An Interpolation Algorithm to Speed up Nonlinear Modal Testing using Force Appropriation

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Abstract

Force appropriation testing has long been used for ground vibration testing of aircraft, where it is critical to estimate the modal parameters and especially damping accurately. Recently, extensions were presented that allow systematic identification of the nonlinear normal modes (NNMs) of conservative [1] and non-conservative [2] nonlinear structures. While this method provides accurate results with high confidence, it is unfortunately quite slow and so the structure may be subjected to significant damage over the course of a test. This work proposes a new approach in which the test is performed more quickly by simply acquiring measurements near the nonlinear resonance, but without the time consuming tuning required to reach the resonance precisely. Then, the recently proposed single nonlinear resonant mode method is used to interpolate between test points in order to estimate the NNM from each set of forced responses. The method is first evaluated numerically using a reduced model of a curved clamped-clamped beam that exhibits both softening and hardening response due to geometric nonlinearity. Then the method is employed experimentally to measure the first two NNMs of a curved beam that was manufactured from plastic using a 3D printer and the results are compared to the traditional tuning approach.

Keywords: Nonlinear Normal Modes, Force Appropriation Method, Single Nonlinear Resonant Mode Method, Nonlinear Modal Analysis, Nonlinear Frequency Response Functions

Single Nonlinear Resonant Mode Method

Frequency response functions of vibrating linear mechanical systems can be expressed as a combination of their linear normal modes. This is possible because the linear modes are orthogonal and can be used to diagonalize the mass and stiffness matrices. This principal of superposition, which is an essential property of linear theory, does not strictly apply to nonlinear systems.

The single nonlinear resonant mode (SNRM) method, which was first proposed in [3], assumes that if (*i*) most of the vibration energy is restricted to one nonlinear mode and (*ii*) there are no internal resonances between the modes it might be possible to predict the motion by simulating a single nonlinear mode and then superimposing the linear response of the remaining modes, see Eq. (1),

$$\mathbf{x}(t) = \mathrm{Re}\left\{ \frac{\tilde{\mathbf{\Phi}}_j \tilde{\mathbf{\Phi}}_j^T(-\mathbf{F}i)e^{i\Omega t}}{\tilde{\omega}_{0,j}^2 - \Omega^2 + 2i\tilde{\zeta}_j\tilde{\omega}_{0,j}\Omega} + \sum_{\substack{k=1 \\ k\neq j}}^{N_{\mathrm{lin}}} \frac{\mathbf{\Phi}_k \mathbf{\Phi}_k^T(-\mathbf{F}i)e^{i\Omega t}}{\omega_{0,k}^2 - \Omega^2 + 2i\zeta_k\omega_{0,k}\Omega} \right\}, \tag{1}$$

where: Ω is the forcing frequency; $\mathbf{\Phi}_i$, $\omega_{0,i}$, ζ_i are the mode shape, natural frequency and modal damping ratio of the i-th mode, respectively; $\mathbf{F}$ is the distribution of a sinusoidal force excitation; j is index of the dominant mode, and N_{lin} denotes the number of relevant linear modes. The quantities marked ($\sim$) vary with the vibration level. Indeed, the algorithm based on SNRM presented in [4] shows that the prediction of nonlinear frequency response function is possible when the NNM curve is known.

Nonlinear FRF Interpolation Algorithm Overview

In our work we represented curves in a frequency-velocity plot (v, Ω), where v is the magnitude of velocity complex amplitude at the point of the structure where the maximum displacement occurs and Ω is the aforementioned forcing frequency. Moreover, in the algorithm used here, we assume that the effective NNM frequency $\tilde{\omega}_j$ can vary from the true NNM frequency $\tilde{\omega}_{0,j}$, see Eqs. (2, 3). The method discussed here also simplifies the model by assuming that the dominating mode shape $\tilde{\mathbf{\Phi}}_j$ does not depend on vibration level. Thus, the proposed algorithm is based on the formulas shown in Eqs. (2) and (3), where the models for magnitude and phase of the complex amplitude of the velocity are presented.

