Slightly inconsistent measurements are simulated by adding small masses to each drive point when creating simulated, noise-contaminated measurements. The concepts are also illustrated by applying modal parameter identification to actual measurements from a complicated system with many close natural frequencies, and comparing the results as various numbers of modes are fit to the responses.

2. Review of Relevant Theory

The frequency response of a linear time invariant system with under-damped modes can be expressed in the following general state space form

$$\underset{N_o \times N_i}{H(\omega)} = \sum_{r=1}^{N} \frac{A^{ss}{}_r}{i\omega - \lambda_r} + \frac{\left(A^{ss}{}_r\right)^*}{i\omega - \left(\lambda_r\right)^*} , \tag{1}$$

where λ_r are the eigenvalues of the system with $\lambda_r = -\omega_r\zeta_r + i\omega_r(1-\zeta_r^2)^{1/2}$ in terms of the natural frequency and damping ratio of the rth mode of vibration, $A^{ss}{}_r$ are the state-space modal residues, and the dimensions of the FRF matrix are the number of outputs or response points N_o by the number of inputs or drive points N_i. Various definitions for the residues exist, depending on the normalization scheme employed. For example, using Ginberg's normalization scheme, $A^{ss}{}_r = \lambda_r \{u_o\}_r \{u_i\}_r^{\mathrm{T}}$, where $\{u_o\}_r$ is the complex, state space mode vector at the output points and $\{u_i\}_r$ the mode vector at the input points.

The vast majority of modal parameter identification schemes seek to identify the modal parameters λ_r and $A^{ss}{}_r$ by minimizing some norm of the difference between the measured response $H(\omega)_{meas}$ and the state space representation.

$$\left\| H(\omega)_{meas} - \left(\sum_{r=1}^{N} \frac{A^{ss}{}_r}{i\omega - \lambda_r} + \frac{\left(A^{ss}{}_r\right)^*}{i\omega - \left(\lambda_r\right)^*} \right) \right\| \tag{2}$$

Allemang and Brown showed how most techniques fit in this framework in [18], and the same is true of recent techniques such as the pLSCF or Polymax® algorithm [19, 20], the SMAC algorithm [21-23], the AMI Algorithm [24-26], and the AFPoly approach recently developed by Vold and Richardson [27]. It is important to note that the residue matrices in eq. (1) have rank one. Some schemes enforce this, while others use a common-denominator formulation that allows residues with rank greater than one, such as that discussed in [19].

Most structural systems obey reciprocity, in which case the FRF matrix has the following form

$$\underset{N_o \times N_i}{H(\omega)} = \sum_{r=1}^{N} \frac{A_r}{\omega_r{}^2 - \omega^2 + i2\zeta_r\omega_r\omega} , \tag{3}$$

where A_r are the classical modal residues $A_r = \{\phi_o\}_r\{\phi_i\}_r{}^{\mathrm{T}}$ and $\{\phi\}_r$ denotes the rth real mode vector of the system. However, the state-space identification problem in eq. (2) can be solved with computationally efficient linear techniques where that in eq. (3) cannot, so virtually all parameter identification schemes identify a state space model from the measurements and then reduce that to a classical modal model. The classical residues are related to the state space residues as follows

$$A_r = -2\,\mathrm{Im}\left(\lambda_r\right)\mathrm{Im}\left(A^{ss}{}_r\right). \tag{4}$$

One can then find the mode vector corresponding to each residue under certain conditions. For example, if a Single-Input-Multi-Output (SIMO) identification is performed and the nth response point corresponds to the input point, then $\{\phi_o\}_r = A_r/(A_r)_n{}^{1/2}$.

We shall use structural modification theory [2] to demonstrate how the fit model responds to a structural modification, and contrast that with the true model. Assuming a classical system with symmetric mass and stiffness matrices, and a modification that does not alter the eigenvectors of the system, adding a mass Δm_n to the nth nodal degree of freedom changes each eigenvalue by the following amount [2].

$$\Delta\omega^{mod}{}_r{}^2 = -\omega_r{}^2\phi_{nr}{}^2\Delta m_n \tag{5}$$

This only holds for an infinitesimal modification, and we must be mindful of the assumption regarding the eigenvector remaining constant, yet this equation does capture the fact that the modification affects those modes most strongly whose mode vectors are large at point n, and that phenomenon usually holds for large modifications as well.

