Simple Experiments to Validate Modal Substructure Models

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ABSTRACT

While significant strides have been made in recent years, experimental/analytical substructuring methods can be quite sensitive to seemingly small measurement errors, to modal truncation (for modal methods), small residual terms (for frequency based methods), etc… As a result, one tends to have less confidence in a substructuring prediction than, for example, a finite element model, even though both may have similar accuracy in some situations. This work explores ways of estimating the uncertainty in modal substructure models, seeking to provide the experimentalist with an approach that could be used to evaluate the fidelity of a substructure model. This would allow one to detect cases where the substructuring problem is very sensitive to uncertainty, so a remedy can be sought, and perhaps even provide a measure of the expected scatter in the predictions. Simple experiments are proposed, for example obtaining the natural frequencies of the subcomponent after attaching a well characterized subcomponent at a point, in order to verify the subcomponent model and to estimate the sensitivity of the substructuring predictions to uncertainties. Special attention is paid to the adequacy of the modal basis of the substructure.

1. Introduction

Experimental-analytical substructuring is a procedure in which an experimental model for a subcomponent is obtained, e.g. using modal testing techniques, and then coupled with an analytical model (typically a finite element model) of another subsystem in order to predict the response of the assembled system. By replacing part of the system with a test based model one saves the difficulty and expense required to create a computational model; hence the method is most often desirable when the subcomponent of interest is poorly known (e.g. its material properties, interface stiffnesses, etc…) or difficult to model (intricate geometric features, etc…). On the other hand, although the tests required to create the experimental model are straightforward and have existed for over forty years, there are several issues that must be addressed in order to obtain an accurate subcomponent model. Specifically, one must assure that an adequate number of subcomponent modes are captured to describe the substructure adequately in the assembly of interest (modal truncation), rotational motions may need to be measured, which are generally difficult to obtain and susceptible to noise. As a result of these issues, seemingly insignificant experimental errors can cause dramatic errors in the substructuring predictions in some situations (see, e.g. [1, 2]). Some of these issues are summarized in recent review papers [3, 4]. Because of these and other factors, experimental-analytical substructuring seems to be quite under-utilized today, although there has been a resurgence of interest in recent years and recent studies have shown better success in several situations [5-9].

Because substructuring predictions can sometimes be extremely sensitive to experimental errors, one may be hesitant to rely on them and this inhibits more widespread use of the techniques. (One could, perhaps, argue that a finite element model of a complicated substructure is just as likely to be plagued with severe errors, but that is a discussion for another paper…) This paper takes initial steps towards addressing this lack of confidence by exploring whether some small number of additional measurements can be taken when creating the experimental substructure model in order to assess its accuracy. Specifically, after creating the subcomponent model, a second test is proposed where a known fixture is attached to the subcomponent and its natural frequencies are measured. Those results are then compared with predictions of the natural frequencies of the assembly, obtained using substructuring techniques in order to assess the quality of the subcomponent model. Mode shapes could also be measured but here we presume that those would not be available since they might increase the cost of the test. (If a large channel count system is used with fixed sensors then the mode shapes might be easily obtained and used in this validation as well; this will not be explored here.)

Substructuring predictions can be obtained using modal models for the subcomponents (Modal Substructuring) or the frequency response functions directly (Frequency Based Substructuring). This work focuses on the modal substructuring approach; some of the similarities and differences between these approaches are discussed in [3, 4]. The authors are not aware of any other works where model validation has been applied to substructure models, but the work by Kanda, Wei, Brown & Allemang [10] is closely related. In that work they attached rigid blocks with known properties to a structure in order to verify that the mode shape scale factors had been identified accurately. Several studies have compared substructuring predictions with analytical models and/or experiments on the actual assembly (see, e.g. [1, 8, 11, 12]), but here we treat a more realistic case where the modes of the assembly of interest are not known so one must infer whether the subcomponent model is adequate using some other test.

The following section describes the proposed technique using a simple beam system as an example. Simulated results for this system are then presented in Section 3 followed by the conclusions in Section 4.

