The Measurement of a Nonlinear Resonant Decay using Continuous-scan Laser Doppler Vibrometry

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Abstract
The nonlinear resonant decay of a structure offers much insight into the frequency-amplitude behavior of a structure's dynamic response. The spatial deformation during this decay is especially important since nonlinear responses can cause unexpected stress concentrations necessitating full-field measurements for comparison with a model. In this context, full-field measurement techniques, such as continuous scan laser Doppler vibrometry (CSLDV) and high speed three dimensional digital image correlation (3D-DIC) provide tools to obtain the full-field dynamic response experimentally. While CSLDV has been used to measure the steady state response of linear and nonlinear structures as well as transient responses of linear structures, it is unclear whether the approach can be successful for transient nonlinear measurements where the frequency of the dynamic response is amplitude dependent. In this investigation, the capabilities of CSLDV will be utilized to measure the nonlinear resonant decay of a clamped-clamped flat beam. The response measured using CSLDV will then be compared with the decay response measured with 3D-DIC to validate the CSLDV method and to understand the advantages and disadvantages of each.

Keywords: Continuous-scan Laser Doppler Vibrometry, 3D Digital Image Correlation, Resonant Decay

1. Introduction
The measurement of a structure operating in a dynamic environment requires special considerations, especially when the structure of interest is lightweight. In this context, noncontact sensors are needed so the dynamic measurements are not corrupted by mass loading. Since light weight structures can also be prone to nonlinear behavior, the concept of a roving sensor and averaging cannot be directly applied to a point-by-point measurement of the structure. Therefore, non-contact, full-field measurement techniques are ideal when measuring light-weight structures prone to nonlinear behavior in dynamic environments. In this work Continuous-Scan Laser Doppler Vibrometry (CSLDV) and high-speed Three Dimensional Digital Image Correlation (high-speed 3D-DIC) have been employed to measure the full-field dynamic response of a clamped-clamped flat beam.

Previous works have shown the capability of CSLDV to measure full-field deformations of a structure oscillating in linear response regimes when subjected to sinusoidal [1], random [2, 3], and impulse excitation [4, 5]. The work by Yang et al [6] which used the demodulation of the CSLDV signal coupled with linear time periodic system identification was extended to examine the dynamic response of a structure oscillating in nonlinear response regimes, but was limited to purely sinusoidal excitation [7]. On the other hand, the capability of high-speed 3D-DIC has been explored on structures operating in linear and nonlinear environments when subjected to sinusoidal [7], random [8, 9], and impulse excitation [10, 11], with no major difference in the application of the method.

In this investigation, CSLDV and high-speed 3D-DIC are used to measure the nonlinear resonant decay of a 228.6 mm steel clamped-camped beam. At large amplitudes of vibration, the fundamental frequency of vibration for this setup increases due to the geometric nonlinear stiffness. In this work, the fundamental frequency of vibration is 176% higher than the linear natural frequency of the first bending mode at the start of the decay. Displacements measured using the high-speed 3D-DIC setup are post-processed using a commercial software Aramis [12] and its Real Time Sensor program [13], and velocities measured with CSLDV are post-processed using linear-time periodic algorithms developed in [4].

2. Background

2.1 High-speed 3D-DIC Processing

Accurate 3D displacements can be obtained with digital image correlation using two cameras in a stereo configuration as described in [14]. As shown in Fig. 1, the two cameras (Photron SAZ's) are placed at a specific distance from the test article to allow the surface to be captured simultaneously and an LED panel light is used to illuminate the surface. A frame rate of 7000 Hz is used to image the structure over 15 seconds resulting in 105000 images requiring 15.2 GB of memory.