We obtain a different view of the sensitivity using frequency-response based structural modification theory [10, 28]. For a single mass modification at point n, one obtains the following change in the j-kth element of the frequency response function matrix.

$$\Delta H_{jk}(\omega) = \omega^2 H_{jn} H_{kn}\,\Delta m_n \tag{6}$$

Here we see a competing effect between the small modification and the product of two frequency response functions. If the product of the FRFs is large, then the frequency response function of interest could change substantially due to the modification, even if the modification itself is relatively small. However, this equation gives no indication that a small error in the FRFs may lead to large sensitivity to modification, suggesting that FBS is insensitive to the number of modes fit to each resonance peak, so long as the reconstructed FRFs agree closely with the measurements.

3. Model Order Ambiguity in Modal Parameter Identification

There always exists some potential for ambiguity in the number of modes active in a response, in part because the contribution of any one mode in eq. (3) can often be represented as a sum of two modes. Without loss of generality, we consider the first mode, which will be represented with two identified or fit modes, which are denoted with superscripts f, as follows

$$\frac{A_1}{\omega_1{}^2 - \omega^2 + i2\zeta_1\omega_1\omega} = \frac{A^f{}_1}{\omega^f{}_1{}^2 - \omega^2 + i2\zeta^f{}_1\omega^f{}_1\omega} + \frac{A^f{}_2}{\omega^f{}_2{}^2 - \omega^2 + i2\zeta^f{}_2\omega^f{}_2\omega} , \tag{7}$$

so long as

$$\omega_1 \approx \omega^f{}_1 \approx \omega^f{}_2$$
$$\zeta_1 \approx \zeta^f{}_1 \approx \zeta^f{}_2 \,. \tag{8}$$
$$A_1 \approx A^f{}_1 + A^f{}_2$$

Approximate equality for the modal frequencies means that they are close relative to the half power bandwidth: $|\omega^f{}_1 - \omega^f{}_2| << 2\zeta_1\omega_1$. In the following, we shall consider only two response points denoted 1 and 2, and denote the residues A_{11}, A_{21} and so forth. The discussion is valid even if these are merely a subset of a larger collection of measurement points.

3.1. SIMO Identification

In this section we consider the case where a single-input multi-output test has been performed, and we designate the first response point as the drive point, so the true mode vector for the mode in question is

$$\{\phi\}_1 = \begin{Bmatrix} A_{11}{}^{1/2} \\ A_{21}A_{11}{}^{-1/2} \end{Bmatrix}, \tag{9}$$

where the subscripts $()_{mn}$ denote the residues at the mth response point due to excitation at the nth drive point. This yields a non-physical result if the drive point residue is negative. Under the assumption that the eigenvector does not change, the true modified residue $A_1 = A^{mod}{}_1$, so the frequency response functions of the modified system should have frequency content that is shifted by

$$\Delta\omega^{mod}{}_1 = -\omega_1{}^2\phi_{n1}{}^2\Delta m_n \,, \tag{10}$$

from ω_1, with amplitudes A_1 at each response-drive point pair.

Modal parameter estimation routines sometimes artificially localize the response of two modes when over-fitting occurs. In this case, each of the modes is large in only one response point, so the following set of residues and resulting mode vectors satisfy the constraints in eq. (8).

$$A^f{}_1 = \begin{Bmatrix} A_{1,1} \\ \varepsilon_2 A_{2,1} \end{Bmatrix} \quad A^f{}_2 = \begin{Bmatrix} \varepsilon_1 A_{1,1} \\ A_{2,1} \end{Bmatrix} \quad \{\phi^f\}_1 = \begin{Bmatrix} A_{1,1}{}^{1/2} \\ \varepsilon_2 A_{2,1} A_{1,1}{}^{-1/2} \end{Bmatrix} \quad \{\phi^f\}_2 = \begin{Bmatrix} \left(\varepsilon_1 A_{1,1}\right)^{1/2} \\ \left(\varepsilon_1\right)^{-1/2} A_{2,1} A_{1,1}{}^{-1/2} \end{Bmatrix} \tag{11}$$

The frequency shift for each of these modes can be estimated by eq. (5). For mass modification at point 1, we see that fit mode 1 shifts as it should, while the second does not, so the response predicted by substructure coupling using this fit would contain two modes where one should be. However, the response of mode 2 is small in both H_{11} and H_{12}, so this is of little consequence. On the other hand, modification at point 2 leaves fit mode 1 unchanged and shifts the second mode much more than it should. In this case, mode 1 is of little consequence, but mode 2 has an artificially large response, as well as the wrong frequency shift. These difficulties stem from the fact that this SIMO curve fit for H_{11} and H_{12} results in an erroneous estimate of H_{22}.

Overall, this analysis suggests that, in the case of localized modes, the erroneous curve fit will give adequate results for the measured FRFs, so long as the modifications are applied at a measured drive point. We can extend this reasoning to show that good results are obtained at any point so long as the set of response and drive points is adequate to accurately establish each mode vector.