$$\mathcal{F}(v,\ \Omega;\ \tilde{\omega}_j) = v - \left\| \left(\frac{\mathbf{\Phi}_j \mathbf{\Phi}_j^T \mathbf{F} \Omega e^{i\Omega t}\ e^{i\tilde{\varphi}_j^{\text{vel}}}}{\sqrt{(\tilde{\omega}_j^2 - \Omega^2)^2 + (2\tilde{\zeta}_j \tilde{\omega}_j \Omega)^2}} + \underbrace{\sum_{\substack{k=1 \\ k\neq j}}^{N_{lin}} \frac{\mathbf{\Phi}_k \mathbf{\Phi}_k^T \mathbf{F} \Omega e^{i\Omega t}\ e^{i\varphi_k^{\text{vel}}}}{\sqrt{(\omega_{0,k}^2 - \Omega^2)^2 + (2\zeta_k \omega_{0,k} \Omega)^2}}}_{\mathbf{B}_{\text{lin}}(\Omega)} \right) \right|_{\substack{\text{pt. of max.} \\ \text{deflection}}} \right\| = 0 \tag{2}$$

$$\mathcal{G}(\Omega;\ \tilde{\omega}_j) = \tan(\tilde{\varphi}_j^{\text{vel}}) - \frac{-2\tilde{\zeta}_j \tilde{\omega}_j \Omega}{\tilde{\omega}_j^2 - \Omega^2} = 0 \tag{3}$$

In the numerical experiments we conducted, the nonlinear mechanical system behavior was caused by the presence of nonlinear restoring force (which was a function of the displacement only) in the equation of motion. Values $\tilde{\zeta}_j$ were computed such that the damping matrix is constant, thus $\tilde{\zeta}_j = \zeta_j^{\text{lin}} \omega_{0,j} / \tilde{\omega}_j$.

The algorithm proposed here consists of three stages. In the first stage a $\tilde{\omega}_j$ value is assigned to each (v, Ω) point measured during the simulated experiment or laboratory tests. A value of $\tilde{\omega}_j$ was found for each pair of measured $(v,\ \Omega)$ by using a nonlinear curve fitting routine to minimize Eq. (2). Then, various pairs of $(v,\ \tilde{\omega}_j)$ are used to fit a polynomial to $\tilde{\omega}_j$ as a function of velocity, i.e. $\tilde{\omega}_j = c_0 + c_1 v + \ldots$. The objective of the third stage is to find a point $(v_*,\ \omega_*)$ at which FRF intersects the NNM curve. At this particular point Eqs. (4) and (5) hold (see also Fig. 1).

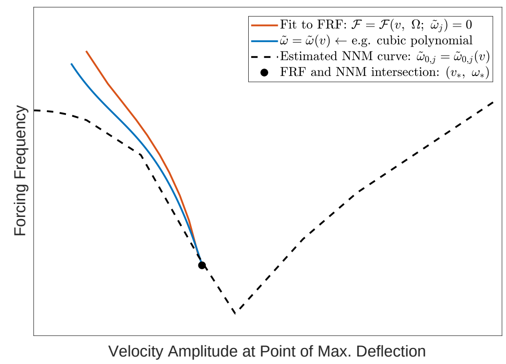


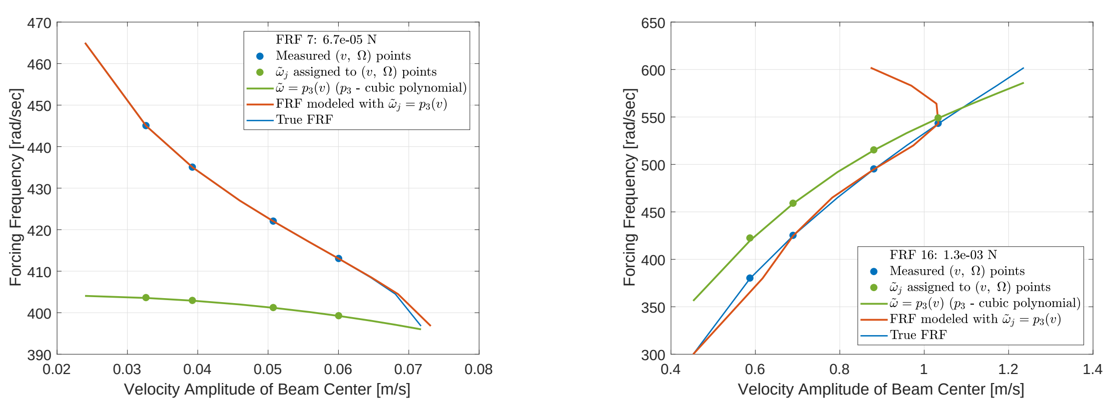
Fig. 1: Plot of interpolated frequency response function, effective NNM frequency $\tilde{\omega}_j(v)$ and estimated NNM curve $\tilde{\omega}_{0,j}(v)$. Those three curves would cross one another at point $(v_*,\ \omega_*)$ marked with a •

