Proceedings of IMECE2006
2006 ASME International Mechanical Engineering Congress and Exposition
November 5-10, 2006, Chicago, Illinois USA

IMECE2006-13175

MODELING AND INPUT OPTIMIZATION UNDER UNCERTAINTY FOR A COLLECTION OF RF MEMS DEVICES

M. S. Allen*
J. E. Massad
R. V. Field, Jr.
Applied Mechanics Development, Org. 1526
Sandia National Laboratories[1]
Albuquerque, New Mexico 87185

ABSTRACT

The dynamic response of an RF MEMS device to a time-varying electrostatic force is optimized to enhance robustness to variations in material properties and geometry. The device functions as an electrical switch, where an applied voltage is used to close a circuit. The objective is to minimize the severity of the mechanical impact that occurs each time the switch closes, because severe impacts have been found to significantly decrease the design life of these switches. The switch is modeled as a classical vibro-impact system: a single degree-of-freedom oscillator subject to mechanical impact with a single rigid barrier. Certain model parameters are described as random variables to represent the significant unit-to-unit variability observed during fabrication and testing of the collection of nominally-identical switches; these models for unit-to-unit variability are calibrated to available experimental data. Our objective is to design the shape and duration of the voltage waveform so that impact velocity at switch closure for the collection of nominally-identical switches is minimized subject to design constraints. The methodology is also applied to search for design changes that reduce the impact velocity and to predict the effect of fabrication process improvements.

INTRODUCTION

Radio Frequency Micro Electro Mechanical System (RF MEMS) switches have been the subject of study for a number of applications because they can potentially provide very low power consumption, high isolation, and greater linearity at low cost and in a compact package [1] [2] [3]. Unfortunately, current designs for RF switches fail to achieve the high reliability demanded for many applications. The high velocity with which the switches can impact the electrical contacts is one contributing factor. Recent research at Sandia has revealed that the actuating voltage pulse can be shaped to limit the velocity with which the plate impacts the electrical contacts, increasing a switch's life by orders of magnitude. Unfortunately, there is considerable unit-to-unit variability in the dimensions and the properties of these switches, so a waveform designed to minimize the contact velocity, or provide a soft landing, for the nominal switch is not effective for a batch of switches manufactured using current processes.

This work demonstrates that the actuating voltage waveform can be optimized for a collection of RF switches with random physical parameters in order to minimize the contact velocity experienced by the ensemble. This can be cast as a problem of optimization under uncertainty or Reliability-Based Design Optimization (RBDO) [4]. The procedure is also used to optimize the design of the RF switches to reduce the contact velocity of the ensemble and to study the effect of reducing the degree of variation due to the manufacturing process.

Reliability-Based Design Optimization has received considerable attention in recent years. A few researchers have applied RBDO to MEMS systems in order to improve the robustness of device designs, in part because MEMS systems tend to suffer from significant manufacturing variation and exhibit complex or uncertain physical phenomena. Optimization under uncertainty methodologies can be classified as Robust Design Optimization (RDO) methods or as Reliability- Based Design Optimization (RBDO) methods [4]. RDO methods use deterministic analysis to attempt to maximize deterministic performance while minimizing the sensitivity of the optimum design to uncertain or random parameters. For example, Han and Kwak [5] used this ap-

*Address all correspondence to this author. Email: msalle@sandia.gov.
[1]Sandia is a multiprogram laboratory operated by Sandia Corporation, a Lockheed Martin Company, for the United States Department of Energy's National Nuclear Security Administration under Contract DE-AC04-94AL85000.

proach to optimize the design of a MEMS accelerometer and a resonant-type micro probe by augmenting their objective function with the gradient of the objective function with respect to the random parameters. Some limitations of RDO methods are that they cannot provide information about the probability that a device will fail and, because they are gradient based, they may not perform properly if the objective function is noisy or highly nonlinear over the range spanned by the uncertain parameters.