Other cases are also of interest. For example, consider a case where the fit residues are not localized, but instead have similar amplitudes in each response coordinate (e.g. $A^f{}_1 = A^f{}_2 = 0.5A_1$). The sensitivity formula predicts that each of the fit modes would be shifted by an insufficient amount, so the response would have the correct amplitude but the wrong frequency. However, the simple model presented in the next section shows that the results are perfectly accurate in this case! The failure of modal sensitivity to produce the correct result can be traced to the assumption of unchanging mode vectors. Small modifications can produce drastically different mode vectors due to small modifications if the system has modes with close natural frequencies [29], so one must use caution when applying this formula, and many common rules of thumb likewise only hold for systems with distinct natural frequencies.

3.2. MIMO Identification

In a MIMO experiment, a matrix of residues is estimated for each mode.

$$A_1 = \begin{Bmatrix} \phi_{11} \\ \phi_{21} \end{Bmatrix} \begin{bmatrix} \phi_{11} & \phi_{21} \end{bmatrix} \approx \begin{bmatrix} A^f{}_{11,1} & A^f{}_{12,1} \\ A^f{}_{21,1} & A^f{}_{22,1} \end{bmatrix} + \begin{bmatrix} A^f{}_{11,2} & A^f{}_{12,2} \\ A^f{}_{21,2} & A^f{}_{22,2} \end{bmatrix} \tag{12}$$

Best practice modal parameter identification routines assure that each of the fit residue matrices above has only rank one, but one can see that it is still possible for the combination of fit modes to artificially localize the true mode, for example.

$$A_1 = \begin{Bmatrix} \phi_{11} \\ \phi_{21} \end{Bmatrix} \begin{bmatrix} \phi_{11} & \phi_{21} \end{bmatrix} \approx \begin{bmatrix} \varepsilon_1 & \varepsilon_1 \\ \phi_{21}\phi_{11} & \phi_{21}{}^2 \end{bmatrix} + \begin{bmatrix} \phi_{11}{}^2 & \phi_{11}\phi_{21} \\ \varepsilon_1 & \varepsilon_1 \end{bmatrix} \tag{13}$$

However, neither of the residue matrices above obeys reciprocity, so one could not reduce these residue matrices to a single mode vector. Unfortunately, most modal parameter identification routines do not provide a way of enforcing reciprocity; it is usually enforced in post processing, after which one can reconstruct the FRFs to see whether this situation has occurred. If reciprocity could be enforced, then the fit mode vectors would obey the following,

$$A_1 = \begin{Bmatrix} \phi_{11} \\ \phi_{21} \end{Bmatrix} \begin{bmatrix} \phi_{11} & \phi_{21} \end{bmatrix} \approx \begin{Bmatrix} \phi^f{}_{11} \\ \phi^f{}_{21} \end{Bmatrix} \begin{bmatrix} \phi^f{}_{11} & \phi^f{}_{21} \end{bmatrix} + \begin{Bmatrix} \phi^f{}_{12} \\ \phi^f{}_{22} \end{Bmatrix} \begin{bmatrix} \phi^f{}_{12} & \phi^f{}_{22} \end{bmatrix} \tag{14}$$

in which case the second identified mode vector must either be uniformly small or else linearly dependent on the first if A_1 is to be near rank-one. Modal parameter sensitivity can be used to show that the response of the modified system would be captured quite well if the second mode is uniformly small, and best practice modal parameter identification procedures would reject adding a mode whose mode shape is not distinct from that of other modes.

3.3. Discussion

The results thus far show that over-fitting should have little effect on the accuracy of substructuring predictions, so long as the over-fit model still reconstructs the measured FRFs correctly. However, there are a few very notable observations:

1.) The SIMO example shows that over fitting can introduce very large errors in the reconstruction of drive points that were not measured. If modifications are applied to those points, then the predictions of all of the modified FRFs may contain very large errors.
2.) If MIMO identification is employed, using standard practices that enforce rank-one but not necessarily reciprocal mode vectors, then one may still artificially localize the modes, yet clear contradictions arise if one tries to express the localized modes in terms of a single mode vector and its transpose. It is important to note that if the identification is used merely to smooth the FRF data, and a modal model is not sought, then this class of MIMO identification may produce erroneous results, as in the SIMO case.
3.) A modal parameter identification routine that enforces reciprocity would be quite useful for substructuring applications, as it would assure that an artificially localized model does not fit the measurements. The SMAC [21] or AMI [24-26] algorithms could potentially provide this capability.

The first observation does not necessarily imply that every point where modifications are applied must be tested, as mentioned previously, only that one must be careful not to fit a model with close modes if the FRF database does not justify it.