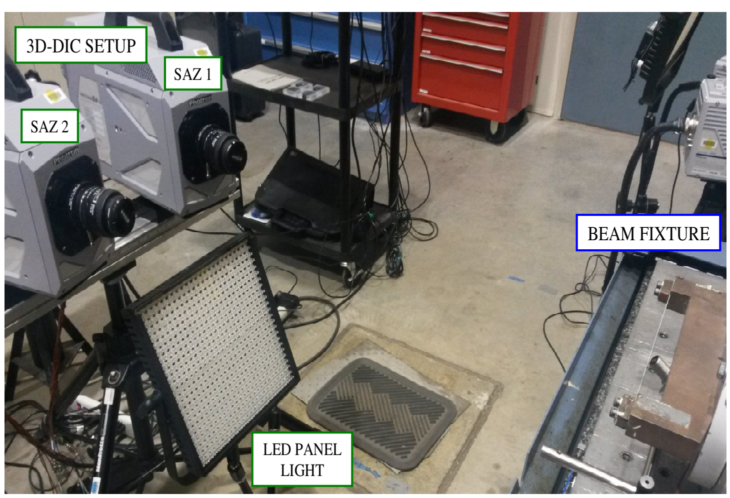


Figure 1: 3D-DIC System

Once the stereo camera setup is assembled and fixed, principles of triangulation are used to establish each camera's position in reference to the global experimental coordinate system. Additionally, lens distortion and variations between the sensor of the camera and the final images can be corrected through a bundle adjustment [14]. The coordinate system transformation matrix is established through the use of images of a known pattern or calibration panel. With this calibration, the accuracy of the coordinate transformation matrix is not limited to the pixel size of the imaged surface of the test specimen, but instead can be interpolated on the sub pixel level (i.e. calibration deviation for this setup is 0.018 pixels). Once the calibration of the 3D-DIC system is established, images of the fully deformable structure can be analyzed to obtain displacements. Prior to testing, a high-contrast random pattern is applied to the measurement surface so the defined subsets can be uniquely and accurately fit. Triangulation is used to determine the coordinate value of each subset using the Real Time Sensor mode in IVIEW [12]. In this work, the subset size is 15pixels x 15pixels laid out manually as shown in Section 2.3. The final displacements are processed with the Hilbert Transform so the instantaneous frequency and amplitude is determined.

2.2 CSLDV Processing

A laser Doppler vibrometer (LDV) is a non-contact measurement technique that detects the Doppler frequency shift in a beam of laser light and converts it to the velocity component of the measurement point along the direction of the incident laser [15]. Continuous-scan laser Doppler vibrometry (CSLDV) extends the LDV measurement by moving the laser beam across the surface of a structure with a pre-defined pattern [16, 17]. In this work, a periodic line pattern is used so that the observed deformation becomes a periodic function of time [2, 18] and hence the deformation shape doesn't need to be approximated with a polynomial in the spatial coordinates as done in [16, 17, 19-21].

The laser beam of a fiber optic LDV is scanned across the beam's surface in a periodic line pattern at 145 Hz using external mirrors, as seen in Fig. 2. The laser beam of a second fiber optic LDV is also set on the beam's surface for comparison to CSLDV measurements. Both lasers are sampled at 20 kHz.