RBDO methods are based on stochastic analysis and are therefore preferred in many applications because they provide an estimate of the reliability of a design and can more accurately account for variability in uncertain parameters. Heo, Yoon and Kim used what may be classified as an RBDO method to optimize the design of a MEMS thermal actuator [6]. Allen *et al* [7], presented an application of RBDO to a variable capacitance MEMS capacitor. They validated the First-Order Reliability Method (FORM) for their application by comparing it to Monte Carlo Simulation (MCS) and then used FORM to optimize the design of the capacitor. FORM can be significantly less computationally expensive than MCS, yet FORM may be inaccurate if the response is non-Gaussian or if the failure boundary is not well approximated by a linear function. Allen *et al* observed that the FORM algorithm worked well even though their system was nonlinear, yet all of the uncertain variables in their system were assumed to be Gaussian with relatively small coefficients of variation. This assumption is often inappropriate for MEMS applications. Maute and Frangopol also used the FORM algorithm as part of an optimization strategy for a MEMS device [4]. One potential limitation of the FORM algorithm is that it includes an iterative search cast as an optimization problem to find the most probable point of failure. As a result, the FORM algorithm becomes much more difficult to apply as the number of uncertain parameters, and hence the dimension of this iterative optimization problem, increases. Also, because FORM is based on optimization, one may encounter situations in which slow or local convergence is obtained, greatly increasing the complexity of implementing FORM and diminishing its computational efficiency. Neither Allen *et al* for Maute and Frangopol mentioned any difficulties obtaining convergence with the FORM algorithm, although their problems were limited to a small number of uncertain variables.

The work discussed here involves an objective function that is highly nonlinear due to mechanical impact and whose uncertainties are large and highly non-Gaussian. For these reasons, Monte Carlo Simulation (MCS) was used to evaluate unit-to-unit variability in these RF MEMS switches. Also, a low order mathematical model exists that captures the physics of the device remarkably well, so the computational burden is low enough that the problem is tractable with MCS.

This paper is organized as follows. We first present a derivation of a reduced order model that provides a good representation of the dynamics of the RF Switch to an actuating voltage. The objective function and optimization procedure are then discussed and some results presented. Finally, the effects of design and process improvement is illustrated followed by some conclusions.

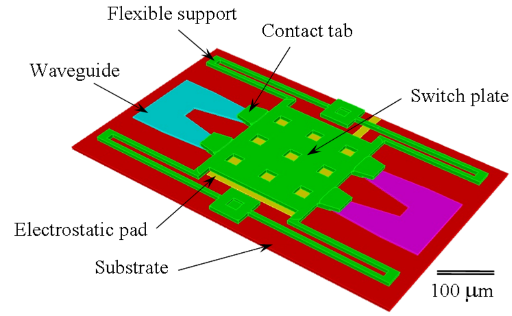

Figure 1. Schematic of RF MEMS switch.

MODEL DEFINITION

The RF MEMS switch design of interest is shown in Fig. 1. The switch consists of a stiff plate supported above a rigid substrate by four flexible supports. A 100 nm thick electrostatic pad is adhered to the substrate below the switch plate to provide electrostatic actuation. When voltage is applied to the pad, the plate deflects downward and the contact tabs make mechanical contact with the waveguide to close the circuit. Dyck *et al* described the design and characterization of this switch in [1].

A single degree-of-freedom model for the RF switch is used for analysis. Previous works have demonstrated the accuracy and utility of this model for these systems, especially when the input is shaped to limit excitation to higher frequency modes [8] [9] [10]. Let $X(t)$ denote the displacement of the contact tabs; the equations of motion are

$$M\ddot{X}(t) + KX(t) = \frac{\varepsilon a}{2}\left[\frac{u(t)}{G - X(t)}\right]^2, \; X(0) = \dot{X}(0) = 0, \quad (1)$$

where M and K denote the effective mass and stiffness of the switch plate, respectively. The right hand side of Eq. (1) defines the applied electrostatic force, where ε and a denote the electric permittivity of air and the surface area of the switch plate, respectively, G is the gap distance between the switch plate and electrostatic pad at $X = 0$, and $u(t)$ is the voltage waveform applied to the pad. Mechanical impact between the contact tabs and waveguide is included by introducing the following kinematic constraint

$$\dot{X}(t^+) = \begin{cases} \dot{X}(t^-) & \text{if } X(t^-) < D, \\ -\eta \dot{X}(t^-) & \text{if } X(t^-) = D, \end{cases} \quad (2)$$

where D denotes the travel distance for switch closure, and $\eta \in (0,1]$ is the (deterministic) coefficient of restitution. A similar model has been used to study the dynamic response of a collection of MEMS inertial switches [11]. Our convention is to denote all deterministic quantities with lower-case letters or symbols and all random quantities with upper-case letters or symbols.