4. Example

This section explores these issues further using a simple two-mass system, which is non-dimensionalzed such that $m_1 = m_2 = 1$, and the spring stiffnesses $k_1 = 1$, $k_2 = 1.02$, $k_{12} = 0.1$, with those stiffnesses corresponding to the stiffness between mass 1 and ground, mass 2 and ground, and between masses 1 and 2 respectively. This system has two relatively close modes, whose damping ratios are set at 0.01 and 0.015 respectively. Slightly inconsistent measurements are simulated by adding a mass of 0.01 to m_1 when simulating H_{11} and H_{21}, and subtracting a mass of 0.02 from m_2 when simulating H_{12} and H_{22}. The simulated FRFs are then contaminated by adding a small amount of Gaussian noise.

4.1. MISO (SIMO) Case

When modal parameter identification is applied to H_{11} and H_{12}, there is some indication that two modes are present at the first peak in the FRFs (where only one mode actually resides), due to the slight inconsistency in

the measurement. If two modes are fit to that peak and one to the second, then one obtains the results shown in Table 1.

True Parameters						
Mode	**ωr**	**ζr**	**A11,r**	**A12,r**	**ϕ1,r**	**ϕ2,r**
1	1.0047	0.010	0.571	0.419	0.741	-0.671
2	1.1002	0.015	0.494	-0.494	0.671	0.741
Identified Model						
Mode	**ωr**	**ζr**	**A11,r**	**A12,r**	**ϕ1,r**	**ϕ2,r**
1	1.0019	0.010	0.548	-0.030	0.74	-0.04
2	1.0097	0.011	0.026	0.522	0.16	3.21
3	1.1027	0.014	0.406	-0.454	0.64	-0.71

Table 1: True parameters and those identified from slightly inconsistent MISO data set.

The reconstruction with three modes agrees very well with the measurements, and improves the fit somewhat as compared to a two-mode fit. However, after modifying the structure by adding a mass at either point 1 or 2, some highly erroneous results are obtained. Figure 1 shows the analytical FRFs H_{11}, H_{12} and H_{22} of the system before modification, the analytical FRFs after modification, and the FRFs of the modified system computed from the identified modal parameters. The column on the left corresponds to a mass modification of 0.5 added to point 1; the one on the right shows the result of adding a mass of 0.5 to point 2. The predictions are quite reasonable for H_{11} and H_{12} when the mass is added to point 1, in spite of the erroneous model order in the curve fit, but H_{22} is not reconstructed accurately (not shown), and after modifying the system the modified H_{22} is totally in error. On the other hand, if the mass modification is applied to point 2, then none of the FRFs is accurately predicted after modification.