2. Theory

Suppose that an experimental model of a certain system is desired. The system of interest might be as complicated as an automotive engine, a rocket payload with exotic, difficult to characterize materials, or it might be as simple as a truss or a basic electronic component. In any event, experiments would be performed to find the modes of the subcomponent and those would constitute a modal model for the substructure. One of two basic approaches are typically used, both of which are illustrated in Figure 1. In the free-free modes approach, the structure of interest (the cyan colored beam in the figure) is suspended from negligibly soft supports [13] and a modal test is performed to extract its modes. This method is known to produce quite inaccurate substructure models since all of the modes in the substructure's modal basis have zero shear and moment at the connection point (the left end of the beam in this example). This can often be remedied by including residual effects of out of band modes [14-16], although additional tests are often required to obtain the residual flexibilities and even then they tend to be weakly represented in the measurements and difficult to accurately extract.

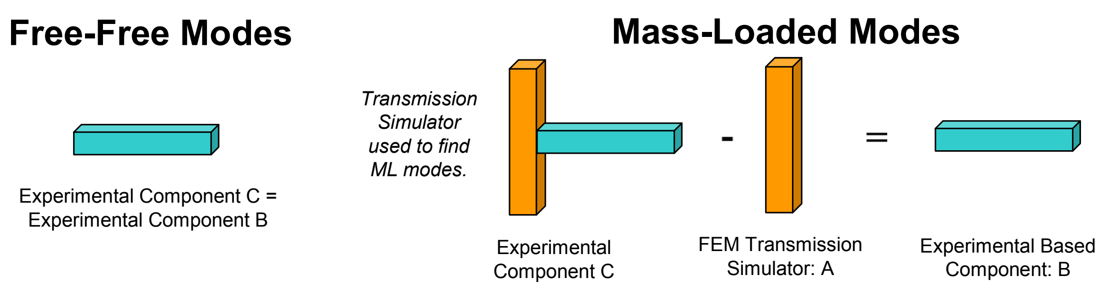

Figure 1: Two common methods for obtaining a modal model of a subcomponent of interest (the cyan colored beam is the substructure of interest in this example).

An attractive alternative is the mass-loaded interface approach, where a well characterized fixture or a transmission simulator (TS) is attached to the subcomponent of interest and the free-free modes of the assembly are measured. The effects of the transmission simulator are then removed from the substructure model in order to estimate the desired modal model for the system of interest. Traditionally the fixture was designed to be rigid in the frequency band of interest so it could be treated as a point mass [10, 17], but the authors recently showed that flexible fixtures could be readily accommodated so long as the proper constraints were applied between the fixture and its analytical model when removing its effects [6]. They introduced the name "transmission simulator" in [18] a few years after their initial work [6]. This method produces a model whose modal basis is enriched with mode shapes that involve nonzero shear and moment at the interface, so it tends to serve as an accurate, efficient basis for the substructure. This approach can also simplify substructure testing when the components of interest are connected at many points. On the other hand, one disadvantage of this approach is that one must subtract one subcomponent from another to estimate the substructure model, and this may introduce negative mass (or stiffness) causing the results to be nonphysical. The authors recent works present some very promising remedies to these difficulties [19, 20].

2.1 Substructure Validation Experiments

Once a substructure model has been obtained, this work proposes to validate it by performing an additional experiment. A well characterized fixture or structure is attached to the system and its natural frequencies are measured. The natural frequencies of the assembly are then predicted using modal substructuring and the frequencies are compared with the measured frequencies. (The details of the substructuring method are reviewed in [6], or also in Chapter 9 of the text by Ginsberg [21].)

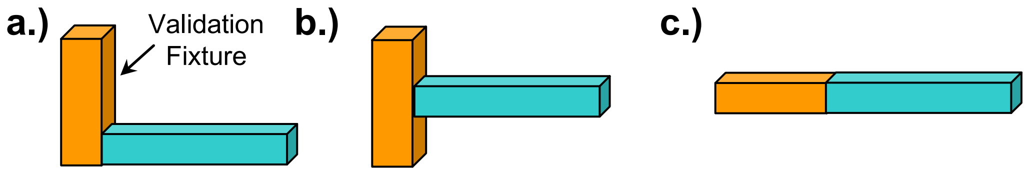

Figure 2: Three potential validation experiments for the beam substructure shown in Figure 1. A fixture with known properties is attached (orange colored beam in the figure) and the natural frequencies of the assembly are measured and compared to predictions.