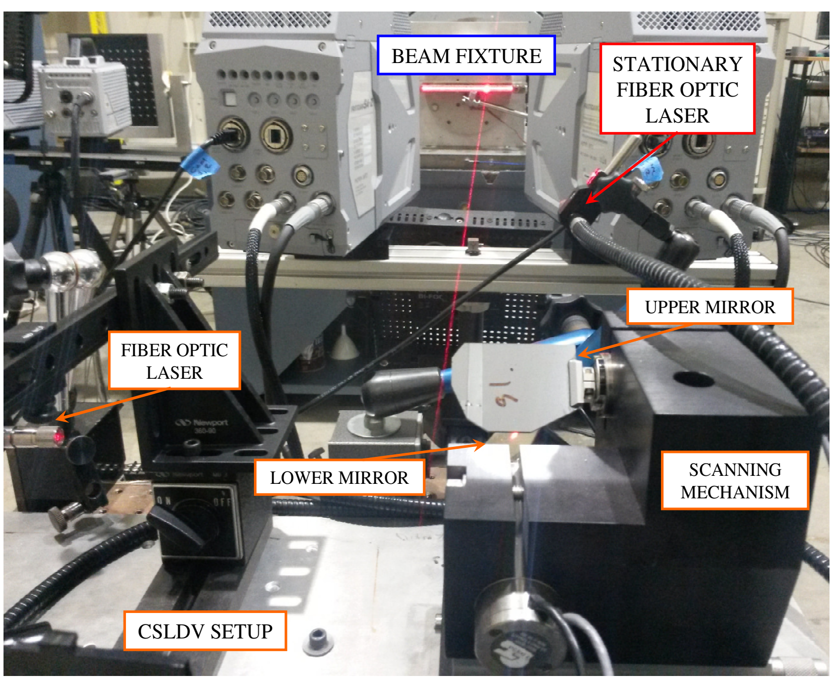
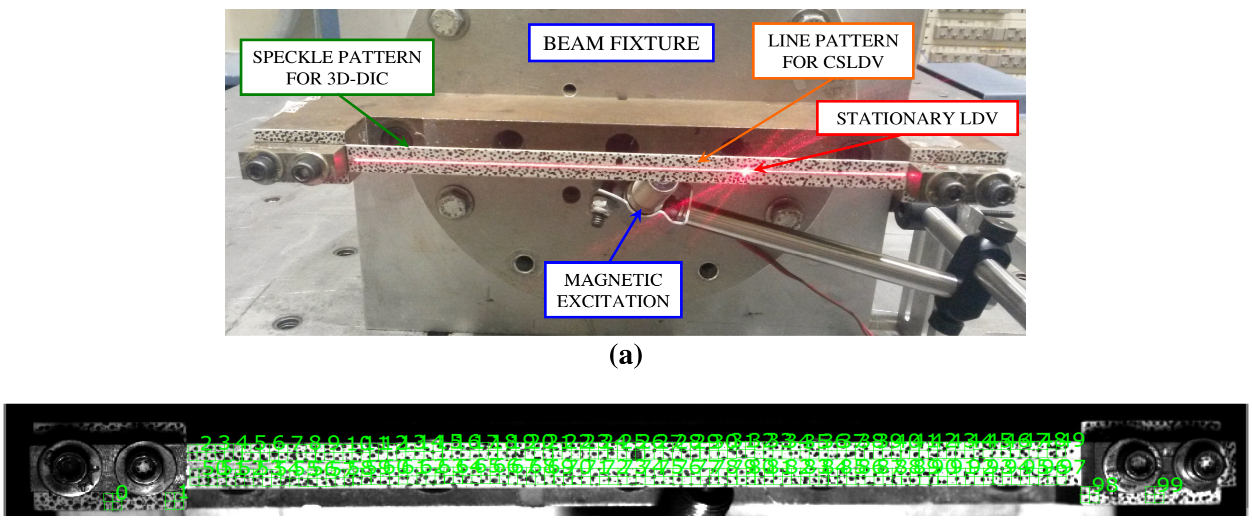


Figure 2: CSLDV System Diagram. The laser beam was redirect by a pair of rotating mirrors.

The implementation of a time-periodic CSLDV scan pattern couples the motion of the measurement point with the deformation of the structure, so the measured velocity appears as an amplitude modulated signal. Therefore, post-processing is required to separate the laser motion from the structural deformation as previously discussed in [22]. In this work, the free decay measured with CSLDV is divided into blocks of 8000 samples so the average frequency and deformation is examined instead of the instantaneous decay. However, results show a favorable comparison.

Structure of Interest

The structure for this investigation is a precision-machined feeler gauge made from high-carbon, spring-steel with clamped boundary conditions. The final clamped-clamped beam configuration has an effective length of 228.6 mm, a nominal width of 12.7 mm, and a thickness of 0.76 mm. Prior to clamping, the beam was painted with a white base coat and a speckle pattern was applied using a marker to allow for tracking positions with the 3D-DIC system. Once the beam had dried, a strip of retro-reflective tape was added to increase feedback for the CSLDV. The final prepared beam is shown in the clamping fixture in Fig. 3.



(a)

(b)

Figure 3: Clamped-clamped beam configuration. a) Physical setup, b) 3D-DIC measurement points

Results

The structure of interest is subjected to a nonlinear resonant decay of the first mode using nonlinear force appropriation as described in [23]. The initial excitation is supplied using a magnetic field generated by a fluctuating voltage through an inductance coil. This method of excitation is beneficial in the measurement of a free decay since the magnetic field can be cut off abruptly and has no lasting effect on the structure.

The magnetic forcing induces an asymmetry in the deformation of the beam, which can be seen in Fig. 4. The asymmetry is most notable in the displacement measurements (Fig. 4a and b), but is also visible in the harmonic distortion observed in the LDV shown in Fig. 4d. Figure 4d also displays the amplitude modulation of the scanning LDV when compared with the stationary LDV. Since the structure is lightly damped, when the force is removed the structure will decay near the fundamental nonlinear normal mode [24]. For this structure, the observable decay occurs in approximately 10 seconds.