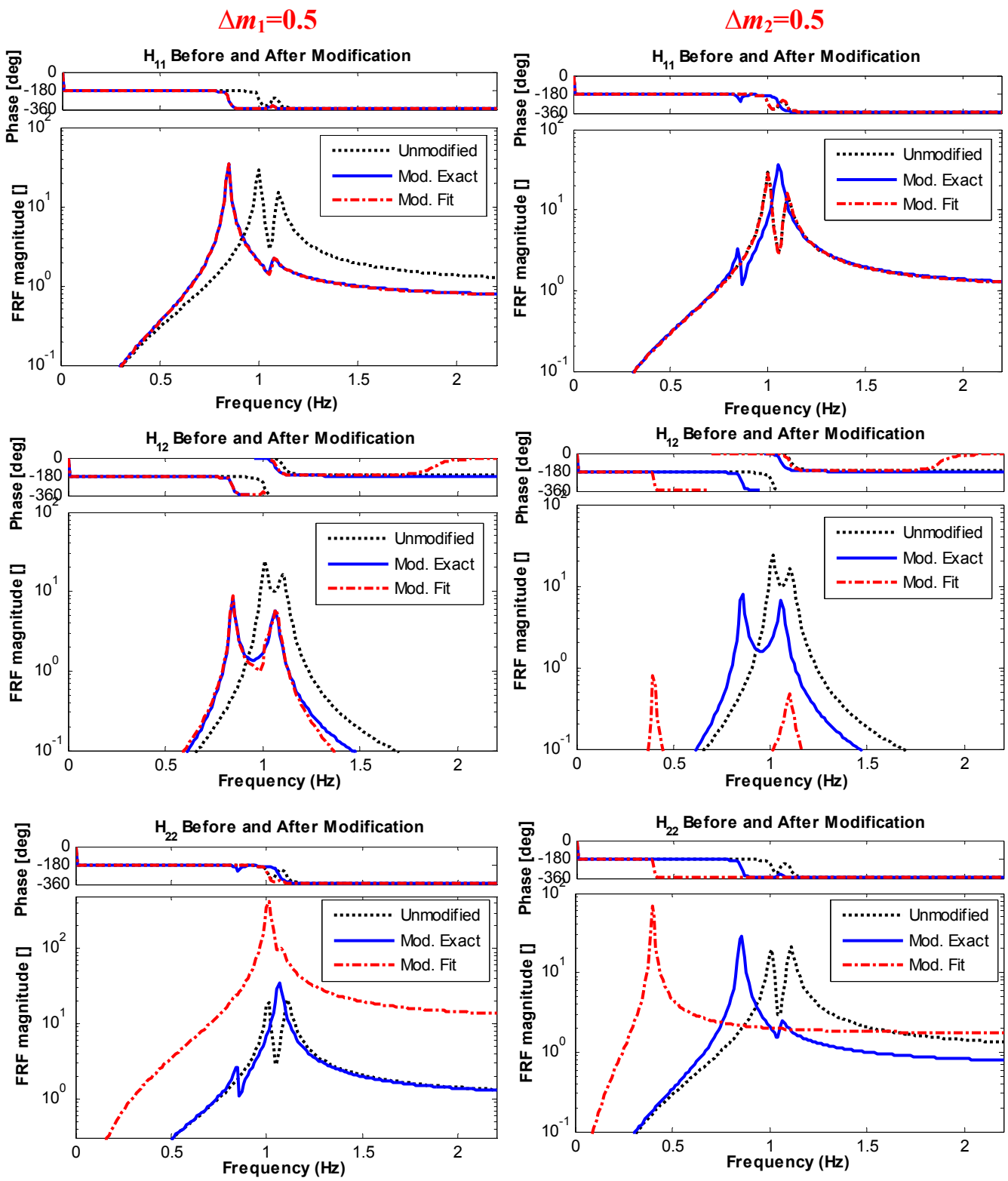

Figure 1: SIMO Identification. (black dotted) Analyitical FRFs of two-mass system before modification. (Identified FRFs agreed well with these for H_{11} and H_{12}, but not for H_{22}.) (blue solid) Analytical FRFs of modified system. (red dash-dot) FRFs of modified system, computed from identified modal parameters.

4.2. MIMO Case

When the full set of MIMO FRFs is employed in modal parameter identification, there remains some indication that two modes might be present at the first peak, but the identification routine does not localize the

modes as heavily due to the rank-one residue constraint (reciprocity was not enforced). The results are shown in Table 2. The first block shows the results when the rank-1 residue at drive point 1 is used to construct the mode vectors. The second (spurious) mode vector is somewhat small, so one would expect that it wouldn't influence the results too severely. This was verified by observing that these modal parameters reconstruct all of the FRFs accurately, and they accurately predict the modified FRFs as well, although those FRFs are not shown here for brevity. The residues are less localized at the second drive point, so there are significant errors in the mode vectors estimated from those residues. However, they still reproduce the FRFs of the modified system quite well, as shown in Figure 2. On the other hand, when a conservative approach was taken, and only two modes were identified for the system, modal model produces the modified FRFs almost perfectly (the FRFs for this case are not shown for brevity).

Identified Model - MIMO Experiment - Drive Pt. 1						
Mode	**ωr**	**ζr**	**A11,r**	**A12,r**	**ϕ1,r**	**ϕ2,r**
1	1.0025	0.009	0.481	0.414	0.69	0.60
2	1.0160	0.008	0.068	0.052	0.26	0.20
3	1.1029	0.015	0.415	-0.486	0.64	-0.76
Identified Model - MIMO Experiment - Drive Pt. 2						
Mode	**ωr**	**ζr**	**A11,r**	**A12,r**	**ϕ1,r**	**ϕ2,r**
1	1.0025	0.009	0.218	0.184	0.51	0.43
2	1.0160	0.008	0.175	0.129	0.49	0.36
3	1.1029	0.015	-0.488	0.578	-0.64	0.76

Table 2: Parameters identified from a slightly inconsistent MIMO dataset.

It is important to note that, based on modal parameter identification theory, one would expect to incorrectly predict the modified FRFs of this system using the fit modes. For example, for the Drive Point 2 case, the new natural frequencies predicted by structural modification theory (assuming the eigenvectors do not change), are 0.94, 0.94, and 0.87, which are significantly different from the true, modified natural frequencies: 0.85, and 1.07. However, the results below show that the fit parameters accurately predict the FRFs of the system after modification.