The implicit assumption is that the error in the natural frequencies found when connecting the subcomponent to a fixture in the validation experiment will be representative of the errors that will be obtained when the substructure model is assembled in the system of interest. This issue should be considered very carefully, as it will determine whether the validation experiment is useful. If the validation experiment is well designed, then good correlation between the test results and the natural frequencies predicted with the substructure model will provide a strong indication of how accurate the substructure model.

Eventually, one might be able to rigorously quantify the accuracy of the substructure model using an approach such as this, but that would probably require more tests than are likely to be practical. On the other hand, if one is willing to make a strong assumption, for example that the errors in the natural frequencies are normally distributed and that each natural frequency has the same standard deviation, then one could estimate bounds on the substructuring predictions. There is no evidence yet to support such an assumption so for the moment this method should be expected to give only a qualitative measure of the accuracy of the substructure model.

3. Results

The proposed validation methodology will be explored using the system pictured in Figure 1, where the substructure of interest is a 12-in long beam with 0.75-in height and 1.0-in length (similar to the system studied in [6]). The transmission simulator for the mass-loaded modes method is a 6.0-in long beam with the same cross section. (The transmission simulator used here differs significantly from that used in [6].) The frequency range of interest for this very stiff structure is 0 to 20 kHz. All of the components were modeled using one dimensional finite element models (three degrees of freedom per node) with 30 nodes for the system of interest and 21 nodes for the transmission simulator.

Two different models were then obtained, one using the Free-Free modes method, denoted FF, and the other using the mass-loaded interface method, denoted ML. The natural frequencies of these two models are shown in the table below. In practice the FF model would have been obtained by performing a modal test on the 12.0-in beam with soft supports simulating free-free boundary conditions. Here such a test was simulated by simply taking the FF model to be a modal model with all of the modes of the beam that were within the testable bandwidth (e.g. assuming that a perfect test was performed). Here the testable bandwidth was taken to be 0 to 20kHz, so ten modes were obtained, three of which are rigid body modes. As discussed previously, a FF model such as this without residual terms is known to be a poor model for a substructure. In the following the goal is to see whether the results of the validation experiment can be used to discriminate between this poor model and the more accurate ML model.

To obtain the ML model for the beam, a test on the T-beam system was simulated by coupling the transmission simulator to the beam (resulting in a 30*3+21*3-3=150 DOF system). Then all of the modes of the system below 20kHz were assumed to be measured perfectly, resulting in a fifteen mode model for the beam. The transmission simulator was then removed using the approach in [6] using only the displacements at each node in the FEA models for each beam (not the rotations) for a total of 42 measurement points. In this process, seven negative TS modes are added to the system and seven constraints applied, so the total number of DOF is still fifteen. Of these fifteen modes, twelve remain below 20 kHz and ten of those \correspond closely with the free modes of the beam. The other two natural frequencies, at 695 and 1037 Hz, are spurious modes that involve very little motion of the beam of interest. (The transmission simulator and its negative model dominate the motion of these modes; they would not make an important contribution to any FRF on the structure of interest.) Finally, the ML mode model has three additional modes above 20 kHz that involve other motions of the beam; these are the modes that allow the ML model to capture motions where the shear and moment at the connection point are nonzero.

Table 1: Natural frequencies (Hz) of FF and ML subcomponent models for the beam.

Mode	FF	ML
1	0	0
2	0	0
3	0	0
-	-	695
-	-	1037
4	1082	1083
5	2982	2993
6	5845	5892
7	8422	8424
8	9663	9809
9	14435	14831
10	16869	16892
11		25954
12		31480
13		2.9E+09

3.1 Validation Experiment: Simulating Perfect Measurements

Now a validation experiment is simulated as follows. A finite element model is created for a validation fixture that is a beam of the same cross section at the transmission simulator, 3.0-in long, modeled in FEA with 11 nodes. This validation fixture is attached to the full FEA of the beam at its end as shown in Figure 2a and all of the modes below 20kHz are extracted to use as validation data. (This assumes that one could extract all of the modes perfectly in a test, hence they are labeled “true” below.) Then the FF and ML substructure models are used with the FEM of the validation fixture to predict the modes the assembly. The natural frequencies obtained are shown below in Table 2. The results show that the ML model much more accurately predicts the first several modes of the system, having a maximum natural frequency error of 1.5% as compared to 10.4% for the FF model. The average error in each case is 0.9% for the ML model and 4.5% for the FF model. Hence, the validation experiment has revealed that the ML model is quite an accurate model for the subcomponent, while the FF model is less accurate although perhaps adequate for some purposes.