MODEL CALIBRATION

Given a small set of experimentally observed quantities, we need to calibrate the model defined by Eqs. (1) and (2). Many of the parameters in Eqs. (1) and (2) cannot be measured directly,

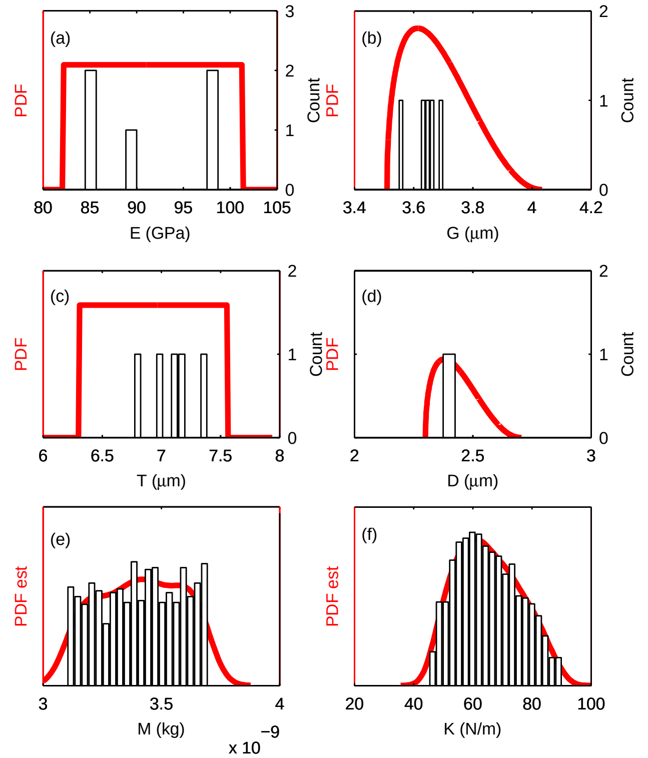

Figure 2. Models and available data for: (a) elastic modulus of plate, (b) electrostatic gap, (c) plate thickness, and (d) travel distance. One thousand sample histogram and PDF estimate of resulting correlated variables (e) effective mass and (f) effective stiffness.

so some effort was required to obtain them from the measured data. The following experimentally observable quantities have a significant effect on the switch model, and have been found to exhibit significant variability: modulus (E), gap (G), thickness (T) and travel distance (D). The data is shown in Figure 2 (a)-(d). (The modulus is actually not directly measurable, yet it can be deduced from the pull in voltage U_{pi} as will be explained.) The model calibration procedure consists of first estimating probabilistic models of E, G, T, and D and then relating these quantities to the parameters of Eqs. (1) and (2).

Expert opinion and past experience with the manufacturing process have found that G and D tend to be skewed to the right. A Beta distribution was fit to the available data for G and D because its parameters can be chosen so that it describes a slightly skewed, yet bounded distribution. Limited information was available for E and T, so these were taken to be uniformly distributed in the interval bounded by $\pm 25\%$ of their nominal values.

These random variables must now be related to the model parameters in Eqs. (1) and (2). The quasi-static voltage at which the switch closes, dubbed the "pull-in voltage" U_{pi} can be related to the model parameters as follows. The system in Eq. (1) exhibits a snapping phenomenon, in which the effective stiffness of the switch becomes negative for sufficiently large voltage U. The snap through point, found by solving for the position of the switch at which the stiffness changes sign is $X = G/3$. Substituting this position for X in Eq. (1) and neglecting inertia since the pull in tests are performed quasi-statically, one obtains teh following expression for U_{pi}.

$$U_{pi} = \sqrt{\frac{8}{27}\frac{K\,G^3}{\varepsilon a}}. \tag{3}$$

A three dimensional static finite element analysis was used to find the effective stiffness of the switch for various values of the switch thicknesses. The following polynomial relationship between the effective stiffness of the switch and its thickness and modulus was then fit to the FEA results for thickness ranging from 5μm to 9μm.

$$K(E,T) = a_2\,E\,T\,(T - a_1) \tag{4}$$

where E is the Young's modulus, T is the switch thickness, and $a_1 = 3.1458$ and $a_2 = 0.027186$ are the coefficients of the polynomial fit. Equation (4) can be substituted into (3) to solve for the modulus in terms of U_{pi}, T, and G. The four parameters defining the switch, D, G, T and E are assumed to be independent, while it is noted that the procedure for determining E could lead to artificial correlation between them if the measurements or static Finite Element model contain large errors.