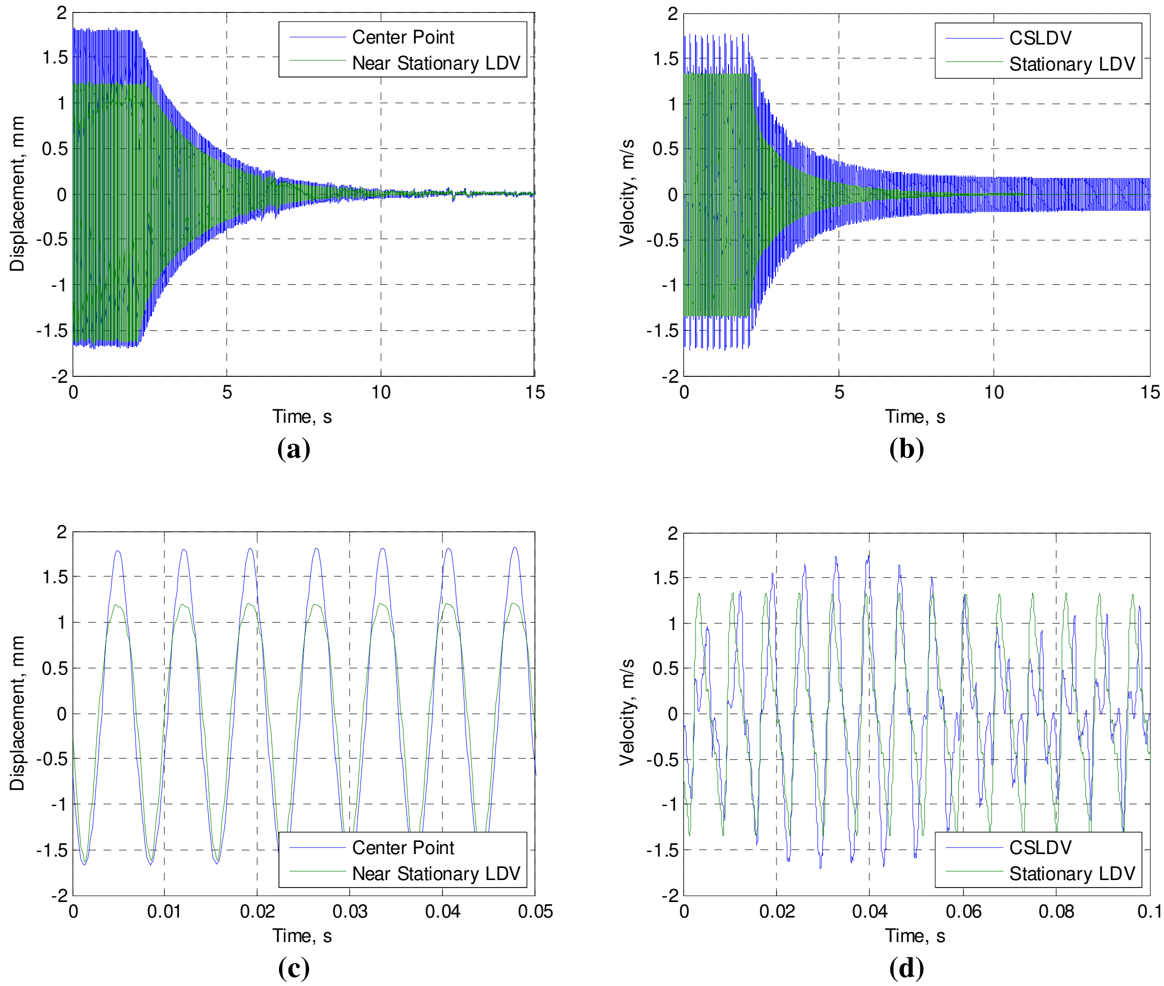
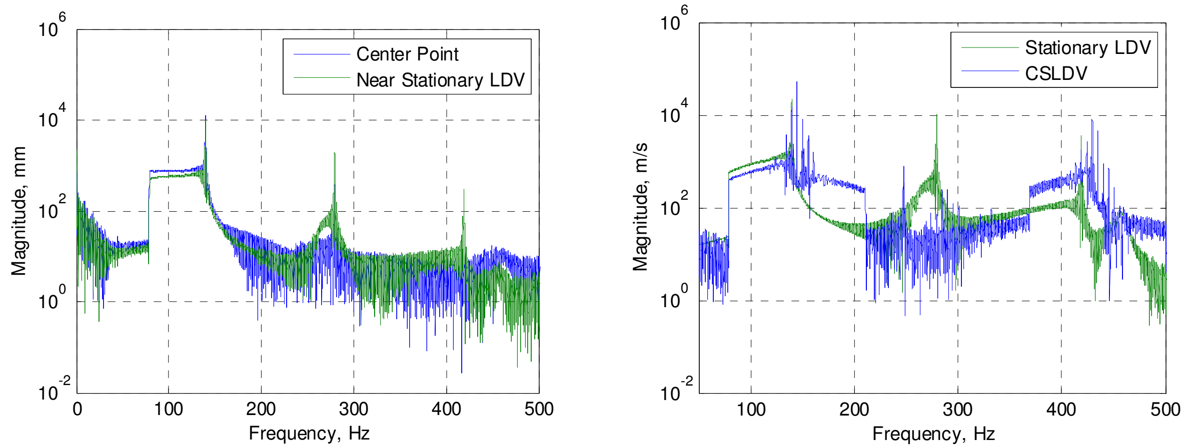