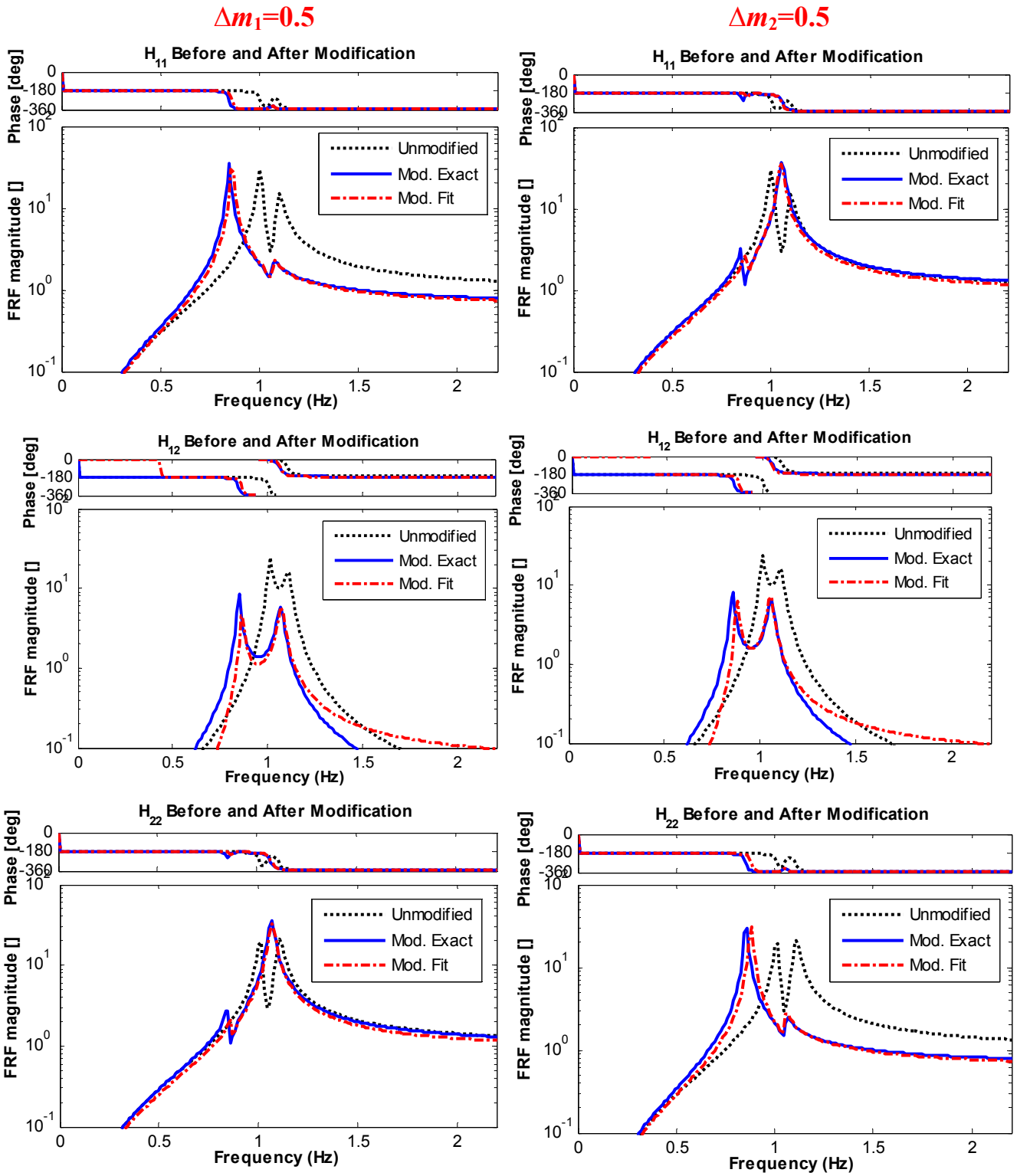

Figure 2: Modal Parameters from MIMO Identification, drive point 2 used to compute modified FRFs. (black dotted) Analyitical FRFs of two-mass system before modification. (Identified FRFs agreed well with these for H_{11}, H_{12} and H_{22}) (blue solid) Analytical FRFs of modified system. (red dash-dot) FRFs of modified system, computed from identified modal parameters.

5. Experimental Application

Experimental measurements from a complicated, high-order system were used to evaluate the effect of over-fitting. The set of measured FRFs contain 32 inputs and 73 outputs, and were measured on a system with many close natural frequencies, some of which are localized while others are not. The measurements were collected in blocks of four inputs at a time over the course of a day, and the system is potentially nonlinear, so it is likely that the measurements are not perfectly consistent. The AMI modal parameter identification routine [24-26] was used identify two modal models for the system. For the first model, a very conservative approach was taken, where the minimum order model that fit the measurements reasonably well was used at each peak. The opposite approach was taken for the second, adding additional modes to try to improve the agreement between the reconstructed and measured FRFs, and this latter case is referred to as "non-conservative fitting" in the following.