The thickness and modulus of the switches both determine the effective stiffness of the single degree of freedom model through Eq. (4). The effective mass is found using the following procedure. A dynamic finite element model was used to determine the following relationship between the natural frequency of the switch plate bounce mode f_b and the plate thickness and modulus

$$f_b = \sqrt{E}\,b_1\,(T + b_0) \tag{5}$$

where the coefficients $b_0 = 0.43613$ and $b_1 = 311.79$ are valid over the same range as a_1 and a_2. Assuming that the static stiffness K is approximately equal to the dynamic stiffness of the system oscillating in its bounce mode only, the effective mass can be found using the familiar relationship $f = 2\pi\sqrt{K/M}$ yielding

$$M = \frac{K}{(2\pi f_b)^2}. \tag{6}$$

This procedure was verified by solving for both the stiffness and mass using the mode shape and natural frequency of the bounce mode and comparing them to the values obtained by the procedure described previously. The stiffness and mass values found using each method were found to agree to within a few percent.

The effective mass is actually a function of the thickness only, as can be seen by substituting the polynomials in Eqs. (4) and (5) into Eq. (6) resulting in

$$M = \frac{a_2\,T\,(T - a_1)}{(2\pi b_1\,(T + b_0))^2}. \tag{7}$$

Histograms of M and K, generated from 1000 samples of the independent random variables D, G, T and E, are shown in Fig. 2 (e) and (f) respectively. Note that, by Eq. (6) and (4), M and K are dependent random variables. Samples of D, G, K and M are used to perform Monte Carlo simulations of Eqs. (1) and (2). The coefficient of restitution is taken to be $\eta = 0.5$, which was found to adequately describe the rebound of an elastic 3D finite element model of the switch that included contact.

PERFORMANCE METRICS

The objective of waveform optimization is to maximize the life of the RF switches while maintaining acceptable time to closure. Previous investigations have found that switch life increases by a few orders of magnitude when the voltage waveform is designed to minimize the velocity with which a switch impacts the electrical contacts. The present study is concerned with minimizing the contact velocity for an ensemble of switches with random parameters. The maximum contact velocity V for a given switch is defined as the maximum velocity $\dot{X}$ at the instants t_k^- just before the switch rebounds from the contacts,

$$V = max\left|\dot{X}\left(t_k^-\right)\right|. \tag{8}$$

The contact velocity for the ensemble was minimized by minimizing the velocity v_u of the switch at the 10% upper quantile defined as

$$P(V > v_u) = 0.1. \tag{9}$$

The velocity v_u represents the worst case impact velocity for 90% of the ensemble and will be referred to as the upper velocity throughout this work. The upper velocity is estimated by as

$$v_u = \hat{F}^{-1}(0.9) \tag{10}$$

where $\hat{F}$ is an approximation for the cumulative distribution function of V [12]. An optimum waveform must also assure that the probability of a switch remaining unclosed p_{nc} for a large time is small This was estimated as the ratio of the number of switches that did not close within 250μs to the total number of switches in the MCS. The following objective function accounts for both of these considerations and was used in the following section to optimize the voltage waveform.

$$g = v_u + c_1 p_{nc} \tag{11}$$

The relative importance of contact velocity and failure to close is specified by the constant c_1. A value of $c_1 = 0.0025$ was used in this study for upper velocities in cm/s.

OPTIMIZATION PROCEDURE

A computational routine was created to solve the equation of motion, Eq. (1), subject to the constraint in Eq. (2) using an adaptive Runge-Kutta time integration routine (Matlab's ode45). The equation of motion was solved for 200 independent realizations of the random variables D, G, K and M sampled from the distributions described previously. The upper velocity v_u and the probability of a switch not closing P_{nc} were estimated for each Monte Carlo simulation, yielding a single value for the objective function g for each Monte Carlo simulation via Eq. (11). The Monte Carlo simulation was repeated for various voltage waveforms in order to arrive at an optimum voltage waveform.