Table 2: Natural frequencies of FF and ML subcomponent models for the beam for Validation Case (a) in Figure 2.

Mode	True (Hz)	FF (Hz)	FF % Error	ML (Hz)	ML % Error
4	768.5	777.9	**1.2**	771.8	**0.4**
5	1699.1	1875.0	**10.4**	1721.1	**1.3**
6	3155.8	3452.8	**9.4**	3196.3	**1.3**
7	5710.8	5919.5	**3.7**	5767.1	**1.0**
8	7752.6	7785.4	**0.4**	7758.2	**0.1**
9	9167.7	9227.7	**0.7**	9240.6	**0.8**
10	12813.4	13310.6	**3.9**	12877.4	**0.5**
11	14534.8	15459.1	**6.4**	14757.8	**1.5**
12	17500.3	20110.2	14.9	17783.5	1.6
13	19691.8	30085.6	52.8	123620.1	527.8

This same procedure was repeated using the configuration shown in Figure 2b, where the validation fixture is attached at its center in much the same way that the transmission simulator was attached, although recall that the TS was 6.0 inches long while the validation fixture is only 3.0 inches long, so its properties are quite different. The results in this case are shown in Table 3. The maximum and mean errors for the FF substructure model are 12.4% and 7.2% for this validation experiment, while the maximum and mean errors for the ML model are 2.2% and 1.2% respectively, significantly lower than those for the FF model.

Table 3: Natural frequencies of FF and ML subcomponent models for the beam for Validation Case (b) in Figure 2.

Mode	True (Hz)	FF (Hz)	FF % Error	ML (Hz)	ML % Error
4	839.9	842.1	**0.3**	840.4	**0.1**
5	2261.5	2334.6	**3.2**	2271.1	**0.4**
6	3982.0	4476.0	**12.4**	4045.3	**1.6**
7	6157.0	6619.5	**7.5**	6246.8	**1.5**
8	7038.7	7689.8	**9.2**	7096.1	**0.8**
9	9805.2	10815.8	**10.3**	9996.7	**2.0**
10	12600.8	13277.4	**5.4**	12702.6	**0.8**
11	14605.8	15998.5	**9.5**	14921.6	**2.2**
12	17293.0	21921.1	26.8	17460.1	1.0
13	20203.7	33475.3	65.7	49402.7	144.5

A third validation experiment similar to the case in Figure 2a was performed, only with a 6.0 inch long validation fixture (twice as long as the fixture used in the results presented in Table 2). Hence, the validation fixture is the same as the transmission simulator except that it is connected at its end rather than at its mid-point. In this case it was observed that the errors in the predictions of the FF substructure model were about 20% smaller than those reported in Table 2, while those of the ML model remained about the same. There were also quite a few additional modes in the testable bandwidth (0 to 20kHz). This suggests, and it seems reasonable that, as the validation fixture becomes larger, that the modes obtained may become more dependent on the (known) properties of the validation fixture and less dependent on the substructure model.

A few other cases were also explored, but they will not be presented in detail since the results presented above are representative of all of the cases studied.

4. Conclusions

This work has proposed a methodology for validating experimentally derived models for substructures by performing an additional experiment with a known fixture attached. The experimentally measured natural frequencies in this configuration are then compared with the predicted natural frequencies to assess the quality of the substructure model. Some simple cases were studied where validation experiments were simulated for two substructure models, one based on free-free modes (and which was known to be fairly inaccurate) and another based on mass-loaded interface modes (which was thought to be superior). In all of the cases shown the validation experiment did suggest that the ML model was superior. The differences between the predicted natural frequencies (using the substructure model) and those obtained in the validation test are thought to be indicative of the quality of the substructure model. This will be explored in more detail in future works.

This work has sought to detect inadequacies in a substructure model caused by modal truncation (i.e. inadequate span of the modal basis of the subcomponent modes). Modal truncation is thought to be a significant contributor to the uncertainty in a modal substructure model, but there are other important sources of uncertainty as well. The same procedure might also be useful for quantifying the error in a substructure model due measurement uncertainty. Another potential source of error is the way in which the connection between two substructures (e.g. the bolted joint) is approximated. This interface is captured experimentally using the transmission simulator method, so this type of uncertainty might also be addressed using this approach. Future works will explore these issues.

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