Figure 4: Measured data. a) Displacement measured with 3D-DIC, b) Velocity measured with CSLDV. The first 0.05 seconds of the 3D-DIC (c) and 0.1 seconds of the CSLDV signal (d).

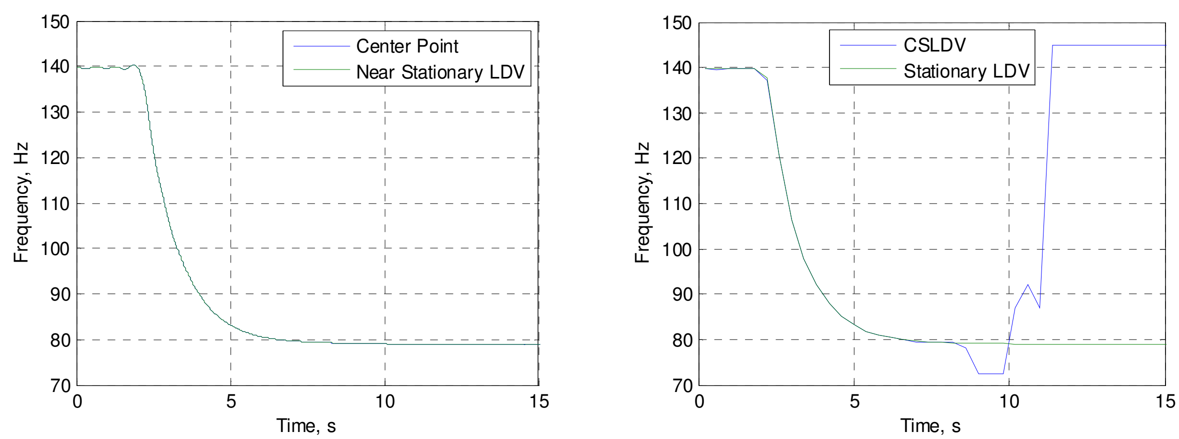
A fast-Fourier Transform (FFT) of the measured time domain signals presented in Fig. 4 provides insight to the frequency content of each signal. The FFT of the displacement measurements (Fig. 5a) shows the fundamental frequency of vibration of the decay starts near 139.7 Hz and ends near 79.3 Hz. Although higher frequencies appear in the response, the focus of this work is on the fundamental frequency. Due to this large shift in the fundamental frequency of vibration, care must be taken in the selection of the scanning frequency used in CSLDV so no overlap of the sideband harmonics of interest occurs. In this investigation 145 Hz is used so the 0th, ±2nd, and ±4th sideband are visible. All odd and higher sideband harmonics are below the noise floor for this setup. Figure 5b shows the FFT of the LDV signals where the 0th and ±2nd sideband harmonics are visible.