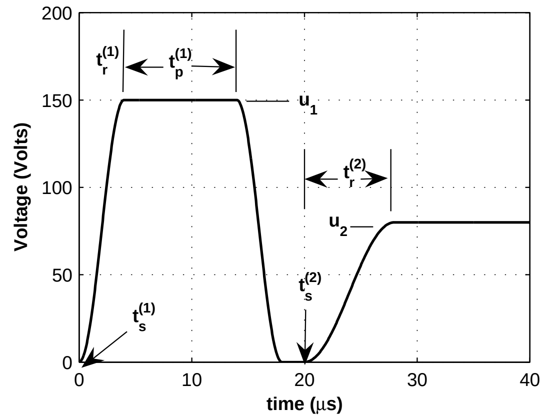

Figure 3. Sample actuation voltage waveform and parameter definitions.

The voltage waveform was parameterized by sets of four parameters per pulse in order to simplify the optimization procedure. These parameters are illustrated in Fig. 3 for a two-pulse waveform. Pulse i is parameterized by its start time $t_s^{(i)}$, rise time $t_r^{(i)}$, peak time $t_p^{(i)}$ and peak voltage u_i. The fall time of each pulse is identical to its rise time. This is a generalization of the pulse/coast waveform used in [8].

Initially, we restrict the analysis to waveforms with two pulses. The optimization procedure was simplified by first considering each pulse independently. The purpose of the first pulse is to bring the ensemble of switches near to the closed position with minimal velocity. Because of the nonlinear dependence of the force applied to the oscillator on $(G - X(t))^{-2}$, one would expect that longer forces will tend to increase the width of the distribution of the ensemble displacement and velocity. For this reason, the voltage of the first pulse was set at $u_1 = 150$ volts, which is near the maximum allowable voltage, so its width could be minimum while imparting the necessary amount of energy to the switch. The rise time for this pulse was set at $t_r^{(1)} = 4\mu$s, which was found previously to be the fastest rise time that could be used without exciting higher modes of the switch. The width of this pulse was increased from zero with the other parameters for this pulse fixed and with the second pulse nullified until a pulse width was found that resulted in a maximum contact velocity of about $10cm/s$ over the ensemble of switches.

Once the peak time of the first pulse had been determined, Monte Carlo analysis was performed for various values of $t_s^{(2)}$, $t_r^{(2)}$ and u_2, typically four values of $t_s^{(2)}$ and three values each of the other two. The set of parameters that minimized the cost function were then used as starting values for a Nelder-Mead simplex algorithm (Matlab's 'fminsearch') [13]. This algorithm varied the values for $t_p^{(1)}$, $t_s^{(2)}$, $t_r^{(2)}$ and u_2 until the objective function defined by Eq. (11) was minimized. This typically entailed 100-200 runs of the Monte Carlo simulation and improved the mean and upper contact velocities by about $1-2\,cm/s$ compared to the starting values.

The DIRECT algorithm in [14] was also applied to this problem in an effort to perform the optimization in a single step, yet it was abandoned when it failed to obtain the optimum input waveform after 350 evaluations of the Monte Carlo simulation. However, the DIRECT algorithm was helpful in finding optimum parameters for a three pulse waveform, because there were too many unknown parameters to use the simple optimization procedure described previously. Unfortunately, the three pulse waveform did not significantly reduce the contact velocity relative to the two-pulse waveform, and it was significantly more difficult to optimize its parameters, so it was abandoned.

RESULTS

The contact velocity for the ensemble of switches described in Figure 2 was minimized using the procedure described above. Figure 4 displays the optimum voltage waveform $U(t)$ and the displacement $X(t)$ and velocity $\dot{X}(t)$ response of the ensemble when it is applied. The optimum waveform is a single pulse followed by a slowly rising pulse of lower amplitude. The displacement of each switch is shifted such that zero displacement corresponds to the closed position. The switches start with shifted displacements equal to their travel distance D (between 2.3 and 2.7μm). Most of the switches close within 50μs, yet some take up to 140μs to close.