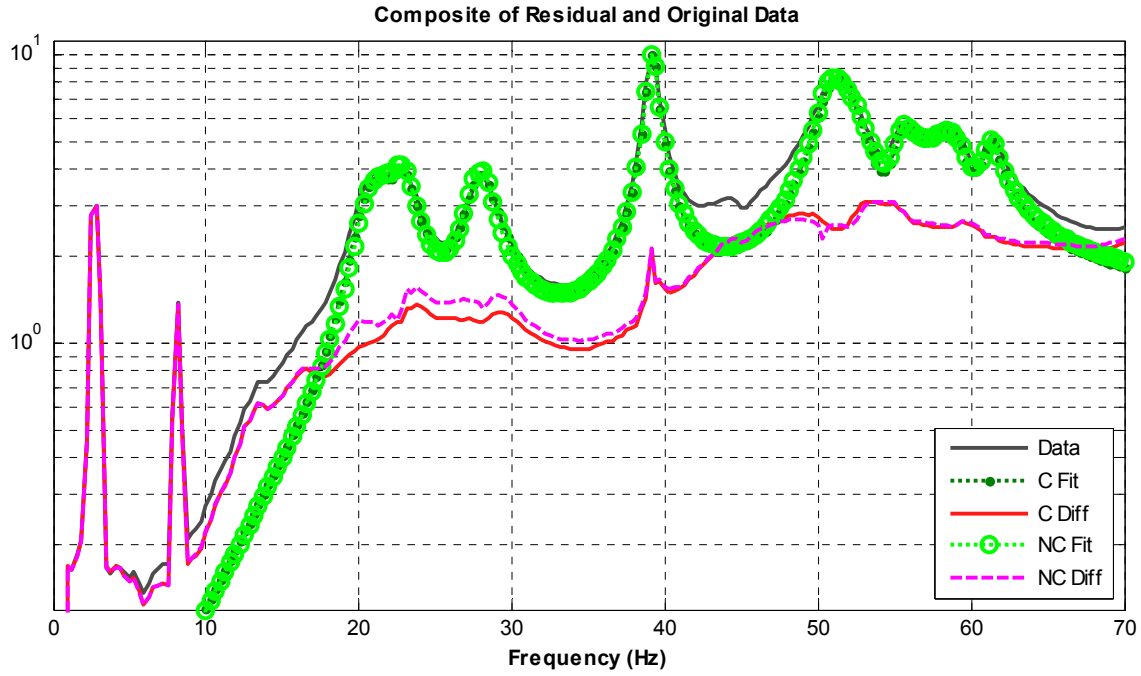

Figure 3: Composite FRF of measurements and two reconstructions of the measurements based on fitting a conservative and non-conservative number of modes to the measurements. A composite of the difference between each reconstruction and the measurements is also shown.

Figure 3 shows composite FRFs of the two curve fits, the measurements, and composites of the difference between the measurements and each curve fit. Both curve fits agree reasonably very well with the measurements above 20 Hz, and only the difference between the data and the fits reveals that the non-conservative fit does agree somewhat more closely with the measurements. In both cases, the AMI algorithm forced rank-one residues on the model, but none of the responses were co-located with drive points, so reciprocity could not be enforced nor checked. The composite FRF above shows only the average agreement between the curve fit and the measurements. Many of the individual FRFs showed much worse agreement, which was the motivation for trying to increase the number of modes. Even then, neither of these curve fits reproduces all of the FRFs well, and it was a laborious task even to look through all of the FRFs to assess the agreement. In most cases, the poor agreement seems to occur at FRFs that had small responses compared to this one, so disagreement there is not manifested very strongly in the composite FRF.