(a) **(b)**

Figure 5: Fourier transform of the a) 3D-DIC and b) CSLDV measurements.

The instantaneous frequency can be approximated for both 3D-DIC and CSLDV measurements using several methods. The displacements measured with 3D-DIC are first filtered around the fundamental frequency of vibration and the Hilbert transform applied to calculate complex amplitude of the signal and approximate the instantaneous frequency. The resulting time vs. frequency plot for 3D-DIC is shown in Fig. 6a. The LDV measurements are split into 8000 sample blocks with 50% overlap and a least-squares sinusoid fit applied to each block. The resulting time vs. frequency plot for CSLDV is shown in Fig. 6b. The stationary LDV point follows the instantaneous frequency with similar accuracy to the 3D-DIC measurements; however, the estimated instantaneous frequency from the CSLDV measurement is corrupted by the noise from scanning the laser.



(a) **(b)**

Figure 6: Instantaneous frequency estimated from: a) 3D-DIC and b) CSLDV measurements.

Although the instantaneous frequency predicted with CSLDV is corrupted by noise the full-field displacements from each measurement technique agree well as shown in Fig. 7. At the initial decay, the CSLDV is shown to slightly under predict the displacement experienced by the beam when compared with 3D-DIC. This is difference is likely from the 'averaged' displacement measurement taken in the blocked CSLDV data as opposed to the predicted instantaneous deformation calculated with the Hilbert transform of the 3D-DIC displacements.

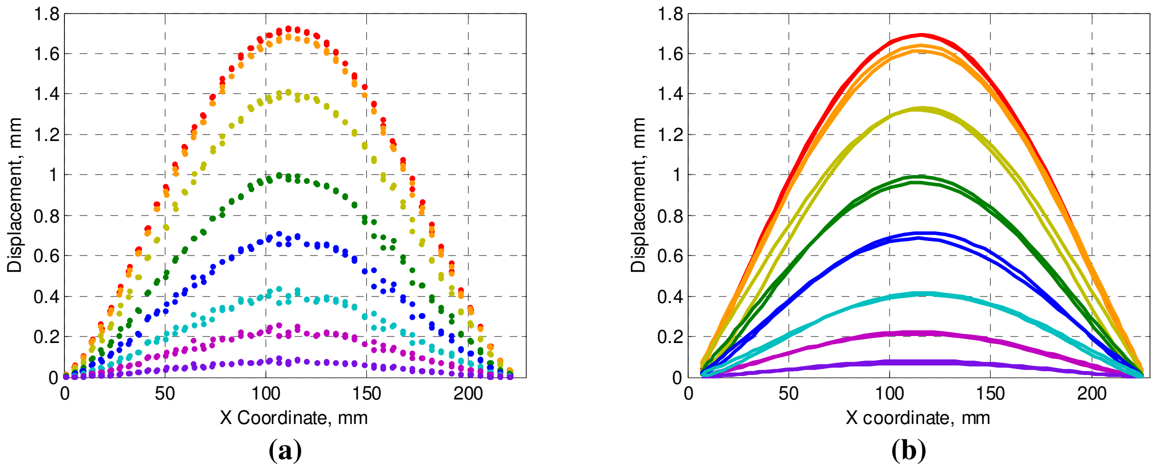


Figure 7: Full-field deformation using a) 3D-DIC and b) CSLDV near 139.7 Hz (-), 136.8 Hz (-), 121.0 Hz (-), 102.6 Hz (-), 91.2 Hz (-), 83.6 Hz (-), 80.6 Hz (-), and 79.34 Hz (-).

Conclusions

In this investigation, a nonlinear resonant decay is measured with 3D-DIC and CSLDV. It is shown that the 3D-DIC can capture the frequency and amplitude of the decay as the structure's response decays into the linear realm; however, 3D-DIC will be subject to more noise at lower response amplitudes. It is discussed that care must be taken when selecting the scanning frequency of the CSLDV so there is no overlap in the desired signal and the sidebands of the response. This work focused on the fundamental frequency of vibration since higher frequencies observed in the response of the beam overlapped the sidebands due to the scanning laser in the CSLDV measurement corrupting the velocity signal. It was shown that CSLDV was able to provide similar amplitude measurements for the range shown; however, the predicted frequency became corrupted by the noise from scanning the laser at lower amplitudes.

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