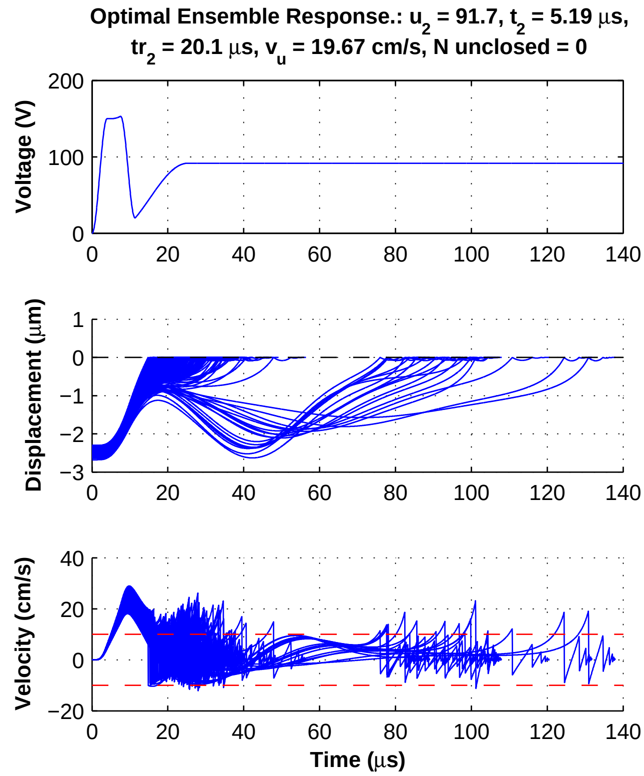

Figure 4. Optimum voltage waveform and ensemble displacement and velocity response to optimum waveform for current design.

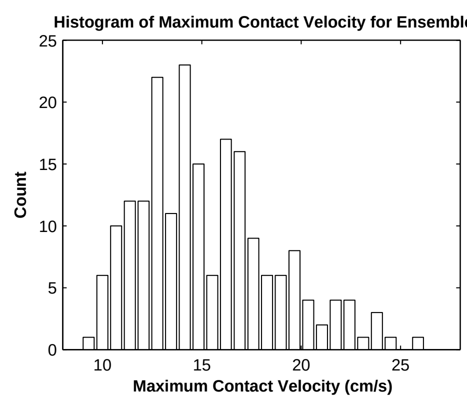

Figure 5. Histogram of maximum contact velocity for the ensemble of switches for current design.

Figure 5 shows a histogram of the maximum contact velocity V in Eq. (8) for the ensemble of 200 switches when the optimum voltage waveform in Figure 4 is used. Ninety percent of the switches close with a contact velocity less than $v_u = 19.7cm/s$ while the mean contact velocity for the ensemble $15.3cm/s$. It is interesting to note that the optimum waveform results in a nonzero contact velocity for all of the switches in the ensemble, suggesting that this waveform wouldn't have been found by designing the waveform to give zero velocity for any individual switch in the ensemble. By way of comparison, previous analysis with an unshaped waveform resulted in upper and mean contact velocities of $v_u = 40.7$ and $34.1cm/s$ respectively, which are twice as high as those obtained by the optimum waveform and unacceptable for RF MEMS applications of interest. The three pulse waveform found by the DIRECT algorithm gave upper and mean contact velocities of 18.8 and $14.0cm/s$, so the improvement obtained by adding a third pulse did not merit the additional complexity that it introduces.

One would expect that it might be possible to reduce the contact velocity by modifying the switch design or by improving the manufacturing process to reduce the variance of the switch's parameters. Either approach changes the distributions for the system parameters. A deterministic design change entails changing the nominal value of the distributions of the parameters defining the switch. In doing so, we assume that changing the design alters only the mean value of a given parameter, leaving the shape of the distribution and the coefficient of variation of the parameter unchanged. On the other hand, one may seek to reduce the coefficient of variation of a parameter by improving the manufacturing process. This may be more costly or difficult than a design change, but it may be necessary to meet demanding performance objectives.

Design Changes

The optimization methodology was also applied to study the effect of design changes on switch contact velocity. Deterministic analysis of Eq. (1) reveals that the system has an unstable equilibrium at $X = G/3$. With the current design, most of the switches close between $0.59 \leq D/G \leq 0.75$. The design was modified to reduce this ratio (resulting in $0.41 \leq D/G \leq 0.52$ for the new design) and a new optimum waveform was found for the modified design. The modified design is described by a new collection of random variables D', G', K' and M', whose mean values have been altered. The new random variables are obtained from the old such that the coefficient of variation of each remains unchanged, i.e.

$$Y' = \left(1 + \frac{\Delta\mu}{\mu}\right) Y, \tag{12}$$

where Y' is the distribution whose mean value μ has shifted by $\Delta\mu$. In order to preserve a feasible design, the values of D and G were increased and decreased respectively by only 0.5μm, and the mean value of the distribution that defines the thickness of the switches was increased by 0.6μm. Samples of D', G', T' and E' were used to generate correlated samples for K' and M'.