$$\mathcal{F}(v_*,\ \omega_*;\ \omega_*) = v_* - \left\| \left(\frac{\mathbf{\Phi}_j \mathbf{\Phi}_j^T \mathbf{F} e^{i\omega_* t}(-i)}{2\tilde{\zeta}_j \omega_*} + \mathbf{B}_{\text{lin}}(\omega_*) \right) \right|_{\substack{\text{pt. of max.} \\ \text{deflection}}} \right\| = 0 \tag{4}$$

$$\tilde{\varphi}_j^{\text{vel}} \longrightarrow -\frac{\pi}{2} \quad \Longrightarrow \quad \tilde{\omega}_j(v_*) = \tilde{\omega}_{0,j}(v_*) = \omega_* \tag{5}$$

Case Study

To test this method, data points $(v,\ \Omega)$ were generated by simulating force appropriation numerically. The test was performed on the 1 mode ROM of a curved beam with clamped-clamped boundary conditions excited with an uniformly distributed sinusoidal force. The beam was 304.8-mm-long, 12.7-mm-wide, 0.508-mm-thick



(a) **(b)**

Fig. 2: Results of the first two algorithm stages for force magnitudes of (a) 6.7e-5 N (the system exhibits softening at this force level) and (b) 1.3e-3 N (hardening)

and had a radius of curvature of 11.43 m. It was constructed of steel with a Young's modulus of 204 GPa, a mass density of 7860 kg/m^3 and Poisson's ratio of 0.29.

Figure 2 shows results obtained in the first two algorithm stages for two regions along the NNM. Using Eq. (2) as a model function with $\tilde{\omega}_j$ expressed as a cubic polynomial of v the frequency response functions could be recreated and validated against the true FRF curves measured by carefully tuning the forcing to obtain phase quadrature. As shown in Fig. 2 they were successfully reconstructed when the forcing magnitudes were low enough. The NNM of the system was then computed using Multi-Harmonic Balance Method (MHB) and is presented in Fig. 3, along with the NNM estimated using the proposed interpolation algorithm and the traditional Force Appropriation Method (FAM).

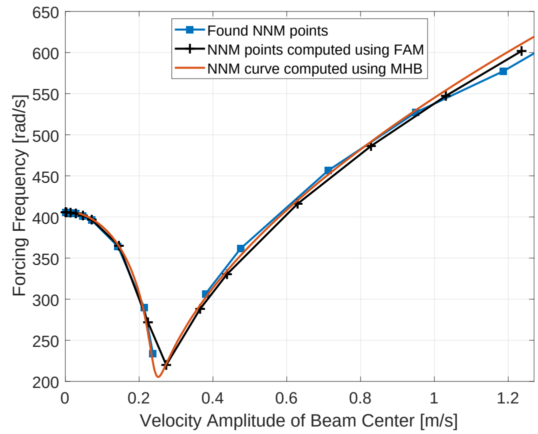


Fig. 3: Comparison of the NNM curve computed with the presented algorithm with solutions obtained using the Force Appropriation and Multi-Harmonic Balance Methods

It is important to note that the only parts of the estimated NNM curve which were successfully found by the algorithm were shown in the chart and that the solution in the vicinity of the turning point was very far from the test data. Moreover, it seems that the algorithm successfully finds the NNM points even if the vibration level is high. Simultaneously, it fails in recreating FRF parts which are close to the nonlinear resonance (see Fig. 2b). Further investigation is needed to determine the causes of this behavior.

Conclusion and Future Work

We proposed an algorithm based on the single nonlinear resonant mode method which interpolates between a few points measured on a nonlinear FRF in order to estimate the NNM curve. Such an algorithm could speed up measurements dramatically.

In the current version of the algorithm we proposed a slight modification to the original SNRM formulation, e.g. we allowed the effective nonlinear natural frequency to vary from the true NNM frequency. The algorithm succeeded in finding the NNM curve for a complicated system that exhibits softening and hardening over most of the range of vibration levels studied. However, it successfully reconstructed the FRFs only for the cases with low forcing magnitudes. Further investigation is needed to determine the causes of this behavior. The algorithm will also be validated against results from other tests, including lab experiments on 3D printed plastic curved beams.

Acknowledgments

This work was supported by the Air Force Office of Scientific Research, Award #FA9550-17-1-0009, under the Multi-Scale Structural Mechanics and Prognosis program managed by Dr. Jaimie Tiley. The authors would also like to thank Christopher Van Damme for his help during the early stages of this project.

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