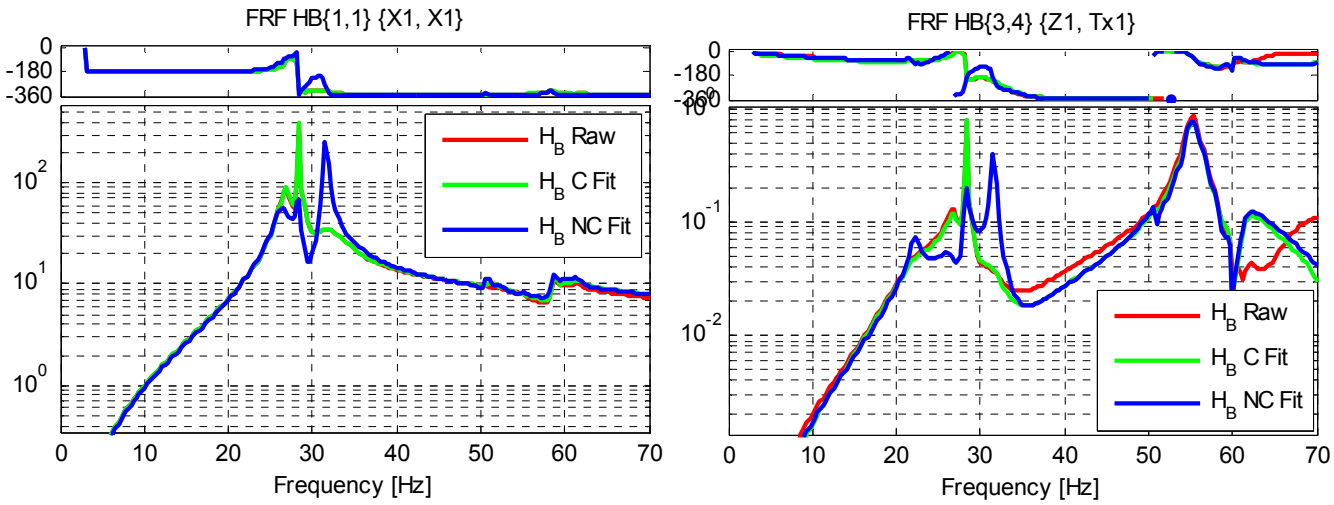

Figure 4: Sample FRFs after admittance uncoupling using measurements, conservative fit and non-conservative fit. (left) x-x drive point FRF (right) z-θ_x FRF.

Figure 4 shows the result of removing four substructures from the system that was tested, using an FBS uncoupling procedure [30]. Although none of the points at which the substructures were removed was directly measured, their motion was determined indirectly using an array of nearby measurements, so this is essentially a case where the response has been measured at the point at which the structural modification is applied. Two FRFs are shown for three cases using the: actual measurements, a conservative fit, and a non-conservative fit in the admittance procedure. The drive point plot on the left shows that all three results capture the general trend of that FRF, although the non-conservative fit is erratic near the peak, with what seems to be reasonable frequency content but large amplitude errors. The situation is worse for the z-θ_x FRF on the right, although in this case there are some significant differences between all three curves at higher frequencies. At low frequencies, the conservative fit matches the result obtained with the raw measurements most closely, although one does not know whether the measurements themselves are producing the correct answer.

6. Conclusions

This work does not even come close to fully addressing the difficult question of model order and its effect on substructure coupling / uncoupling, but it hopefully sheds some light on the subject and perhaps dispels some potential misconceptions. The effect of over-fitting on substructure coupling predictions was investigated, showing that over-fitting can induce serious errors in the predictions. Some situations were suggested theoretically in which this might occur, helping to reveal how these situations might be detected. A simple example showed that the potential for dramatic sensitivity is certainly severe when SIMO fitting is used and modifications are applied at points that were not drive points in the original measurements, because the modal parameter identification package may artificially localize the modes if too many modes are fit to a particular peak in the response. The collection of localized modes may respond to a modification in an entirely different way than the true mode would. However, some limitations of structural modification theory were discussed, revealing that it may give misleading results when the identified system has modes with close natural frequencies. One should use care when applying modification theory or any other related reasoning to systems with close natural frequencies, or even to measurements that, due to small inconsistencies, may appear to have modes with close natural frequencies.

One can minimize the potential for artificially localizing modes during parameter identification by applying a MIMO algorithm and assuring that the fit model has reciprocal, rank-one residue matrices. In that case, it was shown that any spurious modes must be either linearly dependent on other identified modes or small relative to the true modes. Unfortunately, the modal parameter estimation routines currently available do not allow enforce reciprocity as an integral part of the identification; it can usually only be applied in post-processing. However, application to a simulated two DOF system demonstrated that an over-fit modal model (with an extra mode) identified using a MIMO algorithm gave good predictions of the modified system's response. This seems to be attributable to the fact that the reconstructed FRFs agreed well with the measurements for each point at which a modification was applied. This may be explained by the concept of FRF based sensitivity, which asserts that the modified FRFs should be predicted accurately so long as FRFs are accurate at the point where the modification is applied, irrespective of how many modes are used to represent those FRFs.

An application to experimental measurements illustrates the difficulty of estimating the model order of some real-world systems; it is sometimes very difficult to determine whether one is over- or under-fitting the response, and different models may give greatly differing predictions. After the number of modes identified was increase, the predictions of the uncoupled response differed greatly from those obtained using the raw measurements, suggesting that over-fitting had occurred. However, neither the conservative nor non-conservative approaches gave a model that represented all of the hundreds of available FRFs accurately. Fortunately, the substructuring predictions did capture the general trends of the response of the coupled system no matter which method was employed.

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