Figure 6 shows the response of an ensemble of switches with this modified design to its optimum waveform. Figure 7 shows a histogram of the maximum contact velocity. The upper and mean contact velocity have reduced to $v_u = 12.5cm/s$ and $10.7cm/s$ respectively, a reduction of more than 30%. Most of the switches close within 50μs.

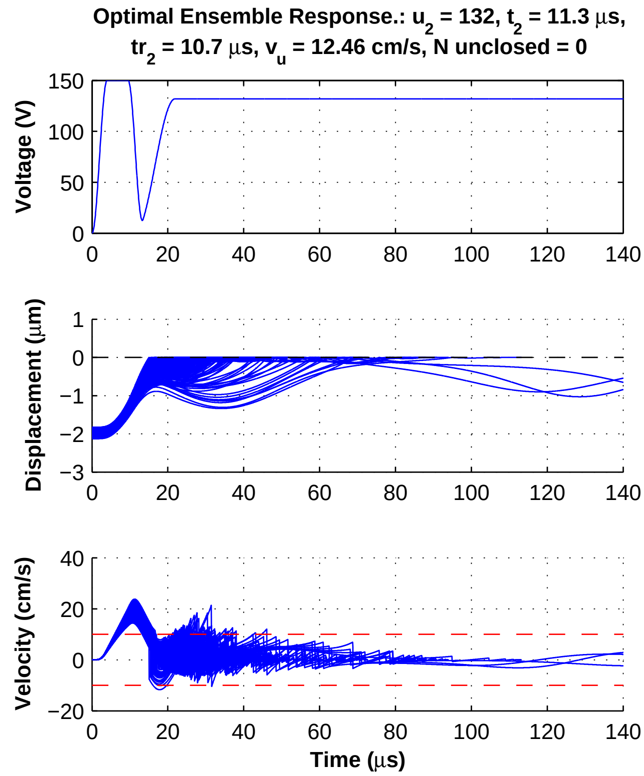

Figure 6. Optimum voltage waveform and ensemble displacement and velocity response to optimum waveform for modified design.

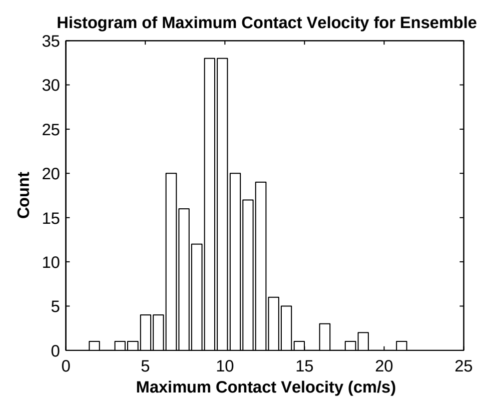

Figure 7. Histogram of maximum contact velocity for the ensemble of switches for modified design.

Process Improvement

The effect of manufacturing process repeatability on impact velocity was also investigated. This information was sought to assess the cost-versus-benefit realized by improving process repeatability. This was studied by changing the coefficient of variation (COV) of each random variable and then finding a new optimum close voltage for the improved process. Let Y be a random variable with mean $\mu \neq 0$, standard deviation $\sigma > 0$ and coefficient of variation (COV) σ/μ. The COV can be modified by $0 < \Delta\sigma < 1$ using the following change of variables:

$$Y'' = \mu(1 - \Delta\sigma) + \Delta\sigma Y \tag{13}$$

New distributions for the switch model parameters D'', G'', K'' and M'' were generated by assuming a 50% reduction in the COV of the switch thickness, gap distance and travel distance ($\Delta\sigma = 0.5$). The optimal waveform for this ensemble has upper and mean maximum contact velocities of $12.8cm/s$ and $9.6cm/s$ respectively, an improvement of 35% over the base design illustrated in Figures 4 and 5.

CONCLUSIONS

This work has demonstrated input waveform optimization under uncertainty for a highly nonlinear, electro-statically actuated radio-frequency MEMS switch. An uncertainty model was derived from experimental data and expert opinion for some of the important parameters of a reduced order model and used to drive a Monte Carlo simulation that predicted the maximum impact velocity experienced by an ensemble of switches subjected to a certain input waveform. The waveform was then optimized to minimize the contact velocity for the ensemble of switches, resulting in a 50% reduction in the contact velocity when compared to an unshaped waveform. Care was taken to assure that the majority of switches closed in a reasonable amount of time.

The procedure was then used to predict the reduction in contact velocity that could be obtained by modifying the design of the switch, and one design was presented that reduced the contact velocity by 30%. Other modifications of this class are currently being investigated considering all of the manufacturing and electrical performance constraints on the switches. Finally, the procedure was used to predict the effect of improving the process on the contact velocity, revealing that a 50% reduction in the coefficient of variation of the process resulted in a 35% in the contact velocity of the ensemble of switches. This information is valuable when performing cost-benefit analyses to justify future investments to improve the fabrication process and to allocate project resources between design and process improvement.

ACKNOWLEDGEMENTS

The authors gratefully acknowledge the technical contributions of Christopher W. Dyck and William D. Cowan in this work.

REFERENCES

[1] Dyck, C., Plut, T. A., Nordquist, C. D., Finnegan, P. S., Austin, F., and Reines, I., 2004. "Fabrication and characterization of ohmic contacting rf mems switches". In SPIE Conference on Micromachining and Microfabrication, Vol. 5344, pp. 79–88.

[2] Marchetti, B., Cannella, F., Caso, T., and Margesin, B., 2006. "Experimental numerical dynamic characterization of series rf mems". In 24th International Modal Analysis Conference (IMAC XXIV).

[3] Yao, J., 2000. "Rf mems from a device perspective". *J. Micromech. Microeng.,* **10**, pp. R9–R38.

[4] Maute, K., and Frangopol, D. M., 2003. "Reliability-based design of mems mechanisms by topology optimization". *Computers and Structures,* **81**, pp. 813–824.

[5] Han, J. S., and Kwak, B. M., 2004. "Robust optimization using a gradient index: Mems applications". *Struct Multidisc Optim,* **27**, pp. 469–478.

[6] Heo, S., Yoon, G. H., and Kim, Y. Y., 2004. "The robust design for micro electro-thermal actuators". In Smart Structures and Materials 2004: Smart Electronics, MEMS, BioMEMS, and Nanotechnology, V. K. Varadan, ed., Proceedings of SPIE Vol. 5389.

[7] Allen, M., Raullin, M., Maute, K., and Frangopol, D. M., 2004. "Reliability-based analysis and design optimization of electrostatically actuated mems". *Computers and Structures,* **82**, pp. 1007–1020.

[8] Czaplewski, D., Dyck, C., Sumali, H., Massad, J., Kuppers, J., Reines, I., Cowan, W., and Tigges, C., 2006. "A soft landing waveform for actuation of a single pole single throw ohmic rf mems switch". *Journal of Micromech. Microeng.*(accepted).

[9] Sumali, H., Kuppers, J., Czaplewski, D., Massad, J., and Dyck, C., 2005. "Structural dynamics of an rf mems switch". In 2005 ASME International Mechanical Engineering Congress and Exposition (IMECE).

[10] Massad, J., Sumali, H., Epp, D. S., and Dyck, C., 2005. "Modeling, simulation, and testing of the mechanical dynamics of an rf mems switch". In Proceedings of the 2005 International Conference on MEMS, NANO and Smart Systems (ICMENS05), pp. 237–240.

[11] Field, Jr., R., and Reese, S., 2005. "Probabilistic analysis and design of a MEMS deceleration switch". In Proceedings of the 2005 ASME International Mechanical Engineering Conference.

[12] Ang, A., and Tang, W., 1975. *Probability Concepts in Engineering Planning and Design: Vol. 1 - Basic Principles*. John Wiley and Sons, Inc., New York, NY.

[13] Lagarias, J. C., Reeds, J. A., Wright, M. H., and Wright, P. E., 1998. "Convergence properties of the nelder-mead simplex method in low dimensions". *SIAM J. OPTIM.,* **9**(1), pp. 112–147.

[14] Perttunen, C., Jones, D., and Stuckman., B., 1993. "Lipschitzian optimization without the lipschitz constant". *Journal of Optimization Theory and Application,* **79**(1), pp. 157–181.

NOTE: M. Allen in [7] is not the primary author of this work